Devision

1st order.

$\dot{x} + kx = u \rightarrow x(t) = e^{-kt} x_0 + e^{-kt} \int_0^t e^{kt_1} u(t_1) \, dt_1$

stability? $k > 0$ stable

Converge? if $u = const.$

$x(t) = \frac{u}{k}(1 - e^{-kt})$

$x_{ss} = \lim_{t \to +\infty} x(t) = \frac{u}{k}$

$u = 0 \quad \dot{x} + kx = 0$ $\quad$ $r + k = 0 \rightarrow$ C.E.

eigenvalue

Assume $x = e^{rt}$

$r = -k$

- $k > 0 \Rightarrow r < 0$ stable
- $k < 0 \Rightarrow r > 0$ unst.

S.S.

$x_{ss} = \ldots$

$\dot{x}_{ss} = 0$

$0 + k \cdot x_{ss} = u$

$x_{ss} = u/k$

$\ddot{x} + A\dot{x} + Bx = u.$

$$\left.\begin{array}{l} \ddot{x} + A\dot{x} + Bx = 0 \\ x = e^{rt} \end{array}\right\} \Rightarrow r^2 + Ar + B = 0$$

$r < 0$

$x \to 0$

$$\Delta = A^2 - 4.B$$

- $\Delta > 0$ $\begin{array}{l} \nearrow r_1 \in \mathbb{R} \\ \searrow r_2 \in \mathbb{R} \end{array} \Bigg\} \Rightarrow$

$x_1 = e^{r_1 t}$

$x_2 = e^{r_2 t}$

$$x = C_1 x_1 + C_2 \cdot x_2$$

$$W(x_1, x_2) = \begin{bmatrix} x_1 & x_2 \\ \dot{x}_1 & \dot{x}_2 \end{bmatrix}$$

if $|W| \neq 0$,

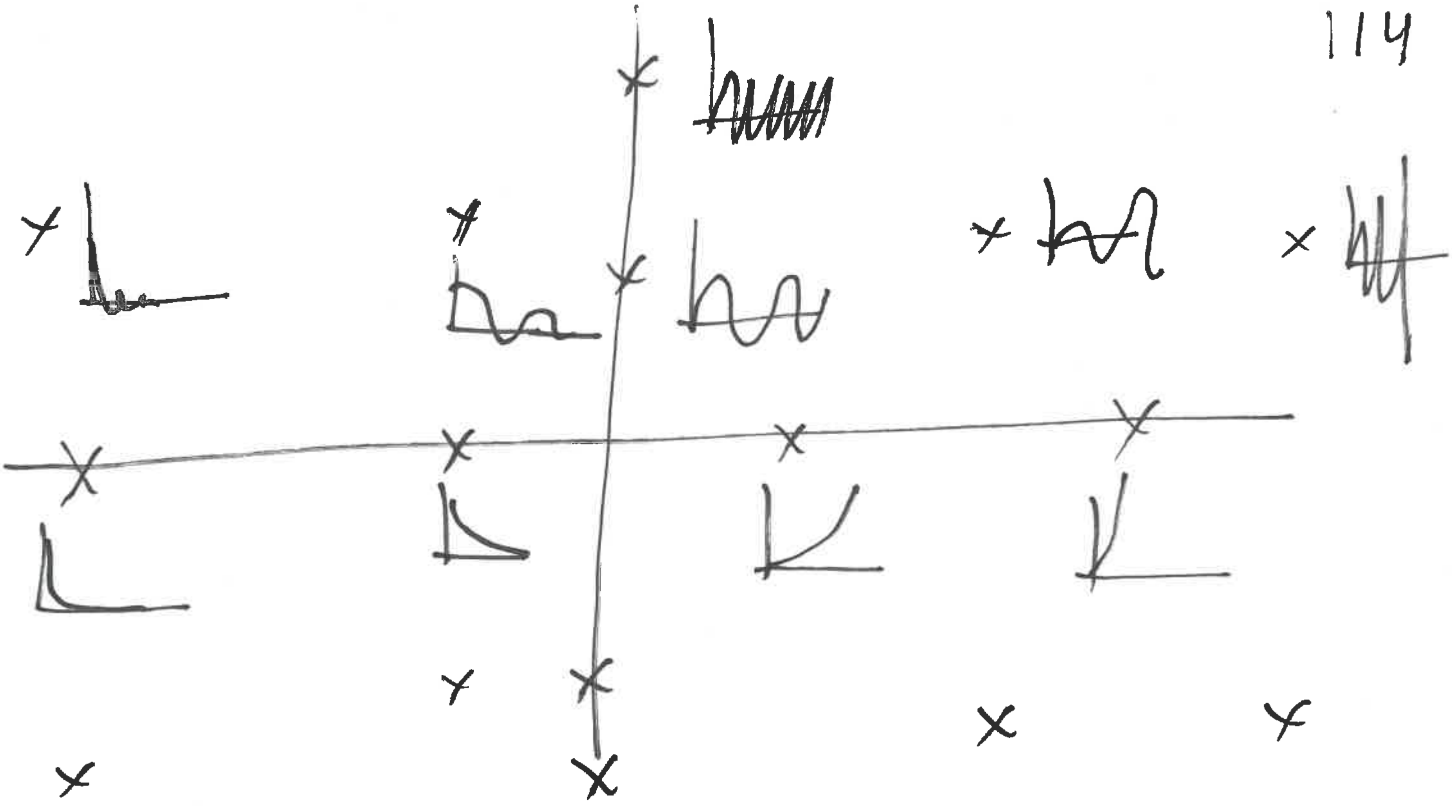
- $\Delta = 0$ $\quad r_1 = r_2 = r \in \mathbb{R}.$

