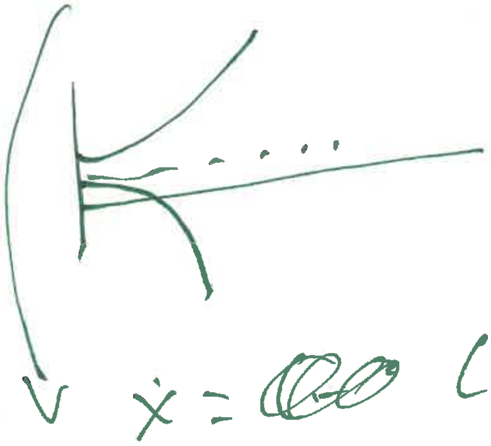
# Revision

$$\left.\begin{array}{l}\dot{x} = Ax + Bu \\ u = -K\cdot x\end{array}\right\} \Rightarrow \dot{x} = (A - BK)\cdot x = A_{CL}\cdot x$$

$K = ?$ : $x \to 0$

$\dot{x} = 3x + u.$ unstable $x_0 = 1$

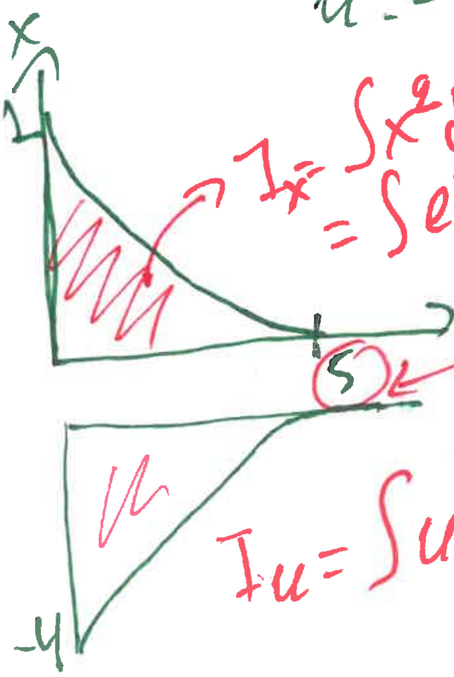


$u = -K\cdot x$

$\dot{x} = (3 - K)\cdot x$

- $K = 4$ $\quad \dot{x} = -x$
  $u = 4\cdot x$
  pole at $-1$.
  $x = e^{-t}$
  $u = -4\cdot e^{-t}$

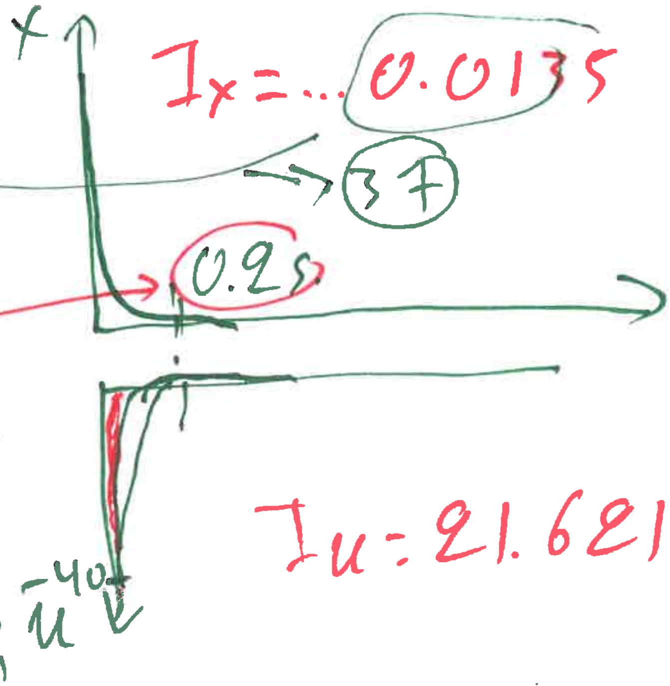
- $K = 40$ $\quad \dot{x} = -37\cdot x$
  pole $= -37$
  $x = e^{-37\cdot t}$
  $u = -40\cdot x$
  $= -40\cdot e^{-37\cdot t}$




