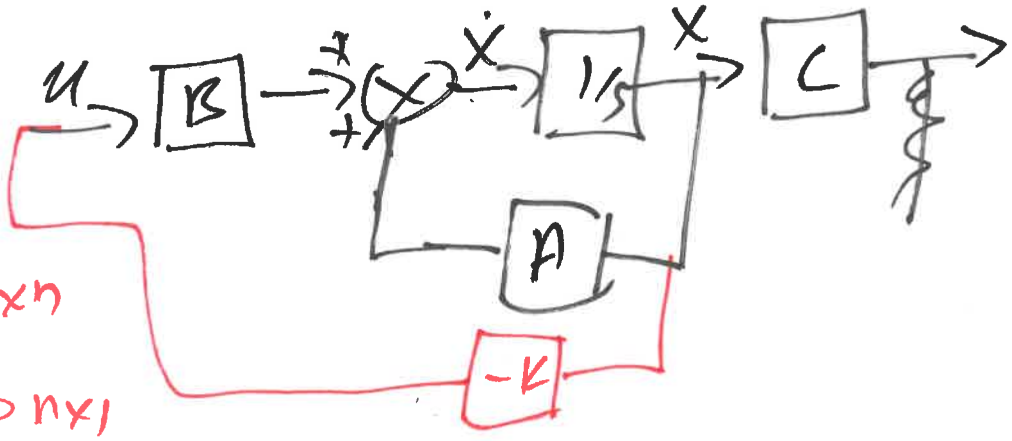
Revision

$\dot{x} = Ax + Bu$

$y = C\cdot x$



1x1

1xn

$u = -K\cdot x$ → nx1

n - unknowns.

Check CTRB

$\dot{x} = (A - B\cdot K)\cdot x$

C.L.S.M.

nxn

Target n poles
eigs

place(A,B,P)

I use this n-times

$|\lambda I - (A - BK)| = 0$

⇒ nxn L.S.

→ pole placement

?. Further in $-\infty$

Faster sys

Increase $|K|$ → $u = -K\cdot x$, $u\uparrow$

volts

N

Nm

kg

I want $\cancel{X} \to 0$ asap

AND $|u|$ as small as possible

$\to$ conflicting arguments

Quantify:

