<u>Cascade Control of DC machines</u>

Permanent magnet dc motor

Armature circuit:

$$v(t) = i(t)R + L\frac{di(t)}{dt} + E(t) \Rightarrow$$

$$v(t) = i(t)R + L\frac{di(t)}{dt} + K_E \phi \omega(t) \qquad (1)$$

Mechanical dynamics:

$$\sum T = J\frac{d\omega(t)}{dt} \Rightarrow$$

$$T_e - T_L - T_F = J\frac{d\omega(t)}{dt} \underset{T_F = B\omega}{\overset{T_L = 0}{\Rightarrow}}$$

$$T_e - B\omega = J\frac{d\omega(t)}{dt} \overset{T_e = K_T \phi\, i}{\Rightarrow}$$

$$K_T \phi\, i(t) - B\omega(t) = J\frac{d\omega(t)}{dt} \qquad (2)$$

Laplace transform of (1) and (2):

$$\begin{aligned} V(s) &= I(s)R + LsI(s) + K_E \phi \Omega(s) \\ K_T \phi\, I(s) - B\Omega(s) &= Js\Omega(s) \end{aligned} \qquad (3)$$

Solve (3a) for *I*(*s*):

$$V(s) - K_E \phi \Omega(s) = (R + Ls)I(s) \Rightarrow$$

$$I(s) = \frac{V(s) - K_E \phi \Omega(s)}{(R + Ls)}$$

and replace it in (3b):

$$K_T\phi \frac{V(s) - K_E\phi\Omega(s)}{(R + Ls)} - B\Omega(s) = Js\Omega(s) \Rightarrow$$

$$K_T\phi \frac{V(s)}{(R + Ls)} = K_T\phi \frac{K_E\phi\Omega(s)}{(R + Ls)} + B\Omega(s) + Js\Omega(s) \Rightarrow$$

$$K_T\phi \frac{V(s)}{(R + Ls)} = \left(K_T\phi \frac{K_E\phi}{(R + Ls)} + B + Js \right)\Omega(s) \Rightarrow$$

$$K_T\phi\, V(s) = (K_T\phi\, K_E\phi + (R + Ls)(B + Js))\Omega(s) \Rightarrow$$

$$\frac{\Omega(s)}{V(s)} = \frac{K_T\phi}{(R + Ls)(B + Js) + K_T\phi\, K_E\phi}$$

It is possible to simulate that using the following numerical values:

$K_T\phi = 3.65$ Nm/A

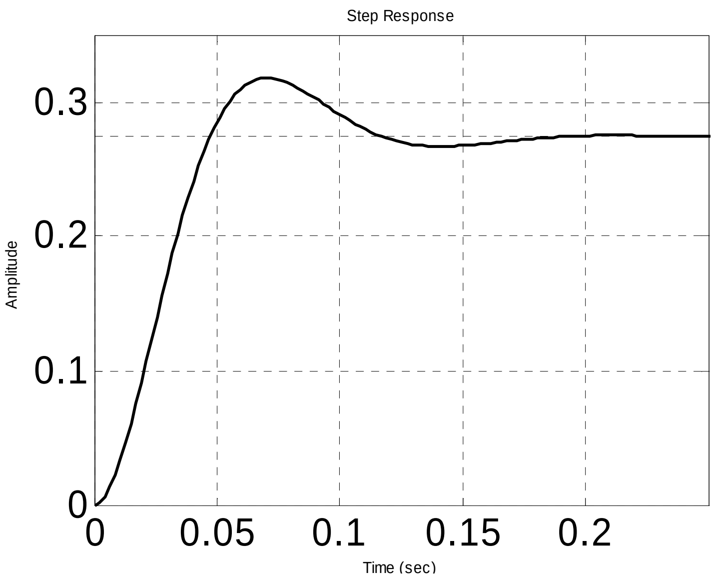
$K_E\phi = 3.65$ Vs / rad

$R = 0.052\ \Omega$

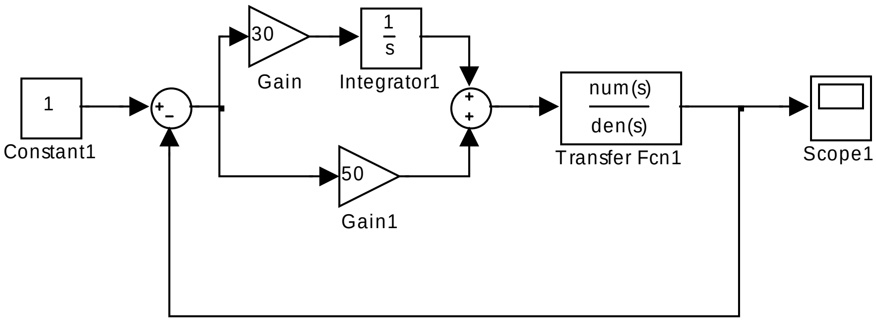
$L = 1$ mH

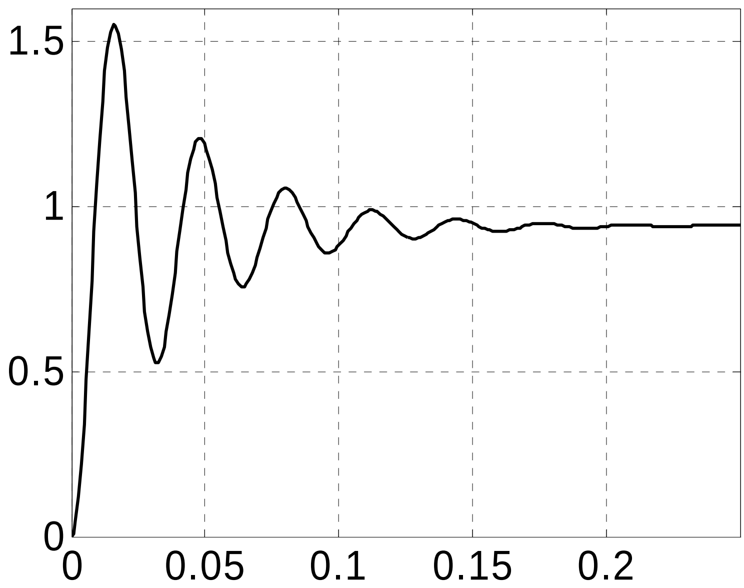
$J = 5$ Kgm^2

$B = 0.0001$ Nms/rad



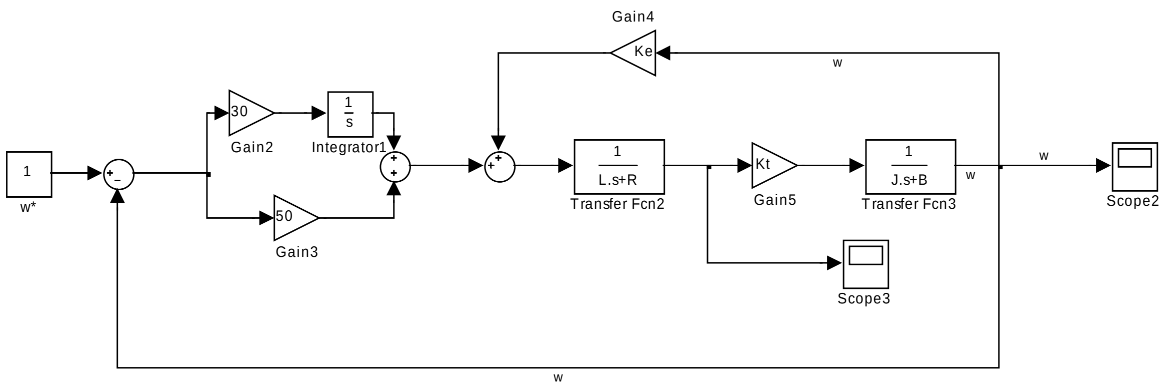
Also we can use a PI controller to improve the motor's performance:



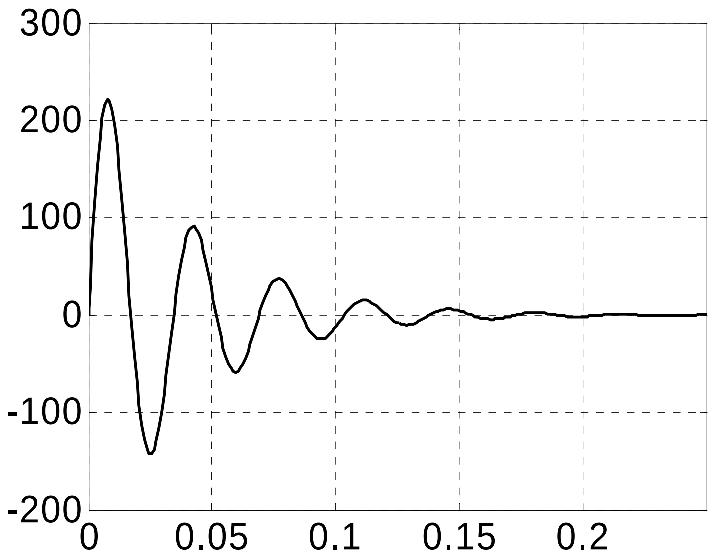


But this system has a peculiar property which can be seen if we slightly change the model:

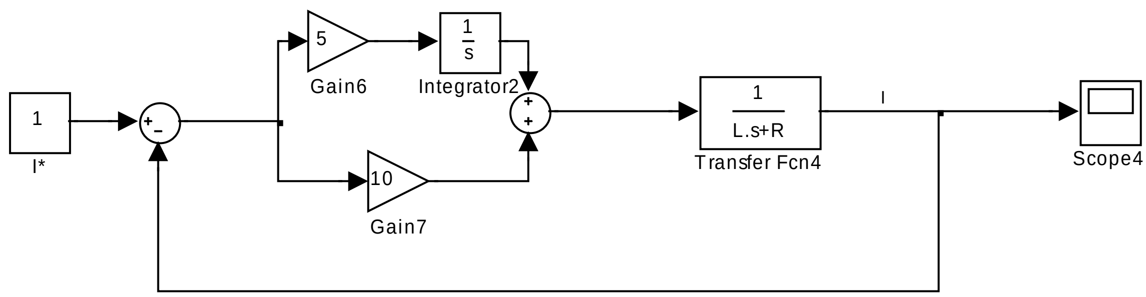
$$V(s) - K_E \phi \Omega(s) = I(s)(R + Ls)$$
$$K_T \phi\, I(s) = (B + Js)\Omega(s)$$

