Chapter 3 Exercises

1. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}-1 & 1\\ 0 & -10\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the particular solution for 2 arbitrary initial conditions. Discuss your results.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

2. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}-1 & -1\\ 1 & -3\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the particular solution for 2 arbitrary initial conditions. Discuss your results.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

3. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}-1 & -1\\ 1 & -1\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the particular solution for 2 arbitrary initial conditions. Discuss your results.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

4. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}0 & -1 & 2\\ 2 & -3 & 2\\ 3 & -3 & 1\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

5. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}-4 & 5 & -3\\ -17/3 & 4/3 & 7/3\\ 23/3 & -25/3 & -4/3\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

6. The state matrix of a homogeneous system is given by $\mathbf{A}=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -5 & -2\end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the fundamental solution matrix using the previously calculated eigenvectors.

7. Prove that the matrix $G=\begin{bmatrix}1-t^2 & t-t^2\\ -2t & 1-2t\end{bmatrix}$ is the STM of

$$A=\begin{bmatrix}0 & 1\\ \dfrac{-2}{(t-1)^2} & \dfrac{2}{t-1}\end{bmatrix}$$

8. Check if the matrix $G = e^{-t}\begin{bmatrix} \cos(t^2) & \sin(t^2) \\ -\sin(t^2) & \cos(t^2) \end{bmatrix}$ is the STM of

$A = \begin{bmatrix} -1 & 2t \\ -2t & -1 \end{bmatrix}$.

9. Check if the matrix $G = e^{-t}\begin{bmatrix} \cos(t^2)\sin(t^2)\int_2^t e^{-38t\sqrt{\cos t}} & \sin(t^2) \\ -\sin(t^2) & \cos(t^2) \end{bmatrix}$ is the

STM of $A = \begin{bmatrix} -1 & 2t \\ -2t & -1 \end{bmatrix}$.

10. The state matrix of a homogeneous system is given by $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$.

- Find the eigenvalues and eigenvectors.
- Find the general solution.
- Find the particular solution for 2 arbitrary initial conditions. Discuss your results.
- Find the fundamental solution matrix using the previously calculated eigenvectors.
- Find the STM using the previously found eigenvectors
- Find the STM using exponential matrix (hint: you will need to use the formula for a geometric series).

11. Find the eigenvalues/eigenvectors of:

$$A_1 = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 2 \\ 0 & -5 \end{bmatrix},\ A_3 = \begin{bmatrix} -1 & 5 \\ -1 & -5 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix}, A_5 = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

12. Find the analytic solution for 5 homogeneous systems described by the 5 state matrices given in the previous question, using both methods (eigenvectors/state transition matrix).

13. Find the particular solutions of the above systems for $x(0) = [1 \quad 2]^T$.