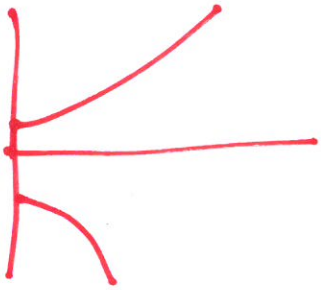
$\dot{x} + k\cdot x = u. \Rightarrow x(t) = e^{-kt}\cdot\left(x_0 + \int_0^t e^{kt_1}\cdot u\cdot dt_1\right)$

$u = \text{const.}$ $\quad x(t) = \frac{u}{k}\left(1 - e^{-kt}\right).$

- $k > 0$ $\quad e^{-kt} \to 0 \Rightarrow$ stable u/k

- $k < 0$ $\quad e^{-kt} \to \pm\infty$ unstable



$x'' + A\cdot x' + B\cdot x = u$

$x'' + Ax' + Bx = 0 \qquad x = e^{rt}$

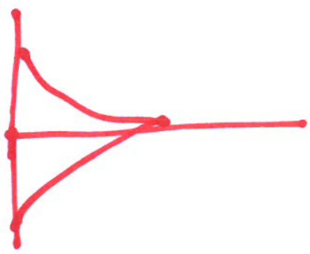
$r^2 + A\cdot r + B = 0 \qquad$ C.E.

$r_{1,2} = \frac{-A \pm \sqrt{\Delta}}{2}$

- $\Delta > 0$ $\quad r_1 \neq r_2 \in \mathbb{R}.$

  $x = \underline{C_1}\, e^{r_1 t} + \underline{C_2}\, e^{r_2 t}$

  r_1 **and** $r_2 < 0$ stable

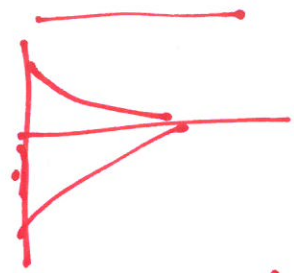


over.

$\Delta = 0 \quad r_1 = r_2 = r \Rightarrow x = c_1 e^{rt} + c_2 \cdot t\, e^{rt}$ (15)

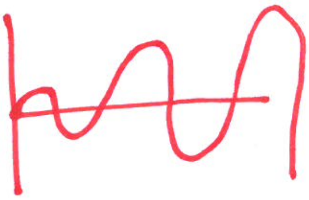
$r < 0 \Rightarrow$ stable

critically



$\Delta < 0 \quad r = \alpha \pm bi \Rightarrow x = e^{\alpha t} \cdot (c_1 \cos bt + c_2 \cdot \sin bt)$

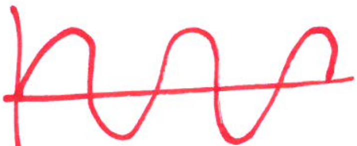
$\alpha > 0$ unst.



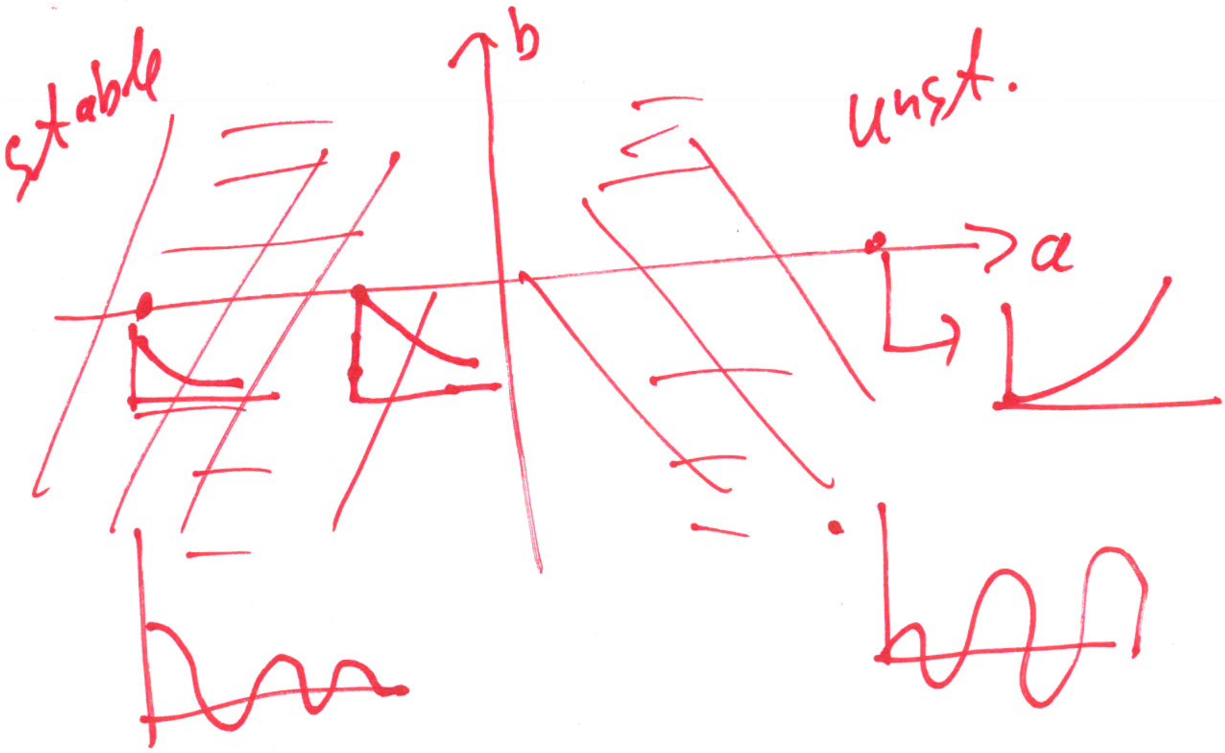
under.

