$$x' + kx = u$$

$$x = e^{-kt} \cdot \left(x_0 + \int_0^t e^{kt_1} \cdot u \, dt_1 \right).$$

assume $u = 0$

$$x(t) = e^{-k \cdot t} \cdot x_0$$

- $k > 0$ $\qquad k = 3, \; x_0 = 1$

$x' + 3k = 0 \qquad x = e^{-3t} \cdot 1.$

$t = 0 \quad x(0) = e^{-3 \cdot 0} \cdot 1 = 1$

$t = 1 \quad x(1) = e^{-3 \cdot 1} \cdot 1 = 0.4$

$t = 2 \quad x(2) = e^{-6} \cdot 1 = 0.14.$

stable system

$\vdots$

$t = \infty \quad x(\infty) = 0$

- $k < 0$ $\qquad k = -3$

$x(0) = 1$

$x(1) = 2.71 \ldots$

$x(2) = 7.3$

$x(3) = 20$

unstable.

$\vdots$

$t = \infty \quad x(\infty) = \infty$

2

$$x = \underbrace{e^{-kt} \cdot (x_0}\ + \int_0^t e^{kt_1} \cdot \underset{\text{const.}}{\underset{\parallel}{u}} \cdot dt_1)$$

$$\int_0^t e^{kt_1} \cdot u\, dt_1$$

$$= u \cdot \int_0^t e^{kt_1}\, dt_1$$

$(e^{kt})' = k e^{kt}$

$$= u \cdot \int_0^t \left(\frac{1}{k} e^{kt}\right)' dt_1$$

$$= u \cdot \left[\frac{1}{k} e^{kt}\right]_0^t$$

$$= \frac{u}{k}\left(e^{kt} - e^{k \cdot 0}\right) = \frac{u}{k}\left(e^{kt} - 1\right) \Rightarrow$$

$$\boxed{x(t) = \frac{u}{k}\left(1 - e^{-kt}\right)}$$

$$\dot{x} + k\cancel{x} = 0$$

$$\hookrightarrow \boxed{x(t) = e^{-kt} \cdot x_0}$$

- $k > 0 \rightarrow$ stable
- $k < 0 \rightarrow$ unstable

$$\ddot{x} + a \cdot \dot{x} + bx = u$$

$$\ddot{x} + a\dot{x} + bx = 0$$

HOPE $x = e^{\text{something} \cdot t}$

$$\left.\begin{aligned} x &= e^{r \cdot t} \\ \dot{x} &= r \cdot e^{rt} \\ \ddot{x} &= r^2 \cdot e^{rt} \end{aligned}\right]$$

$$r^2 \cancel{e^{rt}} + a \cdot r \cdot \cancel{e^{rt}} + b \cancel{e^{rt}} = 0$$

$$r^2 + a \cdot r + b = 0$$

$\ddot{x} + 4 \cdot \dot{x} + 3x = 0 \qquad x_0 = 1 \quad x_0' = 0$

$r^2 + 4 \cdot r + 3 = 0$

$\Delta = 16 - 4 \cdot 3 = 2^2$

$r_{1,2} = \frac{-4 \pm 2}{2}$ → $r_1 = -3 \to e^{-3t}$; $r_2 = -1 \to e^{-t}$

$\boxed{x(t) = c_1 e^{-3t} + c_2 \cdot e^{-t}}$ G.S.

bcs $r_{1,2} < 0$ the system is stable.