$I_x = \int x^2 dt$
$= \int e^{-2t} dt = 0.5$

25

$I_u = \int u^2 dt = 8$




$I_x = \ldots 0.0135$

37

$I_u = 21.621$

$\dot{x} = Ax + Bu \qquad u = -K\cdot x$

$K = ? \qquad x \to 0 \qquad \dot{x} = (A - BK)\cdot x.$

$K\uparrow$ Faster sys. + More Energy (37 times)

---

Perfect: $I_x = \int x^2 dt$ AND $I_u = \int u^2 dt$

$\downarrow \qquad \downarrow$

LQR $\quad K = \dots$

I want to min $I_x$ AND $I_u$

OR min

$I_1 = \int (x^2 + u^2) dt$

$K = 6.1623$

$E = -3.16$

$I_2 = \int (3x^2 + u^2) dt$

$I_3 = \int (x^2 + 3\cdot u^2) dt$

$K = 6.46$

$E = -3.46$

$K = 6.05$

$E = -3.05$

$I_4 = \int (3x^2 + 3u^2) dt$

$K = 6.1623$

$E = -3.16$

$$I = \int \left( q_1 x_1^2 + q_2 x_2^2 + \dots + r_1 u_1^2 + r_2 u_2^2 \dots \right) dt \qquad (110)$$

$n=2$

$$I = \int \left( q_1 \cdot x_1^2 + q_2 \cdot x_2^2 + r_1 u_1^2 + r_2 u_2^2 \right) dt$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad x^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \qquad R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$$

$$I = \int \left( x^T \cdot Q \cdot x + U^T R \cdot U \right) dt$$

---

SYS

$Q=10$, $R=1$: Fast, $K\uparrow$, $u\uparrow$

$Q=1$, $R=10$: Slow, $K\downarrow$, $u\downarrow$

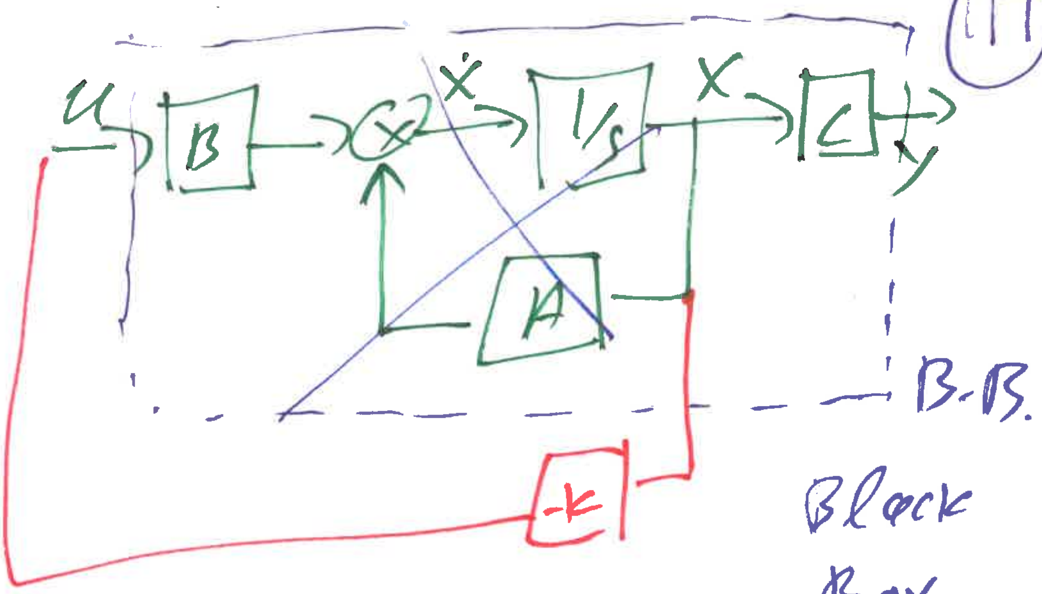
$\boxed{Q=10,\ R=10 \qquad Q=1,\ R=1}$

Fast.