$\alpha < 0$ stable

$\alpha = 0$



marginaly st.



T.F.

$$f(t) \underset{L.T}{\overset{I.L.T}{\rightleftarrows}} F(s)$$

$$\boxed{\begin{array}{l} f(t)=1 \rightarrow F(s)=1/s \\ f(t)=t \rightarrow F(s)=1/s^2 \end{array}}$$

$$f'(t) \rightarrow s\cdot F(s)$$

$$\vdots$$

$$f^{(k)}(t) \rightarrow s^k\, F(s)$$

$$f(t) \rightarrow f_{ss}=\lim_{t\to\infty} f(t)$$

$$\downarrow$$

$$F(s) \rightarrow F_{ss}=\lim_{s\to 0} s\cdot F(s)$$

$$x'''(t)+5\cdot x''+3\cdot x'+2x=u''(t)+3\cdot u'+5\cdot u.$$

$$u(t)=1 \rightarrow U(s)=1/s.$$

$$X(s)\left(s^3+5\cdot s^2+3s+2\right)=U(s)\left(s^2+3s+5\right)$$

$$\frac{X(s)}{U(s)}=\frac{s^2+3s+5 \;\rightarrow \text{zeros}}{s^3+5s^2+3s+2 \;\rightarrow \text{poles}}=G(s)$$

C.E. $s^3+5s^2+3s+2=0$

$s_1<0 \quad s_2<0 \quad s_3<0$

$$X_{ss}=\lim_{s\to 0} s\cdot X(s)=\lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{s^2+3s+5}{s^3+5s^2+3s+2}$$

$$=5/2=2.5$$

$$\xrightarrow{U} \boxed{G(s)} \xrightarrow{X}$$

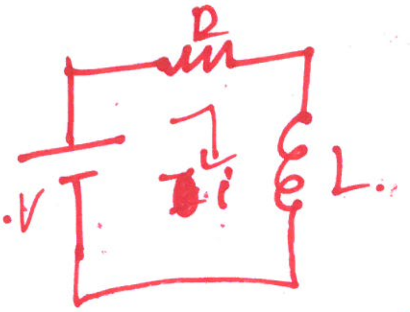
$$x = c_1 e^{-3t} + c_2 e^{-6t}$$

$$x = c_1 e^{-30t} + c_2 e^{-60t}$$

$$x = e^{-3t} \cdot (c_1 \cos 3t + c_2 \sin 3t)$$

$$x = e^{-30t} (c_1 \cos 3t + c_2 \sin 3t)$$

$$x = e^{-3t} (c_1 \cdot \cos 30t + c_2 \sin 30t)$$



$$V = iR + L\frac{di}{dt}$$

$$i(t) = \frac{V}{R}\left(1 - e^{-R/L\,t}\right)$$

$V = \text{const}$

$$i_{ss} = \lim_{t\to\infty} \frac{V}{R}\left(1 - e^{-R/L\,t}\right) = \frac{V}{R}(1-0) = \frac{V}{R}$$

$$V(s) = I(s)\cdot(Ls+R)$$

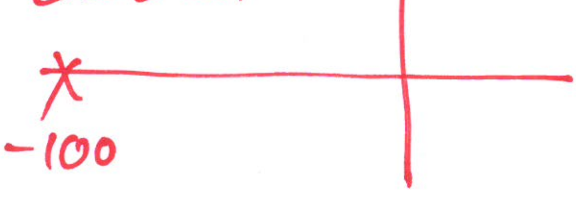
$$\frac{I(s)}{V(s)} = \frac{1}{Ls+R} = \cancel{\frac{1/L}{s}}$$

$$= \frac{1/R}{L/R\,s+1} = \frac{K}{\tau s+1}$$

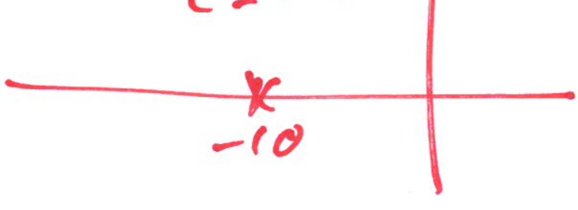
~~pole~~ = ?   C.E. $\tau s + 1 = 0$

pole $s = -1/\tau$.

