Revision

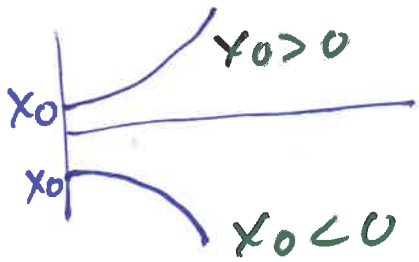
$\dot{x} + kx = u \rightarrow$

$$x = e^{-kt} \cdot x_0 + e^{-kt} \int_0^t e^{kt_1} u(t_1)\, dt_1$$

Trans. (first term) — Input (integral term)

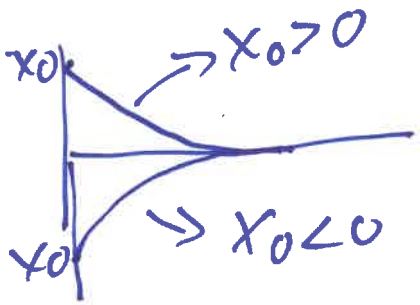
- $u = 0 \qquad x = e^{-kt} \cdot x_0$

☑ $k < 0$



unstable

☑ ~~$k > 0$~~



stable

if $x_0 = 0$

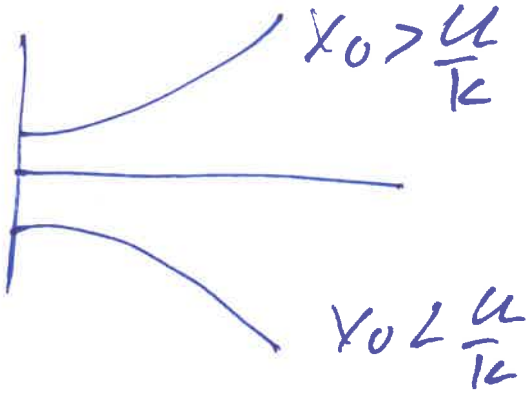
$k = 0 \qquad \dot{x} = 5$

$$\int \dot{x}\, dt = \int 5\, dt \Rightarrow x = 5 \cdot t + C$$

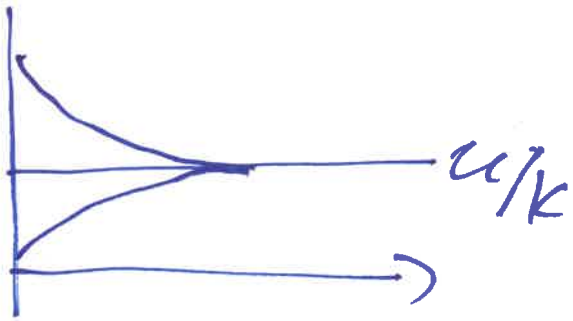
□ $u \neq 0$ but const.

$$x = e^{-kt} \cdot x_0 + \frac{u}{k}(1 - e^{-kt})$$

• $k < 0$ unstable



• $k > 0$ stable

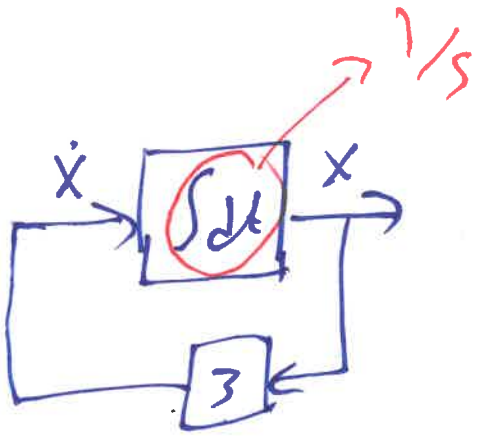


if $x_0 = \frac{u}{k}$

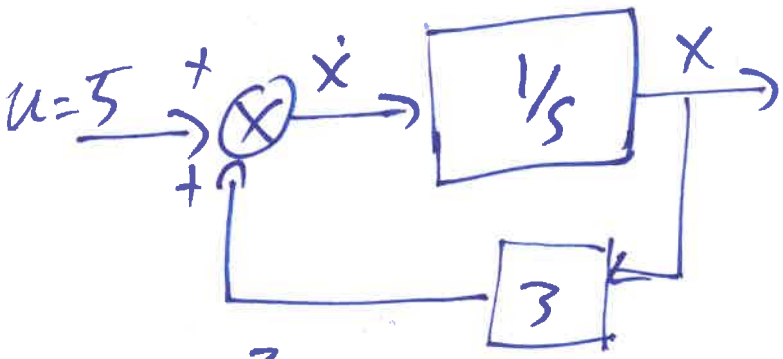
$$x = e^{-kt} \cdot \frac{u}{k} + \frac{u}{k} - \frac{u}{k} e^{-kt} = \frac{u}{k}$$

