Automatic Control – EEE 2002 Tutorial Exercise II

1. Find a polynomial expression for:
   a. y'=-5

$$sY(s) = -5/s \Leftrightarrow Y(s) = \frac{-5}{s^2}$$

   b. y'+3y=1

$$sY(s) + 3Y(s) = 1/s \Leftrightarrow Y(s) = \frac{1}{s(s+3)}$$

   c. y'-0.1y=t

$$sY(s) - 0.1Y(s) = \frac{1}{s^2} \Leftrightarrow Y(s) = \frac{1}{s^2(s-0.1)}$$

   d. y''+y'+6y=cos(t)

$$s^2Y(s) + sY(s) + 6Y(s) = \frac{s}{s^2+1}$$

2. Find in time and s-domain the final value of the signals shown in question. Crosscheck your answers from Simulink and Matlab

We have to solve the ODEs before we find the steady state (if any):

For the first ODE: $y = 0 + e^0 \int_0^t e^0 u dt_1 = \int_0^t (-5) dt_1 = -5t$, the steady state is:

$$y_{ss} = \lim_{t \to +\infty} (-5t) = -\infty$$

$$Y_{ss} = \lim_{s \to \infty} s \frac{-5}{s^2} = \lim_{s \to \infty} \frac{-5}{s} = -\infty$$

For the second ODE:

$$y = e^{-3t}0 + e^{-3t}\int_0^t e^{3t_1}dt_1 = e^{-3t}\frac{1}{3}\left[e^{3t_1}\right]_0^t = e^{-3t}\frac{1}{3}\left(e^{3t}-1\right) = \frac{1}{3}\left(1-e^{-3t}\right)$$

$$y_{ss} = \lim_{t\to+\infty}\left(\frac{1}{3}\left(1-e^{-3t}\right)\right) = \frac{1}{3}(1-0) = \frac{1}{3}$$

$$Y_{ss} = \lim_{s\to 0} s\frac{1}{s(s+3)} = \lim_{s\to 0}\frac{1}{(s+3)} = \frac{1}{3}$$

For the third ODE:

$$y = e^{-3t}0 + e^{-3t}\int_0^t e^{3t_1}dt_1 = e^{-3t}\frac{1}{3}\left[e^{3t_1}\right]_0^t = e^{-3t}\frac{1}{3}\left(e^{3t}-1\right) = \frac{1}{3}\left(1-e^{-3t}\right)$$

$$y = 0 + e^{0.1t}\int_0^t e^{-0.1t_1}t\,dt_1 =$$

$$e^{0.1\,t}\left(100. - 100.\,e^{-0.1000000000\,t} - 10.\,e^{-0.1000000000\,t}\,t\right)$$

$$y_{ss} = \lim_{t\to\infty}\left(e^{0.1t}\left(100-100e^{-0.1t}-10e^{-0.1t}t\right)\right) = \infty$$

$$Y_{ss} = \lim_{s\to 0} s\frac{1}{s^2(s-0.1)} = \lim_{s\to 0}\frac{1}{s(s-0.1)} = \infty$$

There is no steady state for the last ODE.

3. By using Matlab find the TF of the following
   a. G_1 and G_2 are in parallel connection
   b. G_1 and G_2 are in series connection
   c. G_1 and G_2 are in series and this is in parallel with G_2 connection
   d. G_1 and G_2 are in parallel and this is in series with G_1 connection

   Where $G_1(s) = \frac{1}{s+2}$ and $G_2(s) = \frac{s+5}{s^2+3s+6}$

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>> g1=tf([1],[1 2]); g2=tf([1 5],[1 3 6]);
>> ga=g1+g2

Transfer function:
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   2 s^2 + 10 s + 16
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s^3 + 5 s^2 + 12 s + 12

>> gb=g1*g2

Transfer function:
         s + 5
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s^3 + 5 s^2 + 12 s + 12

>> gc=gb+g2

Transfer function:
    s^4 + 11 s^3 + 45 s^2 + 93 s + 90
------------------------------------------
s^5 + 8 s^4 + 33 s^3 + 78 s^2 + 108 s + 72

>> gd=gb*g1

Transfer function:
             s + 5
--------------------------------
s^4 + 7 s^3 + 22 s^2 + 36 s + 24
```

4. Find the order, zeros, poles and plot the results of

$G_1(s) = \frac{1}{s+1}$, $G_2(s) = \frac{2}{s+5}$, $G_3(s) = \frac{s+13}{s^2+s+1}$

$G_4(s) = \frac{s-6}{(s+6)(s+1)}$, $G_5(s) = \frac{s^2}{(s^2+1)(s-10)}$, $G_6(s) = \frac{s^2+1}{s}$

Which system is stable and why?

```
>> g1=tf([1],[1 2]); g2=tf([2],[1 5]);
g3=tf([1 13],[1 1 1]); g4=tf([1 -6],conv([1 6],[1 1]));
g5=tf([1 0 0],conv([1 0 1],[1 -10])); g6=tf([1 0 1],[1 0]);

>> [z1,p1]=pzmap(g1), [z2,p2]=pzmap(g2), [z3,p3]=pzmap(g3),
[z4,p4]=pzmap(g4), [z5,p5]=pzmap(g5), [z6,p6]=pzmap(g6)

z1 =

    -2


p1 =

   Empty matrix: 0-by-1


z2 =
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```
    -5


p2 =

   Empty matrix: 0-by-1


z3 =

  -0.5000 + 0.8660i
  -0.5000 - 0.8660i


p3 =

   -13


z4 =

    -6
    -1


p4 =

     6


z5 =

  10.0000
   0.0000 + 1.0000i
   0.0000 - 1.0000i


p5 =

     0
     0


z6 =

     0


p6 =

        0 + 1.0000i
        0 - 1.0000i

>> pzmap(g1), figure, pzmap(g2), figure, pzmap(g3), figure,
pzmap(g4), figure, pzmap(g5), figure, pzmap(g6)

....
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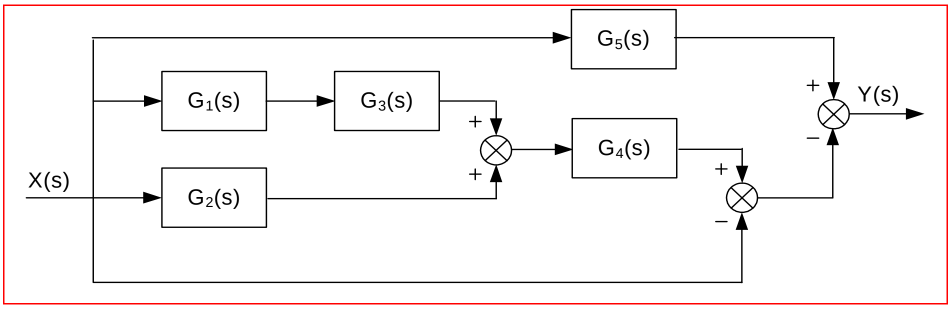
5. Simplify the following block diagram



Done during the lecture!