$x(0) = c_1 \cdot e^{-3 \cdot 0} + c_2 e^{-0}$

$= \boxed{c_1 + c_2 = 1.}$ (1)

$x'(t) = -3 \cdot c_1 \cdot e^{-3t} - c_2 e^{-t}$

$x'(0) = \boxed{-3 \cdot c_1 - c_2 = 0}$ (2)

(1) + (2) $-2 \cdot c_1 + 0 = 1 \Rightarrow \boxed{c_1 = -0.5}$

$\Rightarrow \boxed{c_2 = 1.5.}$

$$r_{1,2} = \frac{-A \pm \sqrt{A^2 - 4B}}{2}$$

$$\boxed{A^2 = 4B}$$

$$r_{1,2} = -A/2. \quad \in \mathbb{R}.$$

$$x_1 = e^{-A/2 \cdot t} \qquad x_2 = e^{-A/2 \cdot t}$$

$$x = c_1 \cdot e^{-A/2 \cdot t} + c_2 e^{-A/2 \cdot t}.$$

$$x = \underbrace{(c_1 + c_2)}_{k} \cdot e^{-A/2 \cdot t}$$

$$\boxed{x = k \cdot e^{-A/2\, t}}$$

$$x_1 = e^{rt}, \quad x_2 = t \cdot e^{rt}$$

$$\boxed{x = c_1 e^{rt} + c_2 t e^{rt}}$$

$r > 0 \rightarrow$ unstable

$r < 0 \rightarrow$ stable

$$r_{1,2} = \frac{-A \pm \sqrt{A^2 - 4B}}{2}$$

$\cancel{A^2 - 4B < 0} \Longrightarrow r_{1,2} = -\frac{1}{2} \pm i$

$\cancel{x(t) = c_1 e^{(-\frac{1}{2}+i)} + c_2}$

$x_1 = e^{(-\frac{1}{2}+i)\cdot t}$ $\quad x_2 = e^{(-\frac{1}{2}-i)t}$

$$r_1 = \mathrm{Re} + \mathrm{Im}\cdot i$$

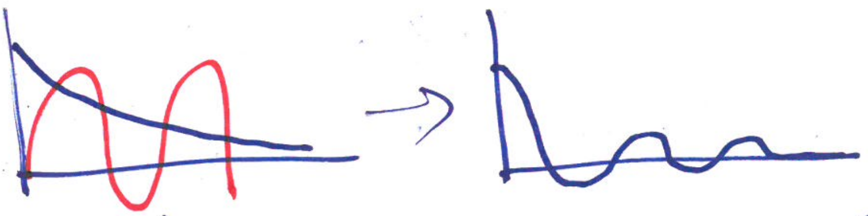
$$\begin{array}{cc} \downarrow & \downarrow \\ r_1 & r_2 \end{array}$$