$I_x = \int x^T . Q . x \, dt$, $I_x \to$ as small as possible

$I_u : \int u^T . R u \, dt$ $\quad I_u \nearrow$

$$I = \int (x^T Q x + u^T . R u) \, dt$$

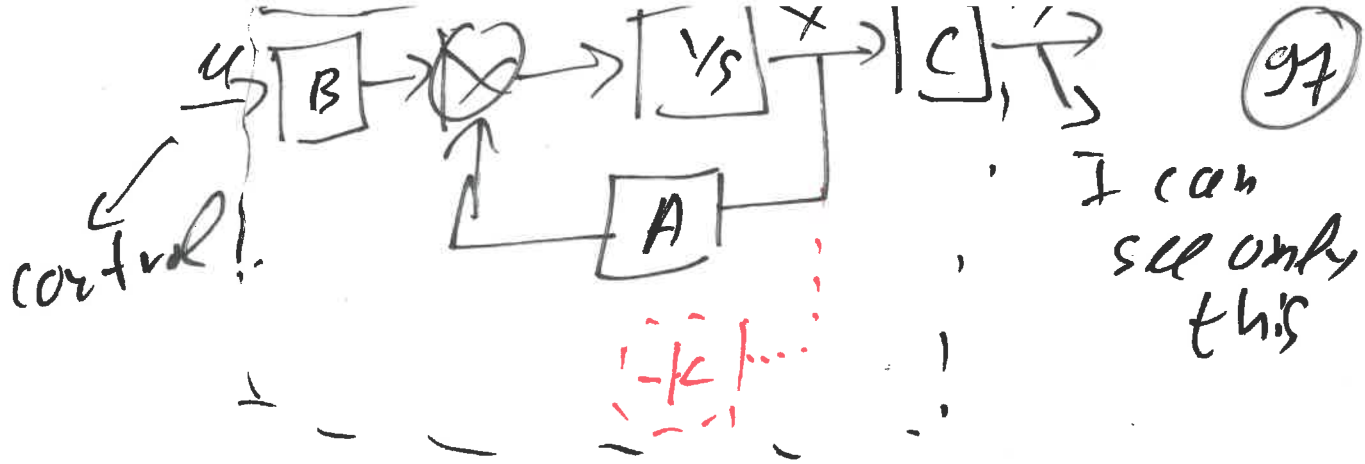
Given Q, R find optimum K : I

$$A^T . P + PA - P B R^{-1} B^T P + Q = 0 \Rightarrow P = ..$$

$$K = R^{-1} . B^T . P \; *$$

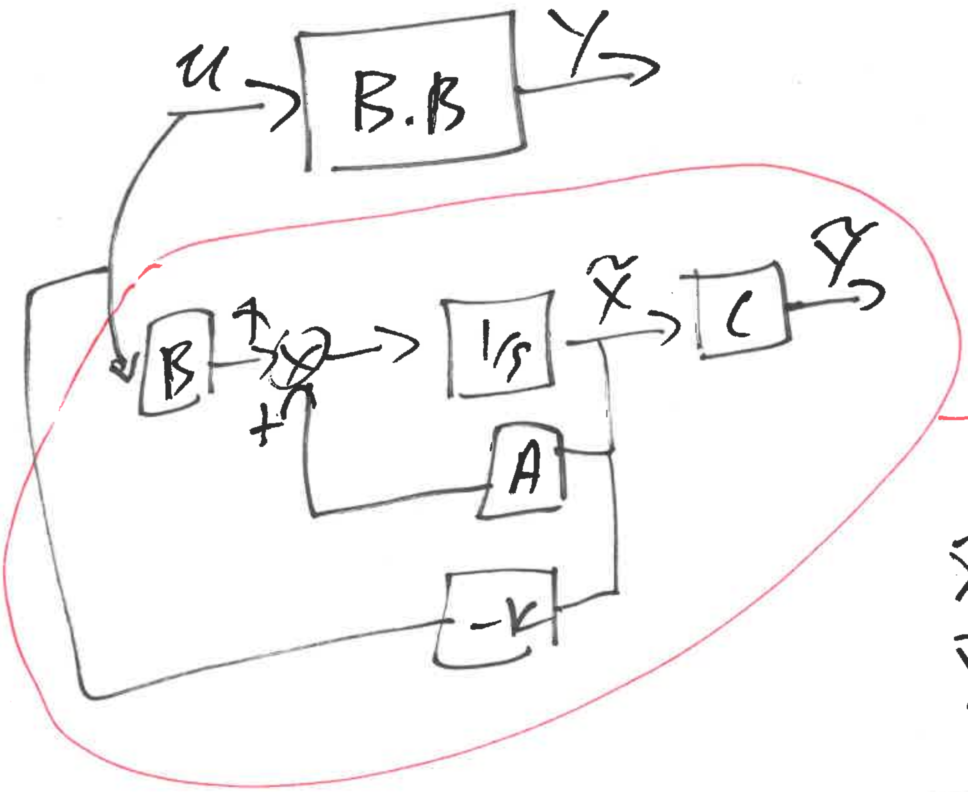
$Q \to$ Imp of speed , $Q \uparrow$ Faster sys

$R \to$ Imp of energy , $R \uparrow$ slower sys



$\dot{x} = Ax + B \cdot u$

$y = C \cdot x$



→ estimator.

$\dot{\tilde{x}} = A\tilde{x} + Bu$

$\tilde{y} = C \cdot \tilde{x}$

$\dot{x} = Ax + Bu$ → real sys

$u = -K \cdot \tilde{x}$ → control signal

$\dot{\tilde{x}} = A\tilde{x} + Bu$ → est.