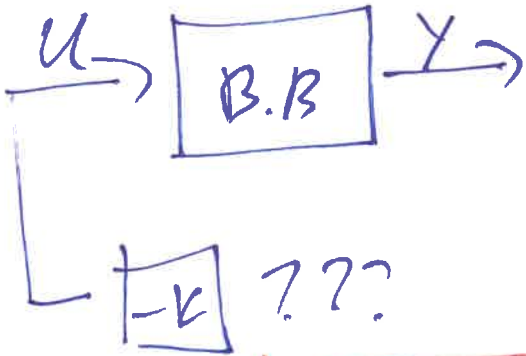
Less

Same.

$\dot{x} = A x + B \cdot u$




Black Box

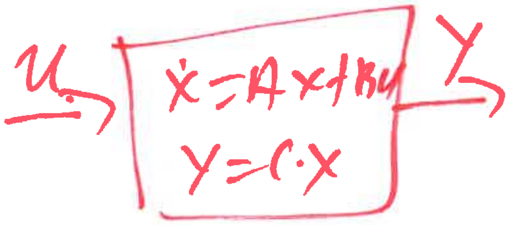


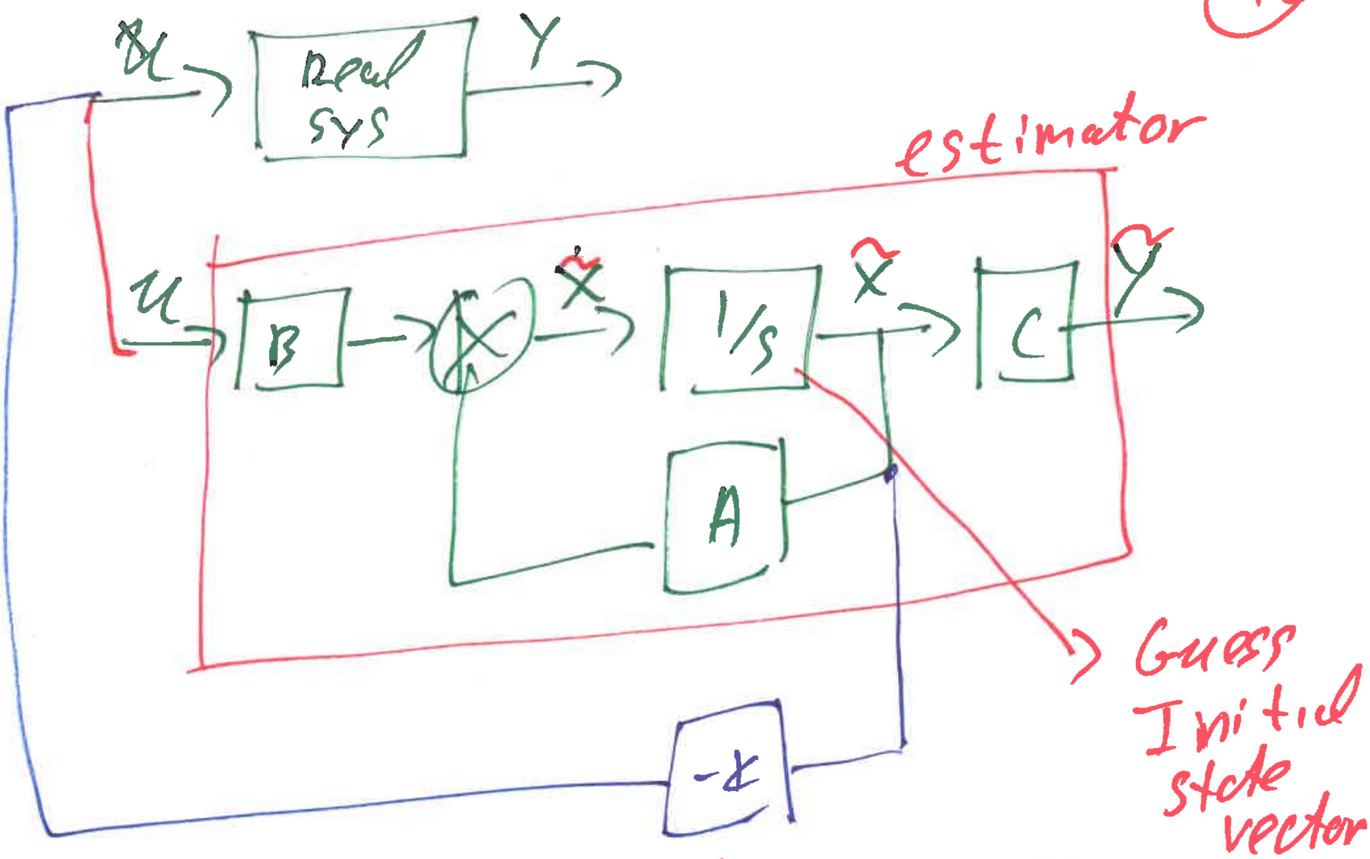


$$\boxed{\begin{aligned} \dot{x} &= A x + B u \\ y &= C \cdot x \end{aligned}}$$

perfect

A, B, C known






Real $\quad \dot{x} = Ax + Bu \qquad \dot{\hat{x}} = A\hat{x} + B\cdot u.$

$\qquad\quad x_0 \qquad\qquad\qquad \hat{x}_0$

