EEE2002 – Supplementary material

Transient Characteristics

A system has the following tf:

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 3s + 5}.$$

Note: The num is not w_n^2 but the w_n and z will be found from the den

1. Find the order of the system.
2. Find the characteristic equation of the system.
3. Find the poles of the system (Use the command roots).
4. What kind of response do you expect from that system just by looking at the pole location?
5. Find the damping factor and the natural frequency of the system by the previous pole location.
6. Find analytically its damping factor and natural frequency.
7. By using Matlab crosscheck your answer.
8. By using the appropriate formulas find the maximum overshoot (Mp), the settling time (2%) and the peak time of the system.
9. By using Matlab find the step response of the system and by using the appropriate tools (in the figure GUI) crosscheck your answer.

Solution:

1.

The order of the system is the order of the denominator: 2.

2.

The CE is the eqn that is created by the denominator of the TF, i.e. $s^2 + 3s + 5 = 0$

3.

The poles can be found by solving the CE:

```
>> roots([1 3 5])

ans =
```

```
  -1.5000 + 1.6583i
  -1.5000 - 1.6583i
Or
>> num=1; den=[1 3 5]; g=tf(num,den), [p,z]=pzmap(g)

Transfer function:
      1
-------------
s^2 + 3 s + 5


p =

  -1.5000 + 1.6583i
  -1.5000 - 1.6583i


z =

   Empty matrix: 0-by-1
```

4.

Since the system has two complex poles the system must be oscillatory.

5.

The angle of a complex pole is $\tan^{-1}$(imaginary part /real part), i.e.

```
>> theta=atan(abs(imag(p(1)))/abs(real(p(1)))), cos(theta)

theta =

    0.8355


ans =

    0.6708


>> zwn=abs(real(p(1))); wn=zwn/z

wn =

    2.2361
```

6.

The classical form of the CE is: $s^2 + 2\zeta\omega_n s + \omega_n{}^2 = 0$ and our CE is $s^2 + 3s + 5 = 0$. Hence by equating the relevant coefficients: $2\zeta\omega_n = 3, \quad \omega_n{}^2 = 5$ so $\omega_n{}^2 = 2.23...,\quad \zeta = {}^{3}\!/_{2\omega_n} = 0.67$

7.

```
>> [wn,z]=damp(g)

wn =

    2.2361
    2.2361


z =

    0.6708
    0.6708
```

8.

The overshoot is given by $Mp = e^{-\left(\zeta/\sqrt{1-\zeta^2}\right)\pi}$, hence:

```
 >> Mp=100*exp(-z*pi/sqrt(1-z^2))

Mp =

    5.8697
```

So theovershoot is 5.8%.

The settling time is $ts_{2\%} = \dfrac{4}{\zeta\omega_n}$, hence:

>> wn=wn(1); z=z(1); % Previously I used the command [wn,z]=damp(g)

```
>> ts=4/z/wn

ts =

    2.6699
```

So the system will enter into an enveolepe of +/-2% of the final value after 2.7s.

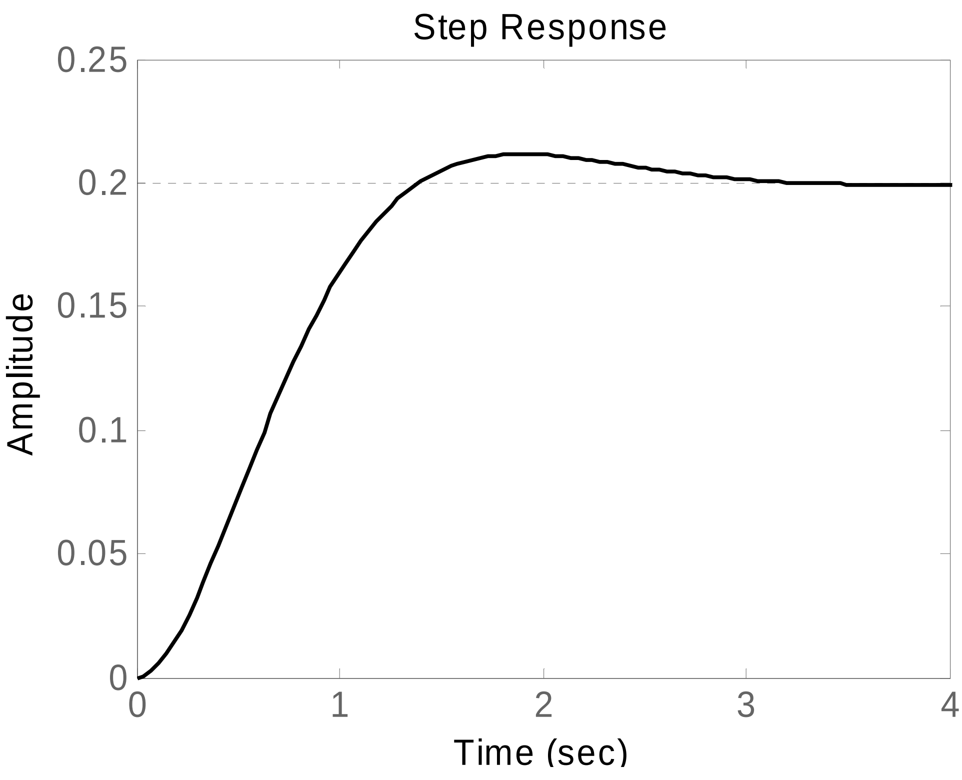
The peak time is $tp = \pi/\omega_d$, hence:

```
>> wd=wn*sqrt(1-z^2), tp=pi/wd

wd =

    1.6600


tp =

    1.8926
```

Hence the system will reach the first (and max) overshoot (of 5.8%) after 1.9 s.

