Control Systems– EEE 3001 8013 Tutorial Exercise Vb

Solutions

1. A system is described by $\mathbf{A}=\begin{bmatrix}-2 & 2\\ 2 & -5\end{bmatrix}, \mathbf{B}=\begin{bmatrix}1\\ 1\end{bmatrix}, \mathbf{C}=\begin{bmatrix}1 & 1\end{bmatrix}$.

    i. Find the system's response (**x**(0)=1, **x'**(0)=0) and comment on the output signal.

    The system is stable but I can only measure/see a linear combination of the 2 states.

    ii. Use an open loop estimator to estimate the 2 states when the initial error between the actual and the estimated states is 0.01. Plot the estimation error. Under what conditions can we use the estimated states in a pole placement control law? Simulate a case where we can use them and a case where we cannot use them.

    See Simulink file. The condition is that the error dynamics must be a lot faster than the system's dynamics so that the state feedback controller will use the correct information as soon as possible. This will minimise the error. To measure that I also calculate $\int |e| dt$

    The error dynamics are given by the eigenvalues of A:

```
>> eig(A)

ans =

    -6
    -1
```

Hence if the desired CL poles of the system are placed in -0.6 -0.1 the system will not converge as soon as I hoped.

```
A=[-2 2;2 -5]; B=[1;1]; C=[1 0];
eig(A)
K=place(A,B,[-10 -60])
K=place(A,B,[-0.5 -3])

ans =
```

```
          -6
          -1


      K =

        114.0000   -51.0000


      K =

          3.1667    -6.6667
```

2. Repeat the previous question when $\mathbf{A}=\begin{bmatrix}2 & 2\\ 2 & -5\end{bmatrix}, \mathbf{B}=\begin{bmatrix}1\\ 1\end{bmatrix}, \mathbf{C}=\begin{bmatrix}1 & 1\end{bmatrix}$.

In that case it is as before but now the system will be unstable even when we use the controller. Notice in the Simulink model that the estimated output will converge to zero while the real output diverges.

```
A=[2 2;2 -5]; B=[1;1]; C=[1 0];
eig(A)
K=place(A,B,[-50 -20])
K=place(A,B,[-4 -2])

ans =

   -5.5311
    2.5311

K =

  144.8571   -77.8571

K =
    3.1429    -0.1429
```

3. Use a closed loop estimator for $\mathbf{A}=\begin{bmatrix}2 & 2\\ 2 & -5\end{bmatrix}, \mathbf{B}=\begin{bmatrix}1\\ 1\end{bmatrix}, \mathbf{C}=\begin{bmatrix}1 & 1\end{bmatrix}$. Place the poles of the estimator to [-10 -11].

See Simulink file

```
>> G=place(A',C',[-10 -11])'
G =

   17.7143
    0.2857
```

4. Create 2 different pole placement control strategies and then use the above estimator:
    i. One with desired closed loop poles at [-5 -6]
    ii. One with desired closed loop poles at [-15 -16].

    See Simulink file. It is obvious that the first control law is more appropriate as the error is smaller even though we have a faster controller.

```
>> K=place(A,B,[-5 -6])

K =

    6.2857    1.7143

>> K=place(A,B,[-15 -16])

K =

   36.2857   -8.2857
```

5. Repeat question 3 when $\mathbf{A} = \begin{bmatrix} 2 & 0 \\ 2 & -5 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

The system is unobservable:

```
>> A=[2 0;2 -5]; B=[1;1]; C=[1 0];
>> rank(obsv(A,C))

ans =

     1
```