$r = a + bi$

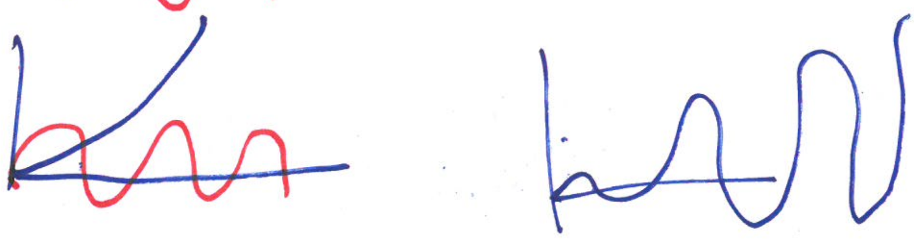
$x_1(t) = e^{at} \cdot \cos bt$

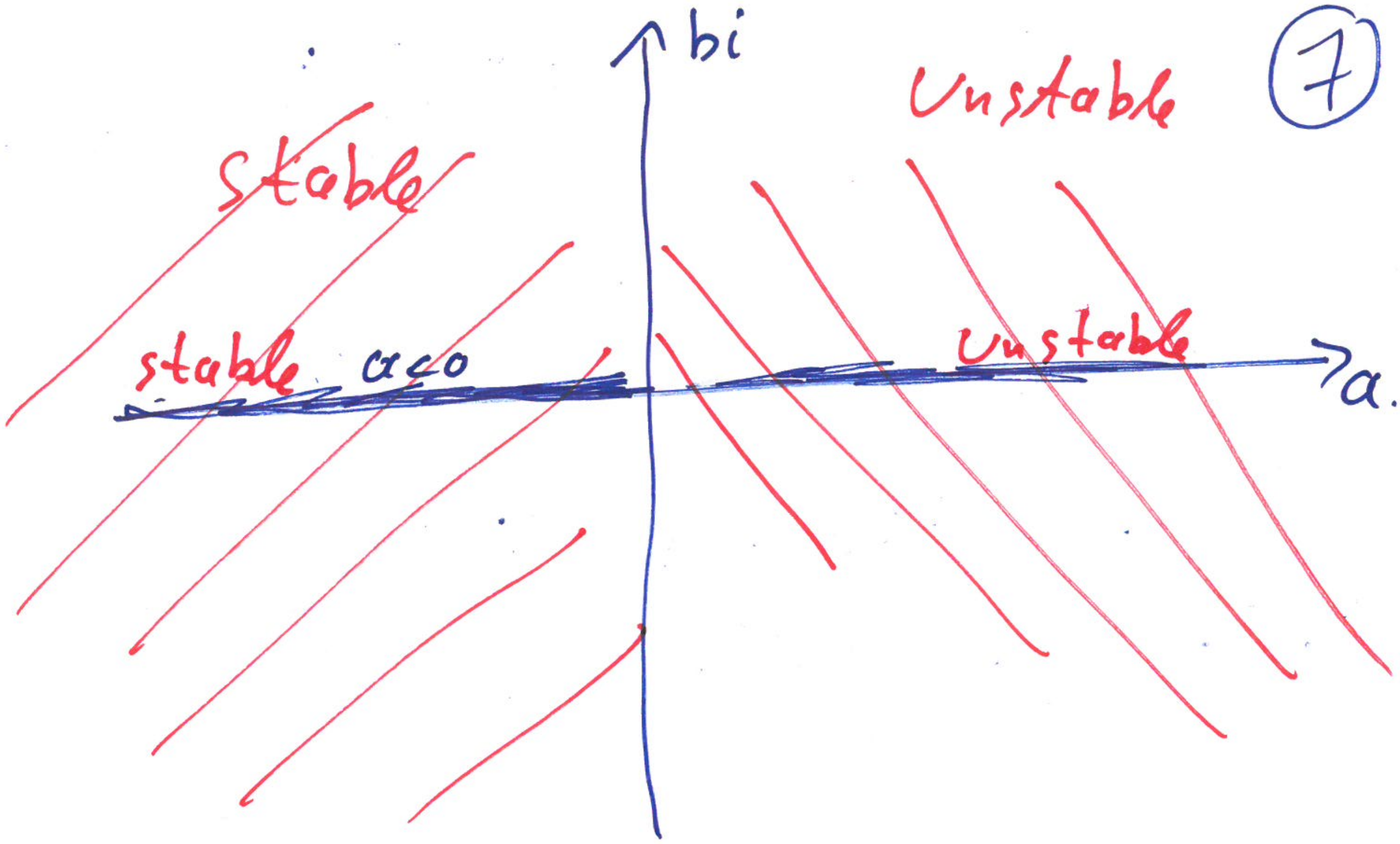
$x_2(t) = e^{at} \cdot \sin bt$

- $a < 0$



- $a > 0$





- $b=0$ real roots.
- $b\neq 0$ complex soln

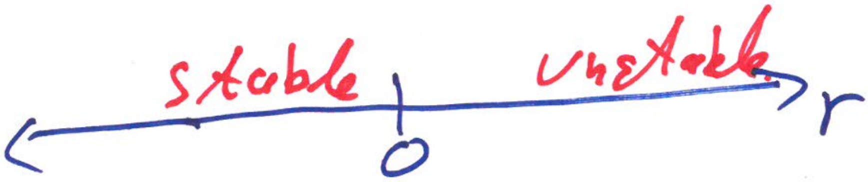
r- plane.

$r=a+bi$

$\Delta > 0 \quad \begin{cases} \to r_1 \\ \to r_2 \end{cases} \in \mathbb{R}.$

$r_1, r_2 < 0 \to$ stable.

r_1 or $r_2 > 0 \to$ unstable.