I want $\boxed{\hat{x} \to x} \Rightarrow u = -k\cdot\hat{x}$

$e = x - \hat{x}, \quad e \to 0$

$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + \cancel{Bu} - (A\hat{x} + \cancel{Bu})$

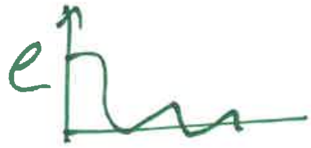
$\dot{e} = Ax - A\hat{x} = A\cdot e$

ODE
Dynamics
of error.

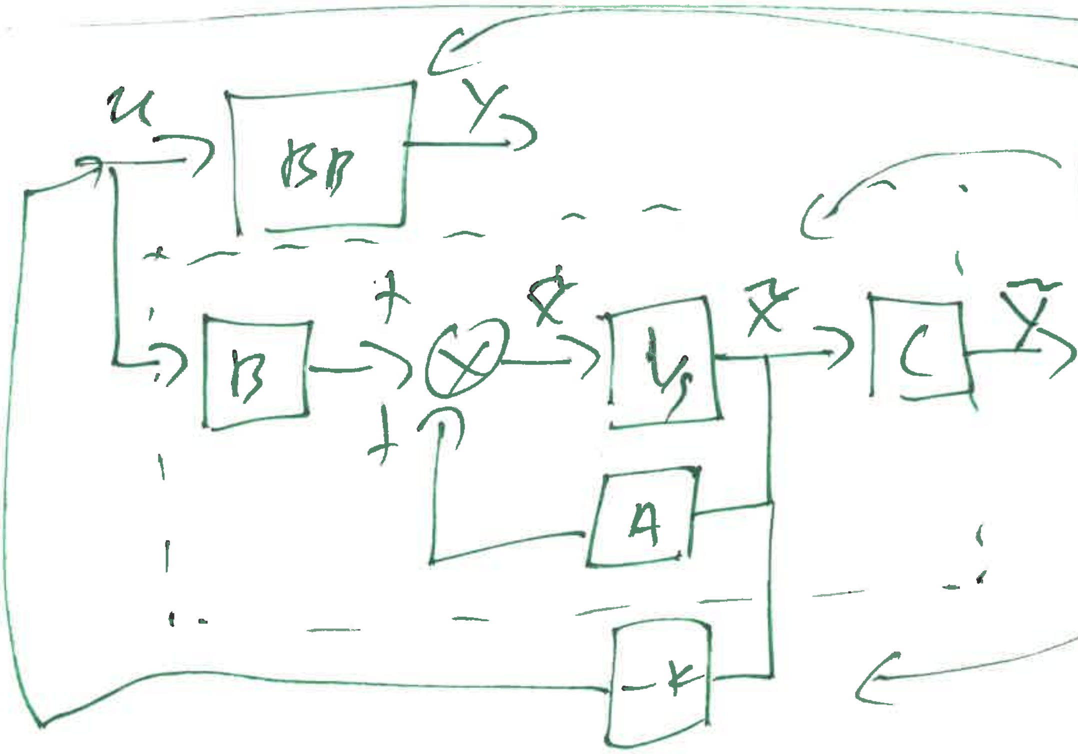
$\dot{e} = Ae \Rightarrow e = e^{At} \cdot e_0$