1. $\tilde{x} \to x$ then $u = -K \cdot x$

will this work???

$x \to \tilde{x}$

$e = x - \tilde{x}$, $e \to 0$ or $\lim_{t \to \infty} e = 0$

$\dot{e} = \dot{x} - \dot{\tilde{x}}$

$\dot{e} = Ax + \cancel{Bu} - (A\tilde{x} + \cancel{Bu})$

$\dot{e} = Ae$ $\quad$ $\dot{x} = Ax$

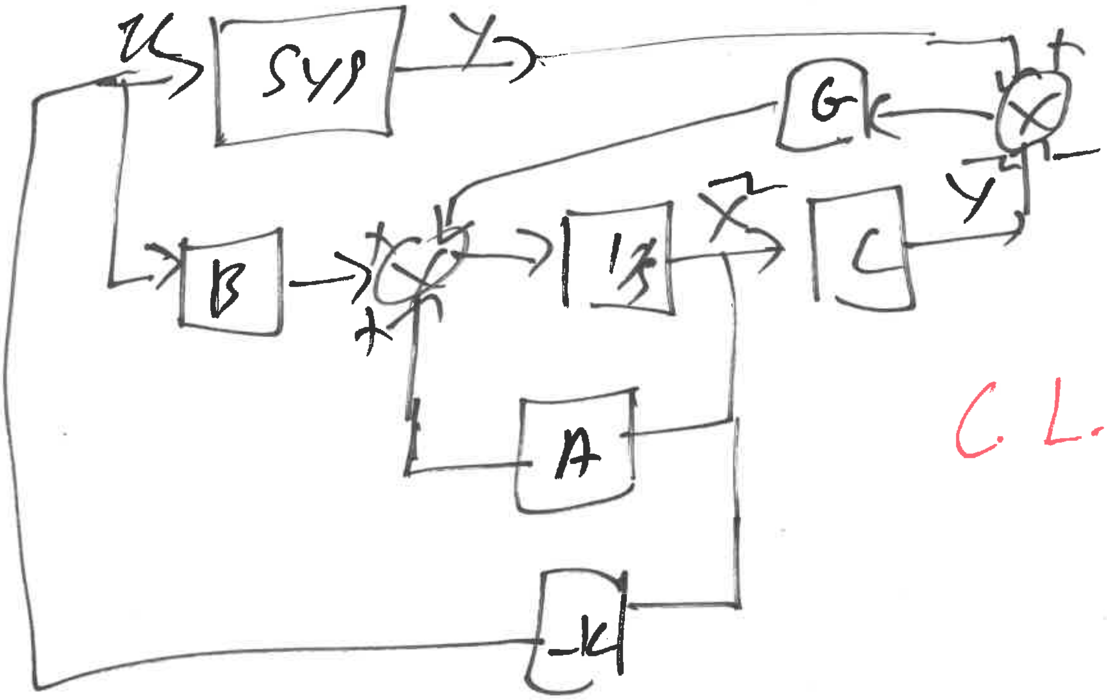
• $e(0) = 0$ $\quad$ $e \to 0$ $\quad \forall t$

• $e(0) \neq 0 \to$ eigs < 0 $\quad$ $e \to 0$

↳ eigs > 0 $\quad$ $e \to \pm\infty$

I want to use $u = -Kx$ when A unst.

problem when O.L.S.M. unstable



C.L. est.

$e \to 0$

if $y - \tilde{y} \to 0 \Rightarrow e \to 0$ ONLY when OBSV

$e = x - \tilde{x}$

$\dot{e} = \dot{x} - \dot{\tilde{x}} = Ax + \cancel{Bu} - (A\tilde{x} + \cancel{Bu} + G(y - \tilde{y}))$

$= A \cdot e - G(y - \tilde{y})$

$= Ae - G \cdot (C \cdot x - C \cdot \tilde{x})$

$= Ae - G \cdot C \cdot e$

$\dot{e} = (A - G \cdot C) \cdot e$

↓ Target "proper" eig. put these eigs as ~~far~~ deep into $-\infty$

5-10 times faster Error dyn. than Cont.

