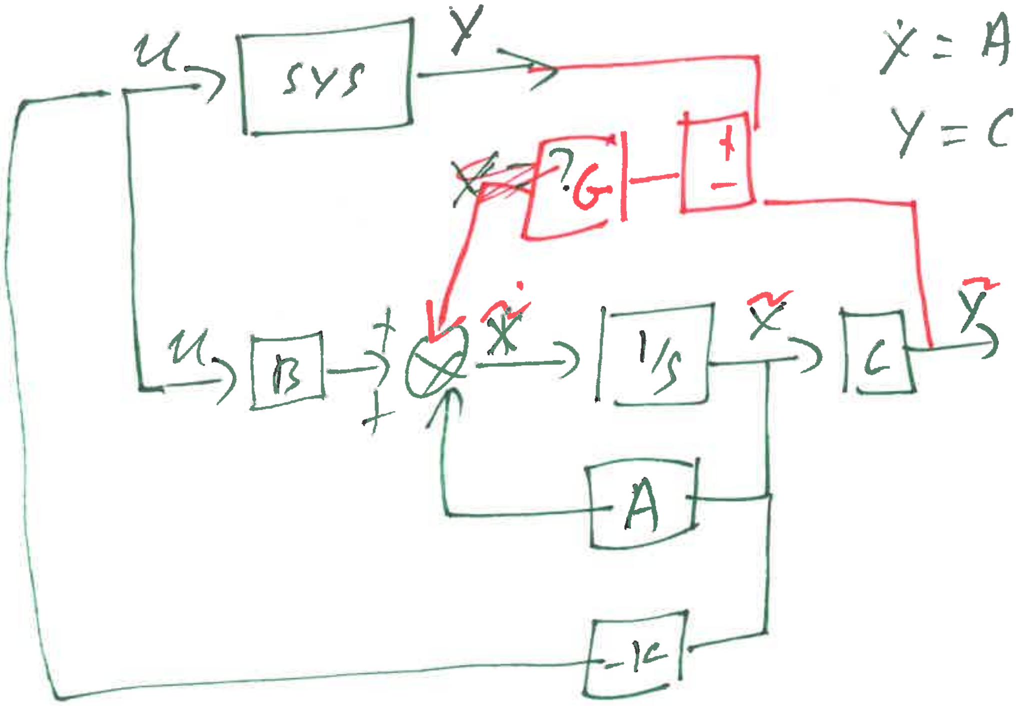
Revision



$\dot{x} = Ax + B \cdot u$

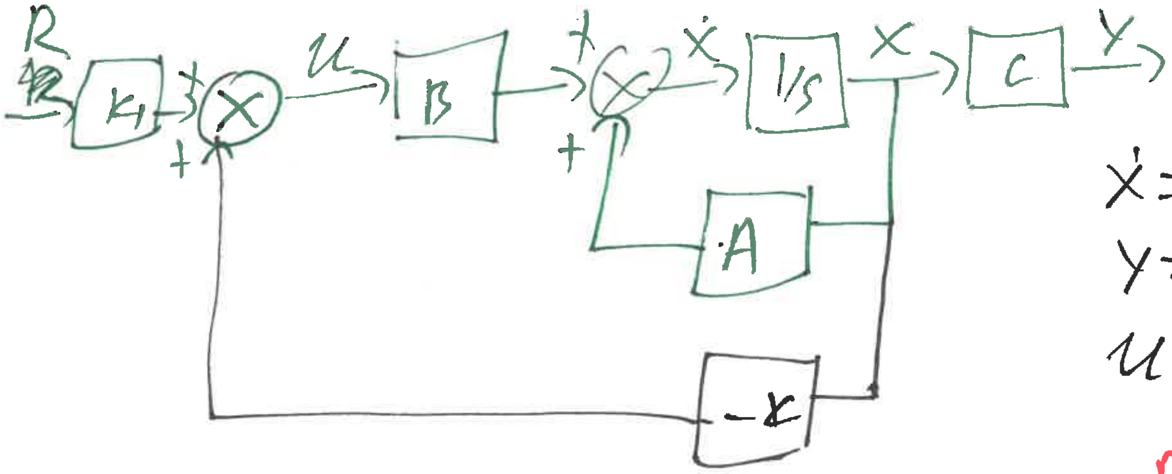
$Y = C \cdot X$

$\dot{e} = A \cdot e \quad , \; e = X - \tilde{X}$

$\dot{e} = (A - GC) \cdot e \quad , \; e \to 0 \qquad X = \tilde{X}$

$\dot{X} = (A - BK) \cdot X + \cancel{B \cdot K \cdot e} \to 0$

Tracking



$\dot{x} = Ax + Bu$

$y = C \cdot x$

$u = -K \cdot x$

P.P.   L.Q.R.

pole placement

$\dot{x} = (A - BK) \cdot x$

u = control signal $= -Kx + K_1 \cdot R$

R = Desired output

Y = Real output

$K_1 = ?$: $Y \to R$

Assume

$R = r_{ss} =$ constant

Single Input
Single Output   SISO

$R \in \mathbb{R}$ 1×1

$Y \in \mathbb{R}$ 1×1

$K_1 = ?$: $Y \to Y_{ss} = r_{ss}$ Given

$k_1 = ?$: $\quad y \to y_{ss} = r_{ss}$ Given

$u \to u_{ss}$
$x \to x_{ss}$ | Not known.

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e.g. $r_{ss} = 5m$, $\quad u_{ss} = 10\,N \quad x_{ss} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$= 10m \quad u_{ss} = \dots \quad x_{ss} = \dots$

$$\boxed{u_{ss} = N_u\, r_{ss} \quad , \quad x_{ss} = N_x\, r_{ss}}$$

$N_u \in \mathbb{R}$
$N_x \in \mathbb{R}^{n \times 1}$

$$\left.\begin{aligned} \dot{x} &= A x + B \cdot u \\ y &= C \cdot x \end{aligned}\right\} \Rightarrow \left.