$x_1 = e^{rt} \qquad x_2 = t\,e^{rt}$

- $\Delta < 0$ $\quad r_1 = a + bi \,, \; a, b \in \mathbb{R}.$

$r_2 = \bar{r}_1$

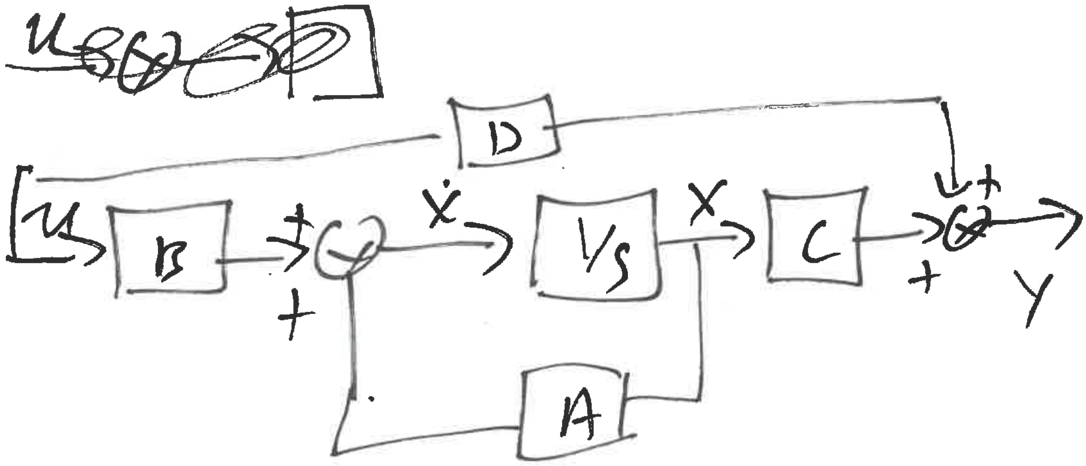
$x_1 = e^{r_1 t} \qquad x_2 = e^{\bar{r}_1 t}$



S.S.

$$\dot{x} = Ax + B\cdot u$$

$$Y = C\cdot x + D\cdot u$$



ODE → S.S.

$$\underbrace{x^{(n)}}\ + \ldots\ldots = u_1 + \ldots\ldots$$

$$x^{(n)} = -a_{n-1}x^{(n-1)} \ldots + u_1 + \ldots\ldots$$

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ &\vdots \\ x_n &= x^{(n-1)} \end{aligned}\right\} \overset{d/dt}{\Rightarrow} \quad \begin{aligned} \dot{x}_1 &= \dot{x} = x_2 \rightarrow \\ \dot{x}_k &= x_{k+1} \rightarrow \\ \dot{x}_n & \end{aligned}$$

