Revision

$\ddot{x} + A\cdot\dot{x} + B\cdot x = 0$

x_1, x_2 are solns

also $y = c_1\cdot x_1 + c_2\cdot x_2$

e.g. $c_1 = 3, \quad c_2 = 5$

$c_1 = j \quad , \quad c_2 = 5\cdot j$

$c_1 = 5+3j \quad , \quad c_2 = -3+5j$

If x_1, x_2 are L.I. $\quad x_1 \neq k\cdot x_2$

$\Downarrow$

ALL other solns

can be given $y = c_1\cdot x_1 + c_2\cdot x_2$

Find $x_1, x_2 = ?$

$$x = e^{rt} \longrightarrow \ddot{x} + A\dot{x} + B\cdot x = 0$$

$$\downarrow$$

C.E.: $r^2 + Ar + B = 0$

$$\downarrow$$

$r_1, \; r_2$ eigenvalues

$$r_{1,2} = \frac{-A \pm \sqrt{\Delta}}{2} \qquad \Delta = A^2 - 4B$$

• $\Delta > 0$ $\quad r_1, r_2 \in \mathbb{R}$ $\quad r_1 \neq r_2$

if r_1 and r_2 $< 0 \rightarrow$ stable

$$x_1 = e^{r_1 t} \qquad x_2 = e^{r_2 t}$$

$$|W(x_1, x_2)| = e^{r_1 t} \cdot r_2 \cdot e^{r_2 t} - e^{r_1 t} \cdot r_1 \cdot e^{r_2 t} \neq 0$$

$$y = c_1 \cdot x_1 + c_2 \cdot x_2$$

$\Delta < 0 \qquad r_{1,2} \in \mathbb{C}$

$$r_1 = a + bi$$
$$r_2 = a - bi$$

$$x_1 = e^{r_1 t}. \quad x_2 = e^{r_2 t}$$

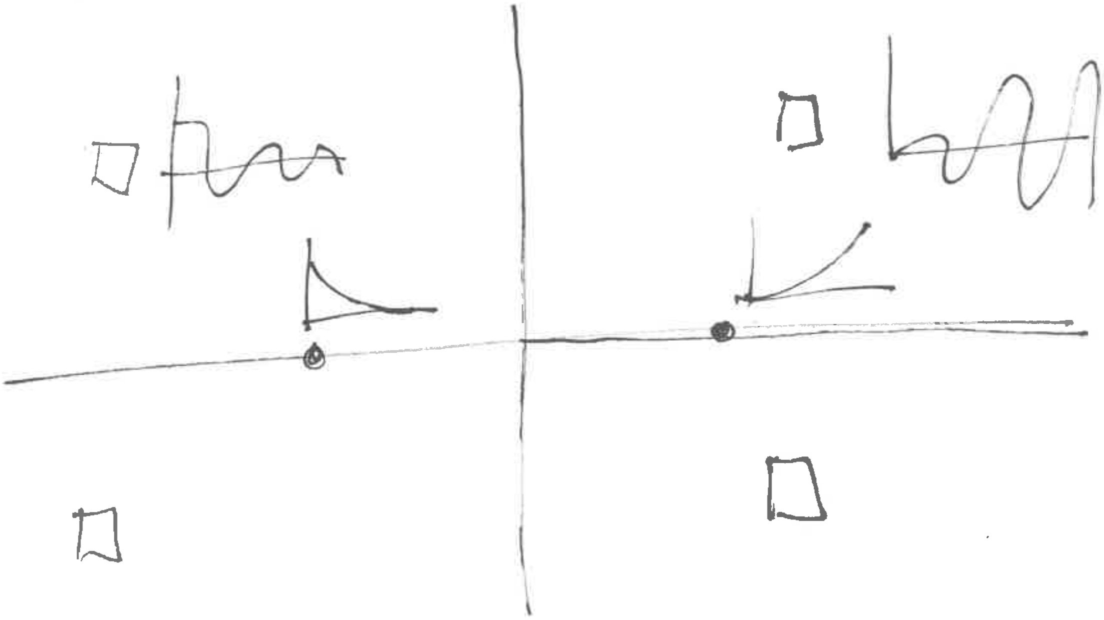
$$|W| = -2 \cdot e^{2at}\, b \cdot i \neq 0$$