\begin{aligned} \cancel{\dot{x}_{ss}}^{\to 0} &= A \cdot x_{ss} + B \cdot u_{ss} \\ y_{ss} &= C \cdot x_{ss} \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} 0 &= A \cdot x_{ss} + B \cdot u_{ss} \\ y_{ss} &= r_{ss} = C \cdot x_{ss} \end{aligned}\right\} \quad \begin{aligned} 0 &= A \cdot N_x \cdot \cancel{r_{ss}} + B\, N_u \cdot \cancel{r_{ss}} \\ \cancel{r_{ss}} &= C \cdot N_x \cdot \cancel{r_{ss}} \end{aligned}$$

I am n-zeros

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \cdot \begin{bmatrix} N_x \\ N_u \end{bmatrix} \begin{matrix} \to n \times 1 \\ \to 1 \times 1 \end{matrix}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow n \times 1$$

$r_{ss} = \ldots$ $\quad N_x, N_u \longrightarrow u_{ss} = , \; x_{ss} =$

$u = K_1 \cdot R - K \cdot X$

$K_1 = ?$

$u \to u_{ss}$

$y \to r_{ss}$

$x \to x_{ss}$

$u = u_{ss} \pm \cancel{\boxed{}}$ $\rightarrow 0$ when $x \to x_{ss}$

$u = u_{ss} - K \cdot (X - X_{ss})$

~~$u = K_1 \cdot r_{ss} - K \cdot (X - X_{ss})$~~

$u = u_{ss} - K(X - X_{ss})$

$u = \underline{N_u \cdot r_{ss}} - K \cdot X + \underline{K N_x \cdot r_{ss}}$

$u = (\underline{N_u + K \cdot N_x}) \cdot r_{ss} - K \cdot X$

$u = K_1 \cdot R - K \cdot X$

$K_1 = N_u + K \cdot N_x$

(12)

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$r_{ss} = 2.$ $K_1 = ?$

$u_{ss} = ?$ $x_{ss} = ?$

$\downarrow$ $\downarrow$

1×1 2×1

$\uparrow$ $\uparrow$

N_u N_x

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 1 & 2 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} =$$

crosscheck.