T. F.

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= C \cdot x \end{aligned}\right\} \Rightarrow \quad G(s) = ?$$

$$= C \cdot (sI - A)^{-1} \cdot B$$

$$G_{i,j}(s) = \frac{\begin{vmatrix} sI - A & -B_i \\ C_j & D \end{vmatrix}}{|sI - A|}$$

B_i = i^{th} colm of B

C_j = j^{th} row of C

OBSV / CTRB

$Y = \dots \quad x = ? \quad M_o = \dots$

$u = \dots \quad x = ? \quad M_c = \dots$

CH3

$\dot{x} = Ax + Bu \qquad x = ?$

eigen value

$u = 0 \qquad \dot{x} = Ax \qquad x = e\, e^{\lambda t}$

eigen vector.

$(A - \lambda I)\cdot e = 0 \qquad (\lambda I - A)\cdot e = 0$

$|A - \lambda I| = 0 \Rightarrow \dots \; \lambda_1 = \dots \Rightarrow e_1 =$

$\lambda_2 = \dots \Rightarrow e_2 = \dots$

- $\lambda_1, \lambda_2 \in \mathbb{R}$. $\quad x_1 = e_1 e^{\lambda_1 t}$
  $x_2 = e_2\, e^{\lambda_2 t}$

- $\lambda_1 = \lambda_2 = \lambda \quad x_1 = e\, e^{\lambda t}$
  $x_2 = (et + b)\, e^{\lambda t}$

- $\lambda = a \pm bi \quad x_1 = e\, e^{\lambda t}$
  $x_2 = \bar{e}\; e^{\bar{\lambda} t}$

$$x = c_1 x_1 + c_2 x_2$$

$$= \underset{\substack{\downarrow \\ [x_1 \quad x_2] \\ \text{F.S.M.}}}{X} \cdot \underset{}{C} \rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

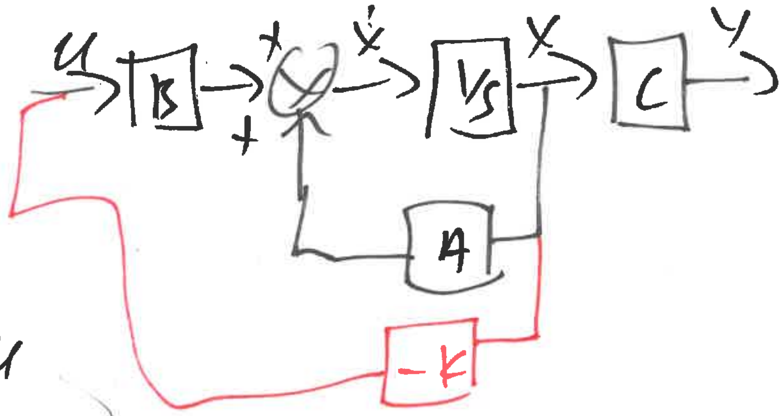
$$x(t) = \underbrace{X(t) \cdot X^{-1}(0)}_{\text{S.T.M.}} \cdot x_0$$

if $A = \text{const}$ L.T.I. $\quad e^{At} = I + At + \dots$

if $\lambda_1 \neq \lambda_2 \quad e^{At} = T e^{\Lambda t} \cdot T^{-1}$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

$$T = [e_1 \quad e_2]$$



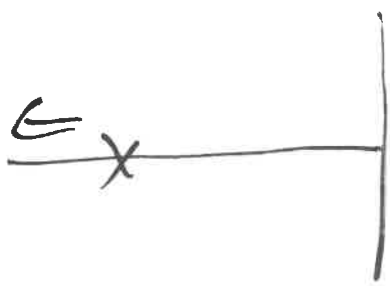
$$\left.\begin{array}{l} \dot{x} = Ax + Bu \\ y = C\cdot x \\ u = -K\cdot x \end{array}\right\} \Rightarrow \dot{x} = \underbrace{(A - BK)}_{A_{CL}}\cdot x$$