9.

```
>> num=1; den=[1 3 5]; g=tf(num,den); step(g)
```



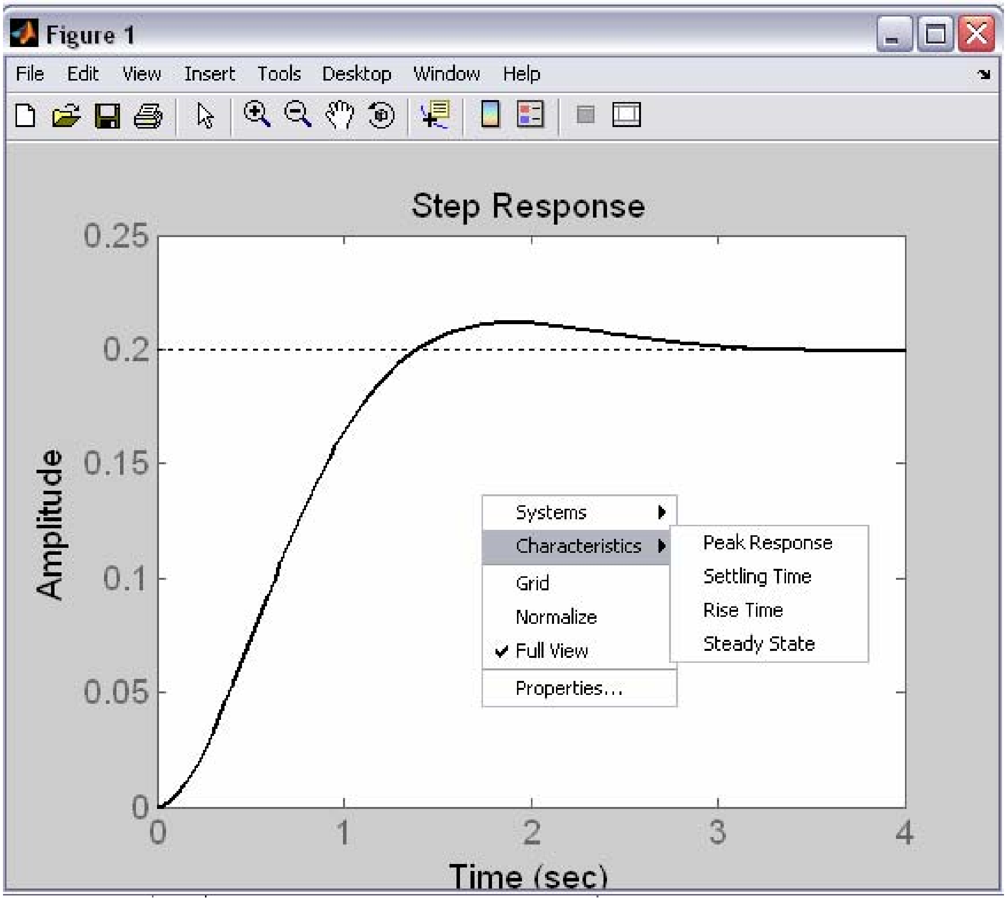
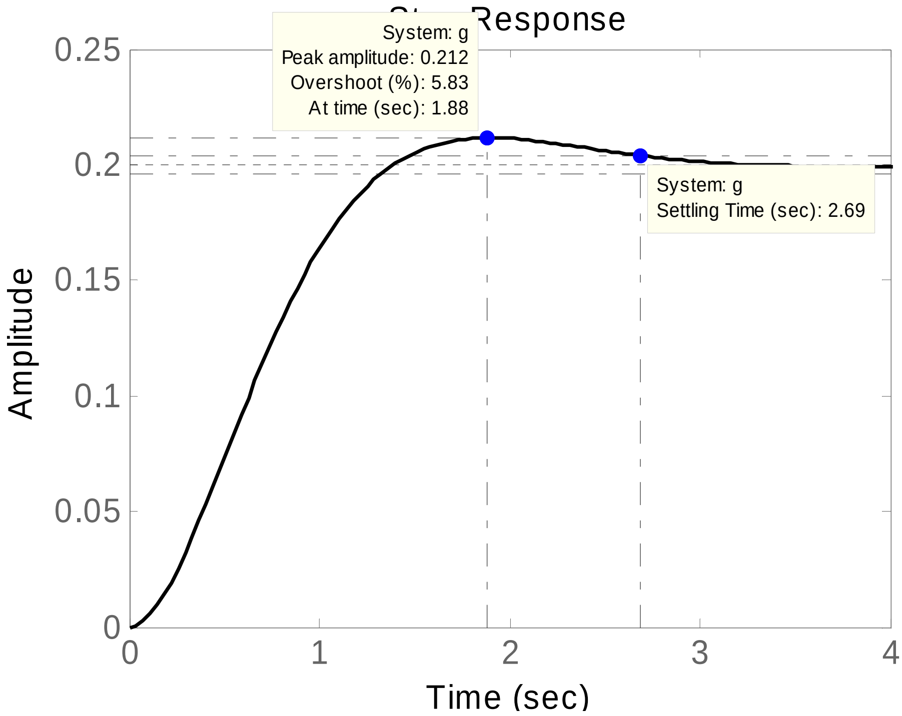


Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response
Amplitude
Time (sec)
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
Systems
Characteristics
Grid
Normalize
Full View
Properties...
Peak Response
Settling Time
Rise Time
Steady State


Response
System: g
Peak amplitude: 0.212
Overshoot (%): 5.83
At time (sec): 1.88
System: g
Settling Time (sec): 2.69
Amplitude
Time (sec)
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4