$$= \ldots \quad N_x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$N_u = -1$$

$$K_1 = N_u + K \cdot N_x.$$

if $K = [-15 \quad 41]$ poles at $[-10, -11]$

$\Rightarrow K_1 = 51$

$$\dot{x} = A\cdot x + B\cdot u \qquad n\times 1$$

$$y = C\cdot x \qquad 1\times 1$$

$$x \to x_{ss}$$

$$\cancel{\dot{x}_{ss}} \to 0 \to n \text{ zeros.}$$

$$\dot{x}_{ss} = A\cdot x_{ss} + B\cdot u_{ss} \Rightarrow = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y_{ss} = C\cdot x_{ss}$$

$n=2$

$$x_{ss} = N_x\cdot r_{ss} \qquad u_{ss} = N_u\cdot r_{ss}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = A\cdot \underbrace{N_x\cdot \cancel{r_{ss}}}_{2\times 1} + B\cdot N_u\cdot \cancel{r_{ss}}$$

$$y_{ss} = \cancel{r_{ss}} = C\cdot N_x\cdot \cancel{r_{ss}} = 1$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}\cdot \begin{bmatrix} N_x \\ N_u \end{bmatrix}$$

$A \to 2\times 2$, $C \to 1\times 2$, $0 \to 1\times 1$, $B \to 2\times 1$

$$\underbrace{}_{3\times 3}$$

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1}\cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\ddot{x} + 2\dot{x} + x = 0 \qquad x_0 = 1 , \dot{x}_0 = 0$

1) specific soln.
2) ODE → S.S.
3) Solve → eigenvectors
→ S.T.M.
→ $\cancel{e^{At}}$

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1) C.E. $r^2 + 2 \cdot r + 1 = 0$

$\Delta = 4 - 4 = 0$

$r_{1,2} = \frac{-2 \pm 0}{2}$

$r_1 = r_2 = -1$

$x_1 = e^{-t}, \quad x_2 = t \cdot e^{-t}$

$x = c_1 \cdot x_1 + c_2 \cdot x_2$

$= c_1 \cdot e^{-t} + c_2 e^{-t} \cdot t$

$x_0 = c_1 \cdot 1 + c_2 \cdot 1 \cdot 0 = 1 \Rightarrow \boxed{c_1 = 1}$

$\dot{x} = -c_1 e^{-t} + c_2 e^{-t} \cdot 1 - c_2 e^{-t} \cdot t$

$\dot{x}_0 = -c_1 + c_2 - 0 \Rightarrow c_1 = \boxed{c_2 = 1}$

$\boxed{x = e^{-t} + t e^{-t}}$

$\dot{x} = \cancel{-e^{-t}} + \cancel{e^{-t}} - t e^{-t} = -t e^{-t}$

S.S.

$$\ddot{x} = -2\dot{x} - x$$

$$\left.\begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \end{aligned}\right\} \xRightarrow{d/dt} \begin{aligned} \dot{x}_1 &= \dot{x} = x_2 \\ \dot{x}_2 &= \ddot{x} = -2\dot{x} - x \\ &= -2 \cdot x_2 - x_1 \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solve

$$|A - \lambda I| = 0 \Leftrightarrow \begin{vmatrix} 0-\lambda & 1 \\ -1 & -2-\lambda \end{vmatrix} = 0$$

$$-\lambda(-2-\lambda) + 1 = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(A - \lambda I) \cdot v = 0 \quad , \lambda = -1$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \left.\begin{aligned} v_1 + v_2 &= 0 \\ v_1 &= 1 \\ v_2 &= -1 \end{aligned}\right\} v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$V=(A-\lambda I)\cdot b$$

$$\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1&1\\-1&-1\end{bmatrix}\cdot\begin{bmatrix}b_1\\b_2\end{bmatrix}$$

$$1=b_1+b_2 \Rightarrow \left.\begin{matrix}b_1=1\\b_2=0\end{matrix}\right\}\Rightarrow b=\begin{bmatrix}1\\0\end{bmatrix}$$