$K = ?$ $eigs(A_{CL}) = \_\_ \quad \_\_$

$\Leftarrow$

P.P. Des pole location $\rightarrow K$

CTRB



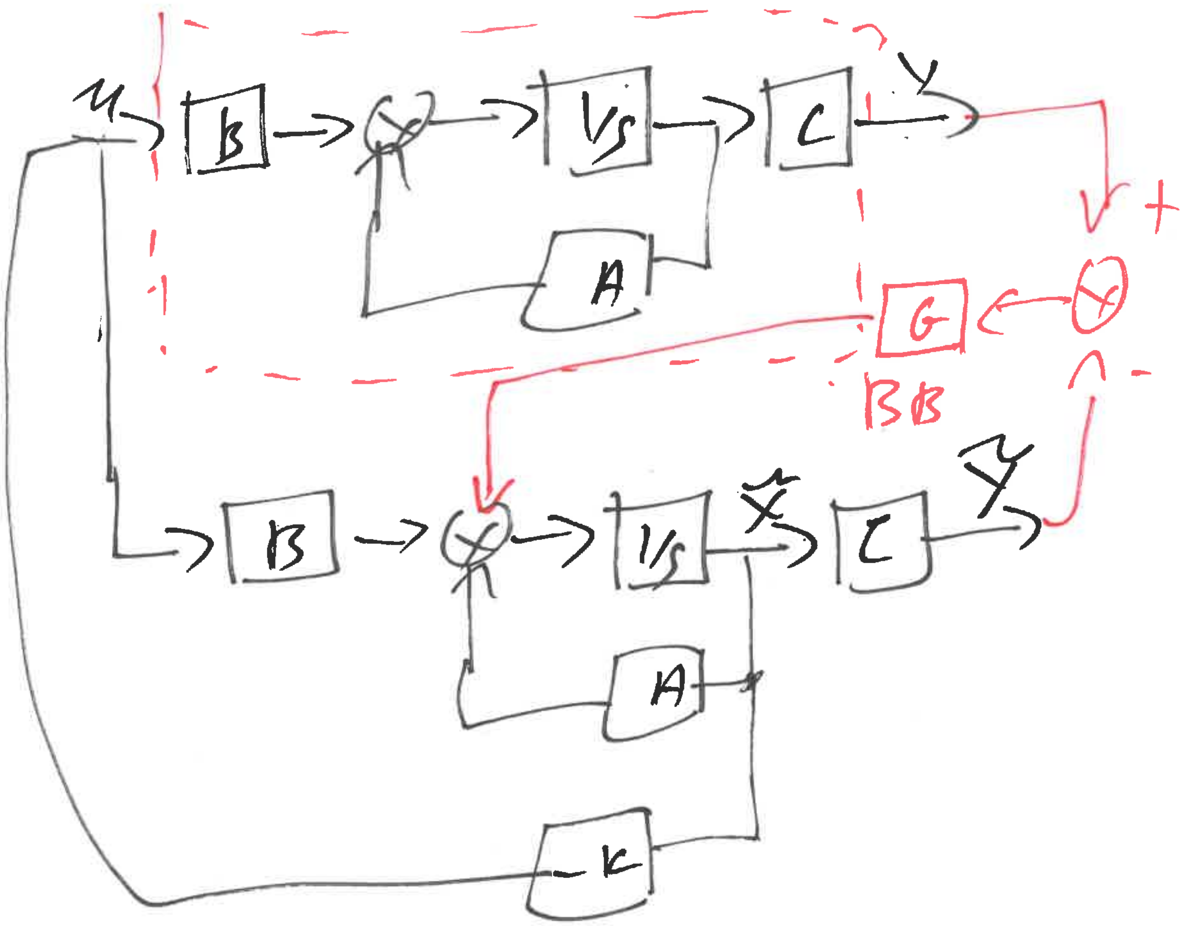
$|K| \uparrow \Rightarrow |u| \uparrow$

LQR

$$J = \int (x^T Q x + u^T R u)\cdot dt \Rightarrow P = \dots$$

$K = ?$ $P = \dots$ $K = R^{-1} B^T \cdot P \Rightarrow ?$

2x2 Lin	$Q_1 = 10\cdot I$	$R_1 = 1.$
	$Q_2 = I$	$R_2 = 10$



$$\dot{x} = Ax + Bu$$
$$y = C \cdot x$$

$$\dot{\tilde{x}} = A\tilde{x} + Bu$$
$$\tilde{y} = C \cdot \tilde{x}$$

$$u = -K \cdot \tilde{x}$$

$\tilde{x} \to x$ ASAP

$$u = -K \cdot x$$

$$\dot{e} = Ae$$

$$e = x - \tilde{x}$$

OBSV

$$\dot{e} = (A - G \cdot C)e$$

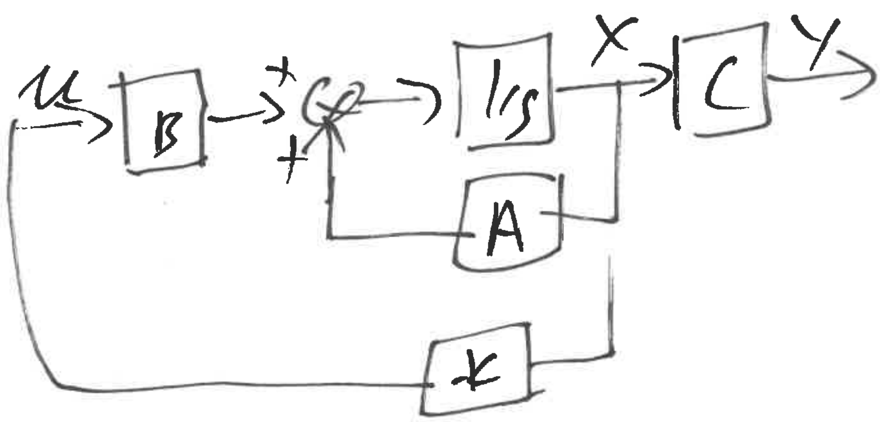