- $e_0 = 0 \Rightarrow e = 0 \quad \forall t$

- $e_0 \neq 0$ • $eig(A) < 0 \quad e \to 0$


- $eig(A) > 0 \quad e \to \pm\infty$




$\dot{x} = Ax + Bu$

$\dot{\tilde{x}} = A\tilde{x} + Bu$

$y = C \cdot x$

$\tilde{y} = C \cdot \tilde{x}$

$u = -K \cdot \tilde{x}$

$e \to 0 \quad \boxed{\text{if } \; y \to \tilde{y} \; \text{ when OBSV.}}$

$$\dot{\tilde{x}} = A\tilde{x} + Bu + G(y - \tilde{y})$$

$$\dot{x} = Ax + B\cdot u$$

$$\dot{\tilde{x}} = A\tilde{x} + B\cdot u + G\cdot(y - \tilde{y})$$

$$e = x - \tilde{x}$$

$$\dot{e} = \dot{x} - \dot{\tilde{x}} = Ax + \cancel{Bu} - (A\tilde{x} + \cancel{Bu} + G(y - \tilde{y}))$$

$$= \underbrace{A\cdot x - A\tilde{x}}_{} - G(y - \tilde{y})$$

$$= A\cdot e - G\cdot(Cx - C\cdot\tilde{x})$$

$$= Ae - G\cdot C\cdot(x - \tilde{x})$$

$$= Ae - G\cdot C\cdot e$$

$$\boxed{\dot{e} = (A - G\cdot C)\cdot e}$$

$$\dot{x} = (A - BK)x$$

ODE of error

$G = ?$ $\quad e \to 0$

Choose $G$ : eig$(A - G\cdot C)$ are 5–10 deeper to $-\inf$ than eig$(A - BK)$

sys $\quad eig(A-BK) = -10 \quad -11$

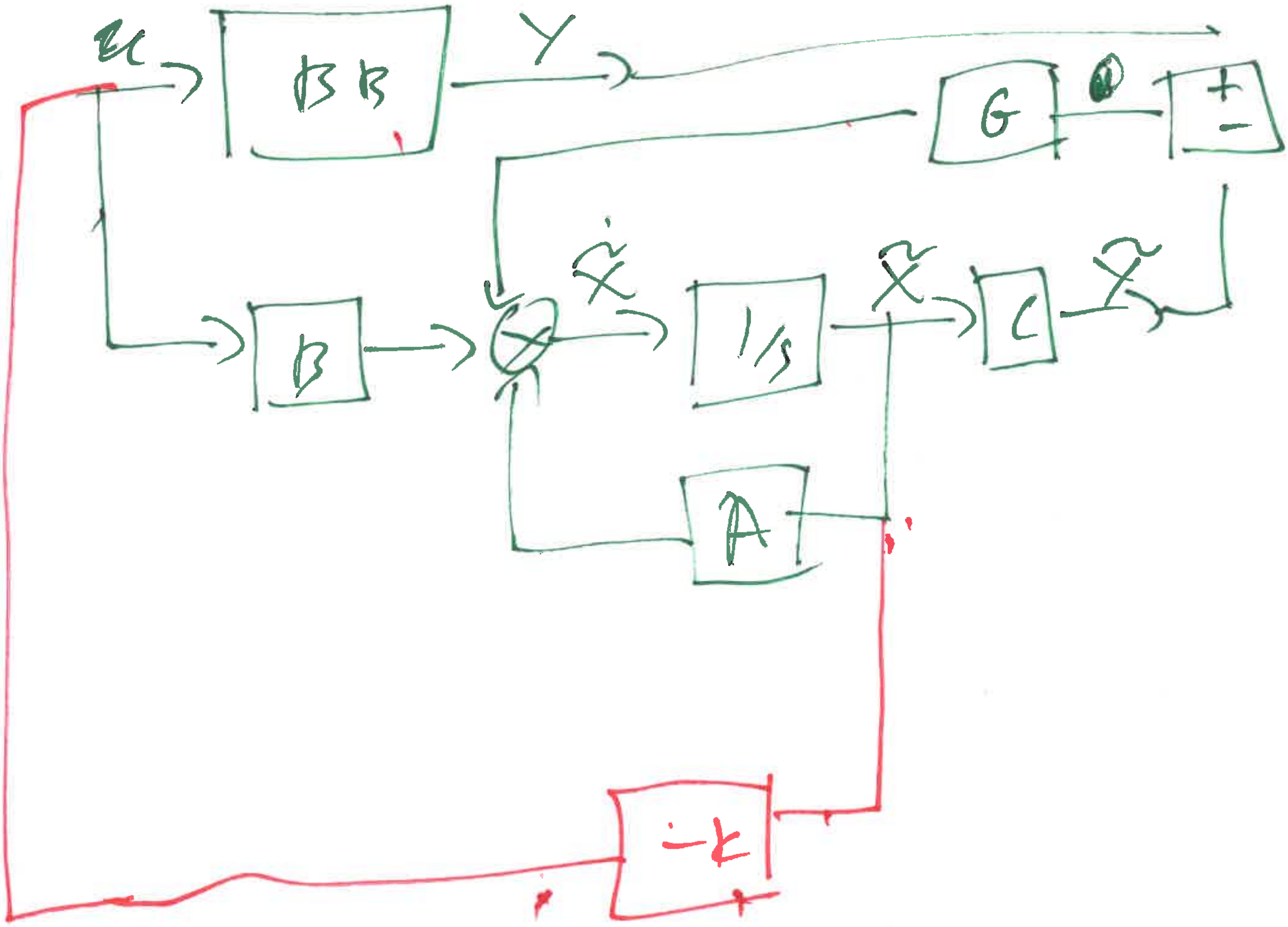
$esA_1 \quad eig(A-GC) = -5 \quad -6$

$esA_2 \quad eig(A-GC) = -1000$

$-2000$

$e^{-1000t}$

$e^{-2000t}$



$$\dot{x} = Ax + Bu$$