$$X=c_1\,(Vt+b)\,e^{-t}+c_2\cdot e^{-t}\cdot V$$

$$\begin{bmatrix}1\\0\end{bmatrix}=c_1\cdot(0+b)\cdot 1+c_2\,V\cdot 1$$

$$\Leftrightarrow\ c_1\cdot b+c_2\cdot V=\begin{bmatrix}1\\0\end{bmatrix}$$

$$c_1\begin{bmatrix}1\\0\end{bmatrix}+c_2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}\Rightarrow \begin{matrix}c_2=0\\c_1=1\end{matrix}$$

$$X=\left(\begin{bmatrix}1\\-1\end{bmatrix}\cdot t+\begin{bmatrix}1\\0\end{bmatrix}\right)e^{-t}$$

$$=\begin{bmatrix}t+1\\-t\end{bmatrix}e^{-t}$$

if $b_2=1$, $b_1=0$

$$X=c_1(Vt+b)\cdot e^{-t}+c_2\cdot Ve^{-t}$$

$$\begin{bmatrix}1\\0\end{bmatrix}=c_1\cdot\left(0+\begin{bmatrix}0\\1\end{bmatrix}\right)e^{-t}+c_2\begin{bmatrix}1\\-1\end{bmatrix}e^{-t}$$

$$C_2 = 1 \quad C_1 = 1.$$

$$x = (v t + b) e^{-t} + v e^{-t}$$

$$= \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) e^{-t} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

$$= \begin{bmatrix} t + 0 \\ -t + 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$$

$$= e^{-t} \begin{bmatrix} t + 0 + 1 \\ -t + 1 - 1 \end{bmatrix} = e^{-t} \begin{bmatrix} t + 1 \\ -t \end{bmatrix}$$

S.T.M.

$$\cancel{X = \begin{bmatrix} (v t + b) e^{-t} & v e^{-t} \end{bmatrix}}$$

$$\cancel{X = \begin{bmatrix} b & v \end{bmatrix}}$$

$$=$$

S.T.M

$$X = \left[v e^{-t} \quad (vt+b) \cdot e^{-t} \right]$$

$$X(0) = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$X^{-1}(0) = \frac{1}{1} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$x = \left[v e^{-t} \quad (vt+b) e^{-t} \right] \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \left[\quad \right] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= e^{-t} \begin{bmatrix} 1 & t+1 \\ -1 & -t+0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= e^{-t} \begin{bmatrix} t+1 \\ -t \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

- T.F. = ?
- CTRB = ?
- $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, CTRB = ?
- K = ? ; closed loop eigenvalues are at $-10, -11$
- G = ? error dynamics at $-1, -2$
- G = ? — " — . . . . at $-50, -60$

Matlab based Q.

Simulink.

$\dot{x} = Ax + Bu$.

Draw sim diagram for a numerical solution

Analytical solution

Symbolic Toolbox

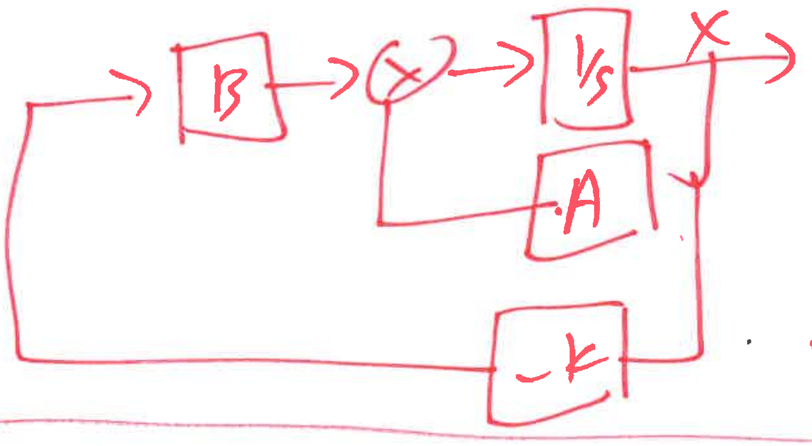
$\dot{x} + 2x = 0$

$x_0 = 1$

Design an LQR controller

• CTRB (rank(ctrb(A, B)))

LQR.



$K = \ldots$

state the necessary matlab commands using dsolve that you will use to solve this ODE.