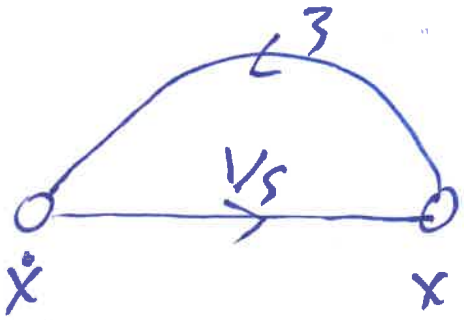
- Isolate the highest t. der.
- place as many int. as the order of OD
- Create the highest T.D.

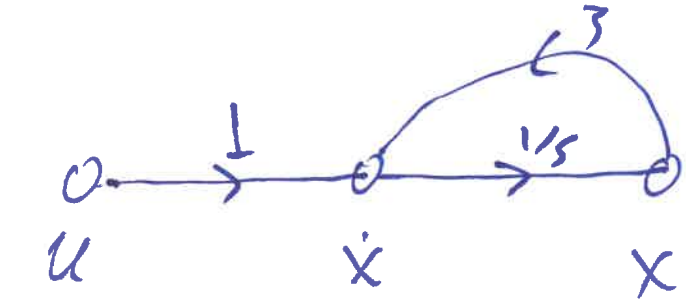
$\dot{X} = 3X$



$\dot{X} = 3X + 5$







$$\ddot{x} + A\dot{x} + Bx = u$$

$\downarrow u = 0$

$$\ddot{x} + A\dot{x} + Bx = 0$$

$\downarrow$ assume

$$x = e^{rt}$$
$$\dot{x} = r e^{rt}$$
$$\ddot{x} = r^2 e^{rt}$$

$$r^2 e^{rt} + A r e^{rt} + B e^{rt} = 0$$

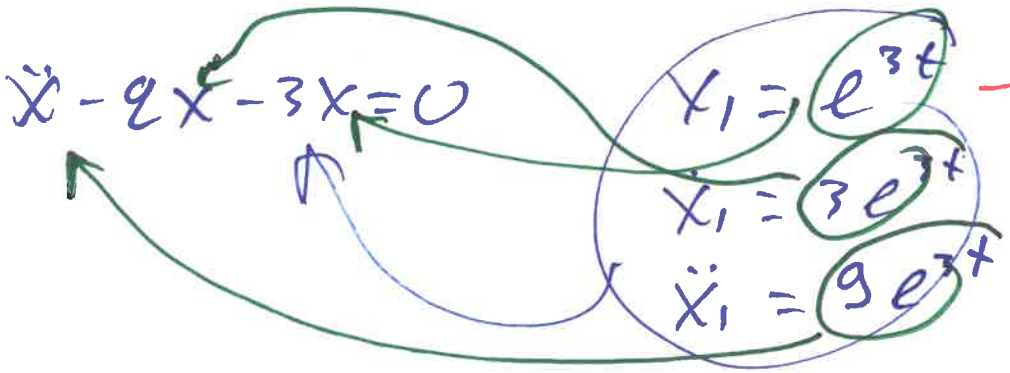
$$r^2 + Ar + B = 0$$

$$r_{1,2} = \frac{-A \pm \sqrt{D}}{2} \qquad D = A^2 - 4 \cdot B$$

- $D > 0 \Rightarrow r_1 \neq r_2 \in \mathbb{R}$
- $D < 0 \Rightarrow r_1 = \bar{r}_2 \in \mathbb{C}$
- $D = 0 \Rightarrow r_1 = r_2 = -A/2$

$$\ddot{x} + kx = 0$$

$$x = e^{rt} \cdot C.$$



→ a soln

$9e^{3t} - 2 \cdot 3 \cdot e^{3t} - 3e^{3t} = 0$

$0 = 0$

$x_2 = 10 \cdot x_1$ → a soln to ODE

So if x_1 is a soln, then $m \cdot x_1$ is also a soln.

$x_3 = e^{-t}$ → Another soln.

Is $x_4 = x_1 + 2x_2$ a soln

$\dot{x}_4 = \ldots$

$\ddot{x}_4 = \ldots$

} x_4 is a soln.

So if x_1, x_2 are soln ⇒ L.C. of x_1, x_2 is also a soln.

$y = c_1 \cdot x_1 + c_2 x_2$