100

$$\dot{x} = Ax + Bu$$
$$\dot{e} = (A - GC)\cdot e$$
$$\dot{\tilde{x}} = A\tilde{x} + B\cdot u$$
$$u = -K\tilde{x}$$

$\Rightarrow$ $\dot{x} = Ax + Bu \xrightarrow{u = -Kx} \dot{x} = (A - BK)\cdot x$

$$\dot{x} = Ax + B(-K\tilde{x})$$
$$= Ax - BK\tilde{x} + (BKx - BKx)$$
$$= \underbrace{Ax - BKx} - BK\tilde{x} + BKx$$
$$= A_{CL}x + BK(x - \tilde{x})$$

$A_{CL} = A - BK$

$$\left.\begin{aligned} \dot{x} &= A_{CL}\cdot x + B\cdot K e. \\ \dot{e} &= 0\cdot x + (A - GC)\cdot e \end{aligned}\right\} \Rightarrow$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - GC \end{bmatrix} \cdot \begin{bmatrix} x \\ \bar{e} \end{bmatrix}$$

$\Downarrow$ tri. matrix

$$\left| \begin{bmatrix} A - BK & BK \\ 0 & A - GC \end{bmatrix} - \lambda I \right| = 0$$

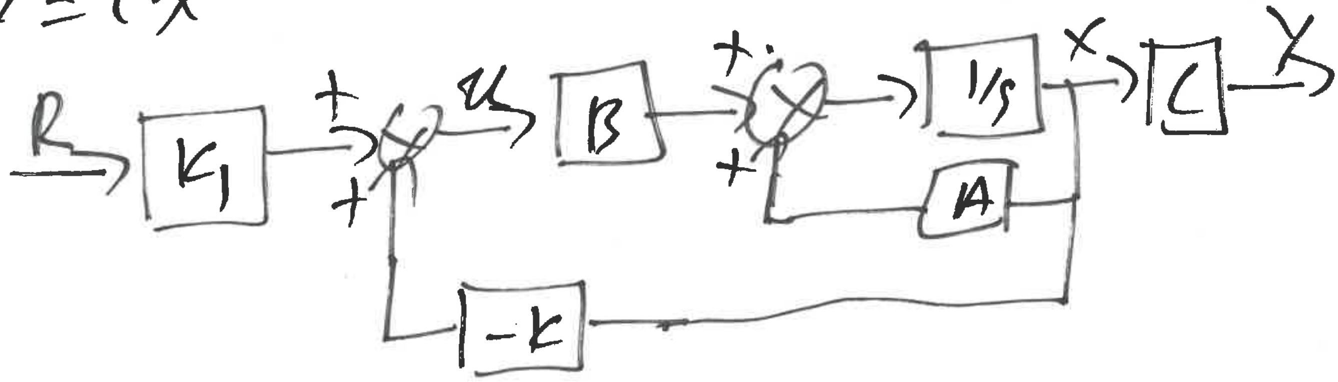
$$|\underbrace{A-BK}_{\text{Dyn. of sys}}-\lambda I|\cdot|\underbrace{A-GC}_{\text{Dyn. of est}}-\lambda I|=0 \qquad (10)$$

$K = ?$ → P.P
→ LQR.

G → poles of est 5, 10 times deeper than eig$(A-BK)$?