$$y = c_1 \cdot e^{r_1 t} + c_2\, e^{r_2 t}$$
$$c_1, c_2 \in \mathbb{C}$$

$$\left.\begin{aligned} y_1 &= \frac{1}{2}(x_1 + x_2) \\ y_2 &= \frac{1}{2i}(x_1 - x_2) \end{aligned}\right\} \Rightarrow |W| \neq 0$$

$$y_1 = e^{at} \cdot \cos bt$$
$$y_2 = e^{at} \cdot \sin bt$$

$$y = e^{at} \cdot (\underbrace{c_1 \cdot \cos bt + c_2 \cdot \sin bt})$$



$\Delta = 0 \qquad A^2 = 4B \qquad r_1 = r_2 = r \in \mathbb{R}$

$x_1 = e^{rt}$

$x_2 = t \cdot e^{rt}$

$y = c_1 \cdot x_1 + c_2 \cdot x_2$

$\ddot{x} + 3\cdot\dot{x} + 2\cdot x = u$, $\underbrace{y = 4\cdot x} = \underbrace{4\cdot x_1 + 0\cdot x_2}$ 26

1. $\ddot{x} = -3\cdot\dot{x} - 2\cdot x + u$

2. $\left.\begin{matrix} x_1 = x \\ x_2 = \dot{x} \end{matrix}\right\} \Rightarrow$ $\boxed{\dot{x}_1 = \dot{x} = x_2}$ $+ 0\cdot u$ 3

$\dot{x}_2 = \ddot{x} = -3\cdot\underbrace{\dot{x}}_{x_2} - 2\cdot\underbrace{x}_{x_1} + u\cdot 1$

$$\boxed{\dot{x}_2 = -3\cdot x_2 - 2\cdot x_1 + u.}$$

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{2\times1} = \underbrace{\underbrace{\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}}_{2\times2\ \cancel{2\cdot2\times2}}\cdot\underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{2\times1}}_{2\times1} + \underbrace{\underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{2\times1}\cdot\underbrace{u.}_{1\times1}}_{2\times1}$$

$y = C\cdot x$

$$\underset{1\times1}{y} = \underset{1\times2}{\begin{bmatrix}4 & 0\end{bmatrix}}\cdot\underset{2\times1}{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}$$

$$\dddot{x} - 3\ddot{x} - 2\dot{x} + 2x = u_1 - 6u_2$$

$$y_1 = \ddot{x} + u_2 \quad = x_3 + u_2$$

$$y_2 = \ddot{x} + 3x + 5u_1 = x_3 + 3x_1 + 5u_1$$

$$y_3 = -3\ddot{x} + x + 5u_2. = -3x_3 + x_1 + 5u_2$$

$$\dddot{x} = 3\ddot{x} + 2\dot{x} - 2x + u_1 - 6u_2$$

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ x_3 &= \ddot{x} \end{aligned}\right\} \overset{d/dt}{\Longrightarrow} \begin{aligned} \dot{x}_1 &= \dot{x} = x_2 \Rightarrow \dot{x}_1 = x_2 \\ \dot{x}_2 &= \ddot{x} = x_3 \Rightarrow \dot{x}_2 = x_3 \\ \dot{x}_3 &= \dddot{x} = 3x_3 + 2x_2 - 2x_1 \\ &\quad + u_1 - 6u_2 \end{aligned}$$

$$\underset{3\times1}{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 2 & 3 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{\underline{3\times2}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -6 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}$$