$\Delta = 0 \quad r = r_1 = r_2 \quad \in \mathbb{R}.$

$r < 0 \to$ stable

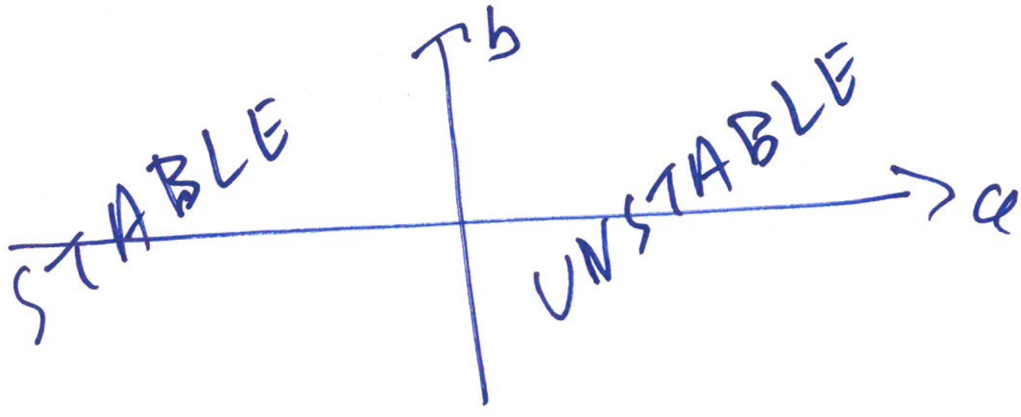
$r > 0 \to$ unstable.

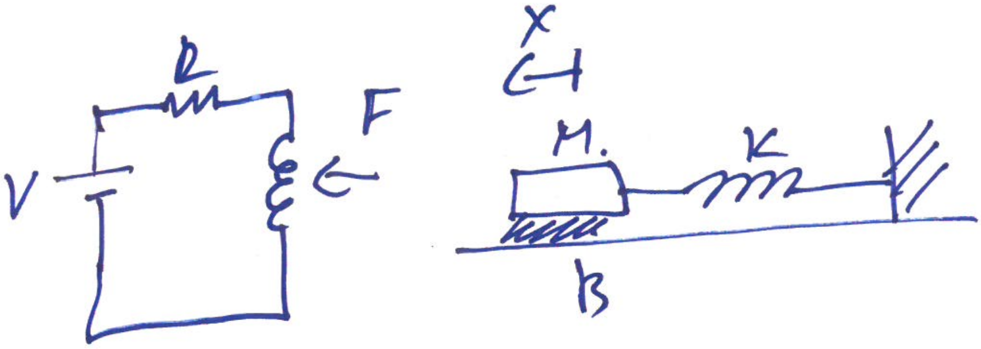
$\Delta < 0 \quad r = a + bi \quad e^{at} \cdot \cos bt$

$e^{at} \cdot \sin bt$

$a < 0 \to$ stable

$a > 0 \to$ unstable.





$V =$ Given $\quad x = ?$

$V = iR + L\, di/dt \qquad i = e^{-R/L\, t} i_0 + \int \ldots$

$u = k \cdot x + x'$

$f - B \cdot \dot{x} - k \cdot x = m \cdot \ddot{x} \qquad f = \cancel{k\, k_A}\, i^2$

$\underbrace{k_A \cdot i}_{u} = \underbrace{m \ddot{x} + kx + B \cdot \dot{x}}$

$$f(t) \rightarrow F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$$ 9

$s = a + bi$

$= \sigma + \omega i$

$$f(t) \rightarrow F(s)$$

e.g.

$$f(t) = 1 \rightarrow F(s) = 1/s$$

$$f(t) = t \rightarrow F(s) = 1/s^2$$

$$\dot{f}(t) \rightarrow s \cdot F(s)$$

$$f''(t) \rightarrow s^2 F(s)$$

$$\vdots$$

$$f^{(k)}(t) \rightarrow s^k F(s)$$

$$\dot{x} + 5 \cdot x = u$$

$\downarrow$ LT.