$$\dot{\tilde{x}} = A\tilde{x} + Bu + G(y - \tilde{y})$$

$$y = C \cdot x$$

$$\tilde{y} = C \cdot \tilde{x}$$

$$\boxed{u = -K\tilde{x}}$$

$$\dot{x} = Ax + B(-K\tilde{x})$$

$$= Ax - BK\tilde{x} + (BKx - BKx)$$

$$= Ax - BKx - BK\tilde{x} + BKx$$

$$\dot{x} = (A - BK) \cdot x + BK \cdot e$$

Dynamics of real sys

$$\dot{e} = (A - G \cdot C) \cdot e$$

$$\dot{e} = 0 \cdot x + (A - GC) \cdot e$$

$$\dot{x} = (A - BK)\,x + BK \cdot e$$

$$\dot{e} = 0 \cdot x + (A - GC) \cdot e$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - GC \end{bmatrix} \cdot \begin{bmatrix} x \\ e \end{bmatrix}$$

118

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

$|A - \lambda I| = 0 \Rightarrow \dots \quad \lambda = 1$

$\lambda = 3$

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\dot{x}_1 = a \cdot x_1 + b \cdot x_2$

$\dot{x}_2 = 0 \cdot x_1 + c \cdot x_2.$

I want to prove that b we don't care

- $c < 0 \quad x_2 = e^{ct} \cdot x_2(0)$

$x_2 \to 0$

$\dot{x}_1 = a x_1 + \cancel{\dots} \to 0 \qquad x_1 = e^{a \cdot t} \cdot x_1(0)$

- $c > 0 \quad x_2 = e^{ct} \cdot x_2(0) \qquad x_2 \to \infty$

$x_1 \to \infty$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - Gc \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$