$$e \to 0$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-GC \end{bmatrix} \cdot \begin{bmatrix} x \\ e \end{bmatrix}$$

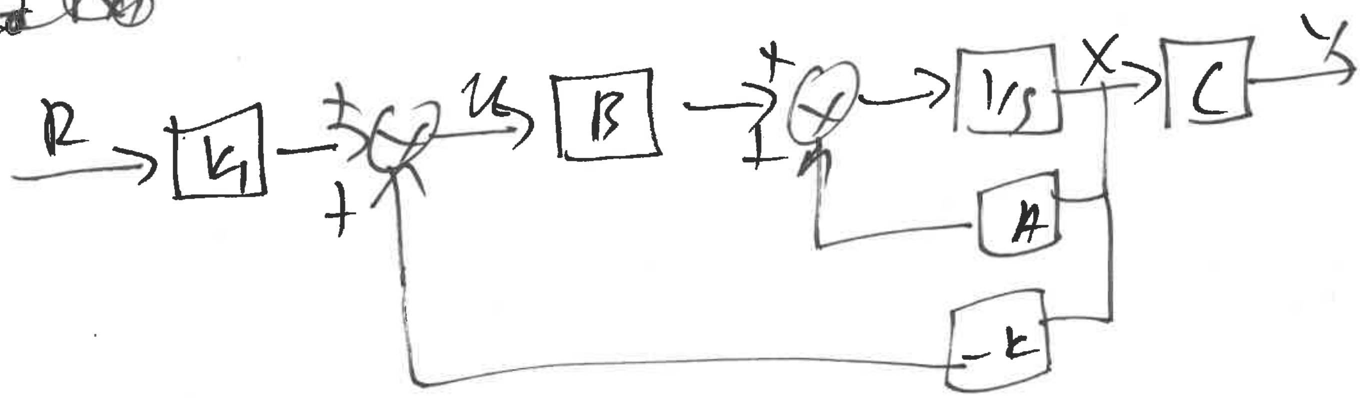
$\dot{x} = Ax + Bu$

$y = C \cdot x$

$\dot{x} = (A - BK) \cdot x$



$y \to y_{ss}$ 1x1 SISO



R = Des out

$K_1 = ?$: $y \to y_{ss} = r_{ss}$

K_1 = Given $x_{ss} = ?$ $u_{ss} = ?$

K_1 = Given $\quad u_{ss} = ?$ $\quad x_{ss} = ?$ 121

r_{ss}

Assume $u_{ss} = N_u \cdot r_{ss}$

$x_{ss} = N_x \cdot r_{ss}$ $\Big\} \Rightarrow$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

→ vector

$$u = R K_1 - \cancel{K} \cdot x = r_{ss} K_1 - K \cdot x$$

$\downarrow K_1 = ?$ $\quad \downarrow$

$$u = u_{ss} - K(x - x_{ss})$$

$$K_1 = N_u + K N_x$$