$x \rightarrow X(s)$

$u \rightarrow U(s)$

$\dot{x} \rightarrow s \cdot X(s)$

$$s \cdot X(s) + 5 \cdot X(s) = U(s)$$

$x'' + 5x' + 6x = U(t) + 3 \cdot U'(t)$

$\downarrow$ L.T.

$s^2 \cdot X(s) + 5 \cdot s \cdot X(s) + 6 \cdot X(s) = U(s) + 3 \cdot s \, U(s)$

$X(s) \cdot (s^2 + 5 \cdot s + 6) = U(s)(3s + 1)$

$$\frac{X(s)}{U(s)} = \frac{3 \cdot s + 1}{s^2 + 5s + 6}$$

zeros.

Transfer Function

poles

$x'' + 5x' + 6 \cdot x = 0$

$r^2 + 5r + 6 = 0$

Det. the stability

C. E.

$$V = iR + L\, di/dt$$

$$K_A\, i = m\ddot{x} + B\cdot\dot{x} + KX$$

$$V(s) = I(s)\cdot R + L\, I(s)\cdot s$$

$$V(s) = I(s)\cdot(Ls + R)$$

$$\frac{I(s)}{V(s)} = \frac{1}{Ls + R} \qquad \text{T.F.}$$

$$I(s) = \frac{V(s)}{Ls + R}$$

$$K_A\cdot I(s) = m\cdot s^2 X(s) + B\cdot s\, X(s) + K\cdot X(s)$$

$$K_A\cdot I(s) = X(s)\cdot(ms^2 + B\cdot s + K)$$

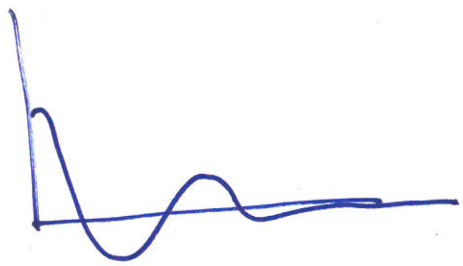
$$K_A \frac{V(s)}{Ls + R} = X(s)\,(ms^2 + Bs + K)$$

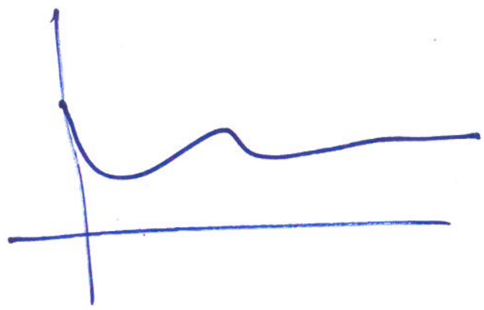
$$\frac{X(s)}{V(s)} = \frac{K_A}{(Ls + R)\cdot(ms^2 + Bs + K)}$$

To test the stability
find the poles of the C.E.

C.E. $(LS+R)\cdot(MS^2+BS+K)=0 \Rightarrow \dots$

.....





$f(t) \longrightarrow F(s).$

$$F_{FV} = \cancel{s}\,\lim_{s\to 0} sF(s).$$

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$$X(S) = V(S) \frac{KA}{(LS+R)(mS^2+BS+K)}$$

$v(t) = 1$ $\quad$ $V(S) = 1/S.$ $\quad$ $X_{SS} = ?$

$$X_{SS} = \lim_{S \to 0} S X(S)$$

$$= \lim_{S \to 0} S \frac{KA \; V(S)}{(LS+R)(mS^2+BS+K)}$$

$$= \lim_{S \to 0} \cancel{S} \frac{1}{\cancel{S}} \frac{KA}{(LS+R)(mS^2+BS+K)}$$

$$= \lim_{S \to 0} \frac{KA}{(LS+R)(mS^2+BS+K)}$$

$$= \frac{KA}{(0+R) \cdot (m \cdot 0 + B0 + K)}$$

$$= \frac{KA}{R \cdot K.}$$