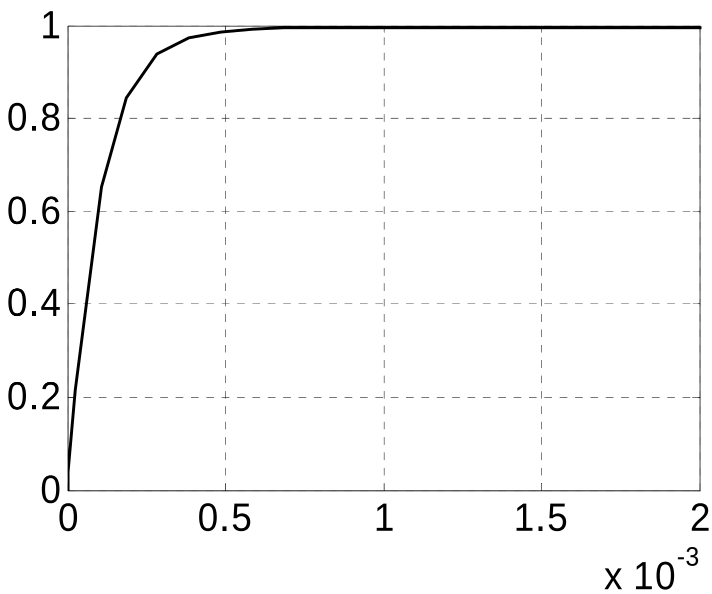


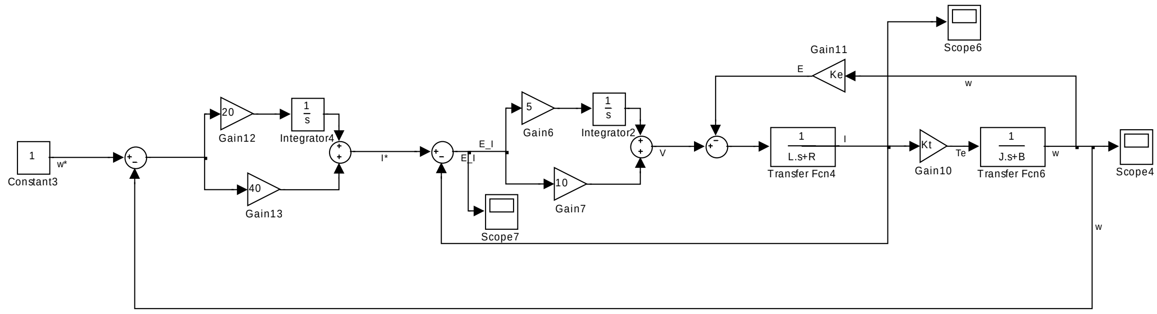
So let’s see the current:

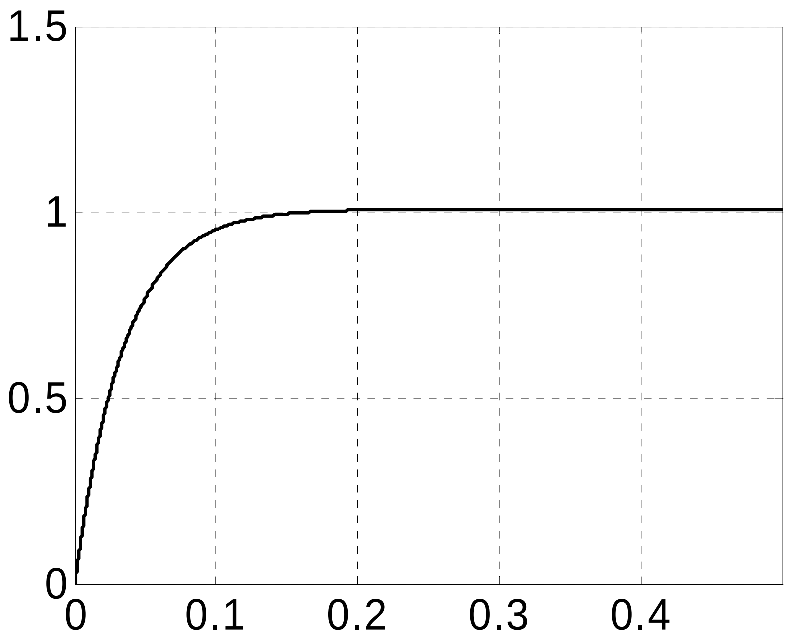


This is happening because the electrical time constant (L/R=0.0192) is much smaller (i.e. faster) than the mechanical one (i.e. J/B=50000).
But we can use that difference to break the system into 2 smaller and separated subsystems, i.e. the electrical (A) and the mechanical (B) part and hence to use 2 PI controllers:

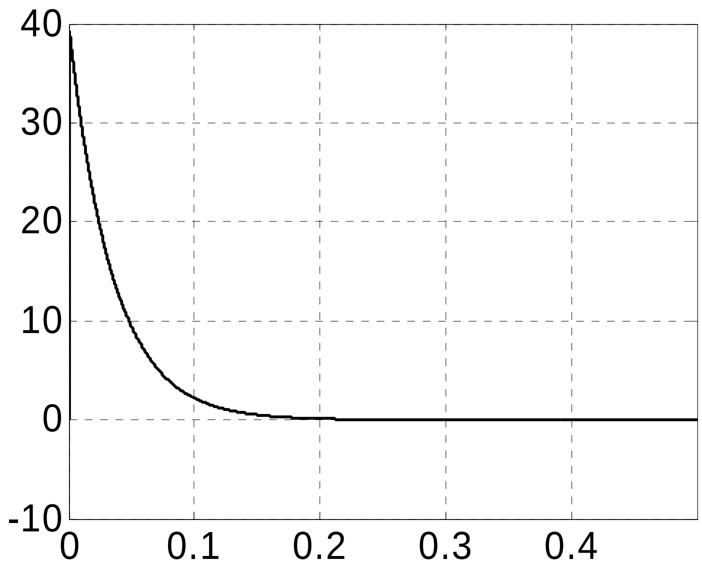








Current:



This type of control is called cascade control and is very popular in electric drives as we first tune the current loop and then we separately tune the speed loop.