$$\underset{3\times1}{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 1 & 0 & -3 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{3\times2}{\begin{bmatrix} 0 & 1 \\ 5 & 0 \\ 0 & 5 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}$$

$$3\dddot{x} + 3\cdot\ddot{x} + 2\cdot\dot{x} - 5\cdot x = u_1 + u_2 + u_3 + u_4 \qquad 22$$

$$\begin{cases} y_1 = \ddot{x} + x + u_1 - 3\cdot u_2 = x_3 + x_1 + u_1 - 3u_2 \\ y_2 = 5\cdot\ddot{x} - x + u_2 - 3u_1 = 5\cdot x_3 - x_1 + u_2 - 3u_1 \end{cases}$$

$$\dddot{x} = \frac{1}{3}\left(-3\ddot{x} - 2\dot{x} + 5\cdot x + u_1 + u_2 + u_3 + u_4\right)$$

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ x_3 &= \ddot{x} \end{aligned}\right\} \xrightarrow{d/dt} \begin{aligned} \dot{x}_1 &= \dot{x} = x_2 \\ \dot{x}_2 &= \ddot{x} = x_3 \\ \dot{x}_3 &= \frac{1}{3}\left(-3x_3 - 2\cdot x_2 + 5\cdot x_1 + u_1 + u_2 + u_3 + u_4\right) \end{aligned}$$

$$\underset{3\times 1}{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5/3 & -2/3 & -3/3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underbrace{\underset{3\times 4}{\left[\begin{array}{c|c|c|c} 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 1/3 & 1/3 & 1/3 & 1/3 \end{array}\right]} \underset{4\times 1}{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}}_{3\times 1}$$

$$\underset{2\times 1}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \underbrace{\underset{2\times 3}{\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 5 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}}_{2\times 1} + \underbrace{\underset{2\times 4}{\begin{bmatrix} 1 & -3 & 0 & 0 \\ -3 & 1 & 0 & 0 \end{bmatrix}} \cdot \underset{4\times 1}{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}}_{2\times 1}$$

Laplace Trans

$$f(t) \longrightarrow F(s)$$

$$F(s) = \int_{-\infty}^{+\infty} e^{-st} f(t)\, dt$$

$$f(t) \longrightarrow F(s)$$

$$\dot{f}(t) \longrightarrow s \cdot F(s) + \cancel{I.C.}^{\to 0}$$

$$f''(t) \longrightarrow s^2 \cdot F(s)$$

$$f''' \longrightarrow s^3 F(s)$$

$$\vdots \qquad \qquad \vdots$$

$$\dot{x} + 5\cdot x = u. \xrightarrow{C.E} r + 5 = 0$$

$$r = -5 < 0 \rightarrow \text{stable}$$

$$x(t) = \dots \quad \dot{x} = \dots \quad u = \dots$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$X(s) \qquad s\cdot X(s) \qquad U(s).$$

$$s\cdot X(s) + 5\cdot X(s) = U(s)$$

$$X(s)\,(s+5) = U(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{s+5}$$

$$\dddot{x} + 3\cdot\ddot{x} + 5\dot{x} + x = U(t) + \dot{u}(t)$$

$$\left.\begin{aligned} x &\rightarrow X(s) \\ \dot{x} &\rightarrow s\cdot X(s) \\ \ddot{x} &\rightarrow s^2 X(s) \\ \dddot{x} &\rightarrow s^3 X(s) \end{aligned} \qquad \begin{aligned} u(t) &\rightarrow U(s) \\ \dot{u}(t) &\rightarrow sU(s) \end{aligned}\right\} \Rightarrow$$

$$s^3\cdot X(s) + 3\cdot s^2\cdot X(s) + 5\cdot s\cdot X(s) + X(s)$$

$$= U(s) + s\,U(s)$$

$$\boxed{\frac{X(s)}{U(s)} = \frac{s+1}{s^3+3s^2+5s+1}}$$