$\dot{x} = Ax + Bu$ $\quad$ $x \rightarrow \bar{x}$

$y = C \cdot x$



$K_1 = ?$: $y \rightarrow R$ → known

$R = \text{Const.}$, $R, y \in \mathbb{R}$

~~x_{ss}~~ $\quad x = x_{ss}$ → I don't know this value

$\dot{x}_{ss} = 0$

$0 = A x_{ss} + B u_{ss}$

$y_{ss} = C \cdot x_{ss}$

x_{ss}, u_{ss} are NOT known

but are fun. of $R_{ss} = y_{ss}$

$u_{ss} = N_u R_{ss}$

$x_{ss} = N_x R_{ss}$

$$0 = A \cdot N_x \cdot \cancel{R_{ss}} + B\, N_u\, \cancel{R_{ss}}$$
$$\cancel{R_{ss}} = C \cdot N_x \cdot \cancel{R_{ss}}$$

$\Rightarrow$

$$0 = A \cdot N_x + B \cdot N_u \quad \rightarrow 1\times1,\ n\times1$$ ($n\times1$)

$$1 = C \cdot N_x \quad \rightarrow 1\times1$$

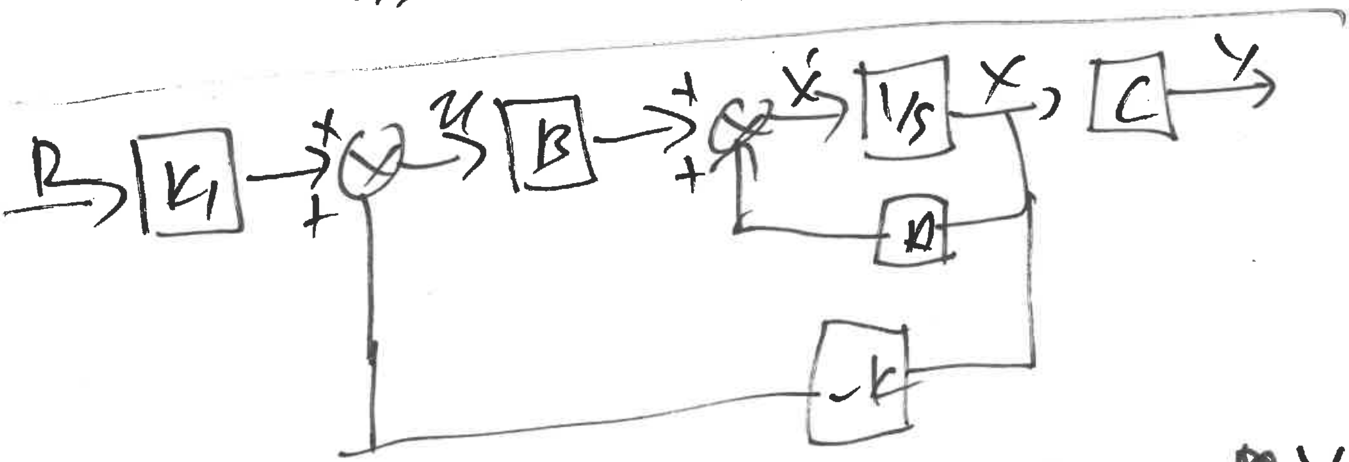
n+1 unknowns
n+1 equns.

I have n^{th} $\quad x \in \mathbb{R}^{n\times1}$

$u \in \mathbb{R}$

~~....~~ $N_x = \ldots \qquad N_u = \ldots$

$u_{ss} \qquad x_{ss}$



R = dem. value = const $\quad K_1 = ? \rightarrow y = R.$

Tracking.

$y \to R_{ss}$ → Given

$x \to x_{ss}$

$u \to u_{ss}$ → Not known

K_1 $\quad u \to u_{ss}$

$u = R \cdot K_1 - Kx \to u_{ss}$

$u_{ss} = RK_1 - Kx_{ss}$

$R_{ss} = \text{const.}$; $x_{ss} = \text{const.}$

$\dot{x}_{ss} = 0$

$\dot{x} = Ax + Bu.$

$$\left.\begin{aligned} 0 &= Ax_{ss} + B \cdot u_{ss} \\ R_{ss} &= C \cdot x_{ss} \end{aligned}\right\} \Rightarrow \begin{aligned} 0 &= A N_x + B N_u \\ 1 &= C \cdot N_x \end{aligned}$$

$(n \times 1)$

$x\ (n \times 1)$