$\tau = 0.01$



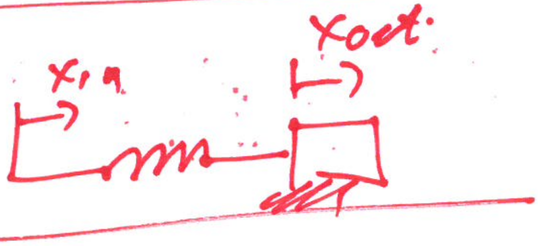
$\tau = 0.1$



$$I_{ss} = \lim_{s\to 0} s\, I(s) = \lim_{s\to 0} s\, V(s)\, \frac{1}{Ls+R}$$

$$= \lim_{s\to 0} s \left(\frac{V}{s}\right) \frac{1}{Ls+R}$$

$$= V/R.$$



$$k(x_{in} - x_{out}) - B\dot{x}_{out} = m \cdot \ddot{x}_{out}$$

$$x_{in}(t) \rightarrow X_{in}(s)$$

$$x_{out}(t) \rightarrow X_{out}(s)$$

$$\dot{x}_{out} \rightarrow s \cdot X_{out}(s)$$

$$\ddot{x}_{out} \rightarrow s^2 X_{out}(s)$$

$$k\left(X_{in}(s) - X_{out}(s)\right) - B \cdot s \cdot X_{out}(s) = m \cdot s^2 X_{out}(s)$$

20

C.E. $s^2 + 2\cdot z\cdot w_n\cdot s + w_n^2 = 0$

$$s_{1,2} = \frac{-2\cdot z\cdot w_n \pm \sqrt{4\cdot z^2 w_n^2 - 4 w_n^2}}{2}$$

$$= -z\cdot w_n \pm \frac{\sqrt{4\cdot w_n^2\cdot(z^2-1)}}{2}$$

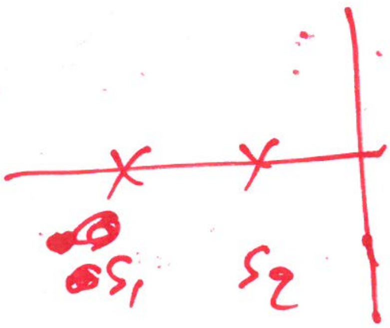
$$= -z\cdot w_n \pm w_n\sqrt{z^2-1}$$

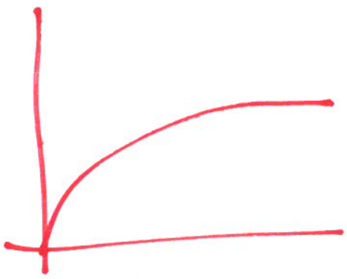
1) $z^2-1>0$ over

$$s_1 = -z\cdot w_n + w_n\sqrt{z^2-1}$$

$$s_2 = -z w_n - w_n\sqrt{z^2-1}$$

$s_1, s_2 < 0$ stable





2) $\cancel{z}$ $z^4 - 1 = 0$ $z = \pm 1$ (21)

$s_1 = s_4 = -u_n$.

$z = -1$.

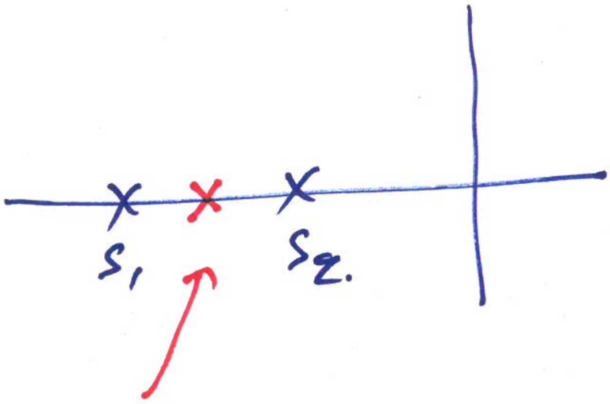
$s_1 = s_2 = u_n$

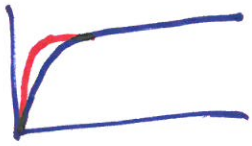
C.E. $s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^2 = 0$

$$s_{1,2} = \frac{-2\cdot\zeta\cdot\omega_n \pm 2\omega_n\sqrt{\zeta^2-1}}{2}$$

$$= -\zeta\cdot\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

- $\zeta > 1$

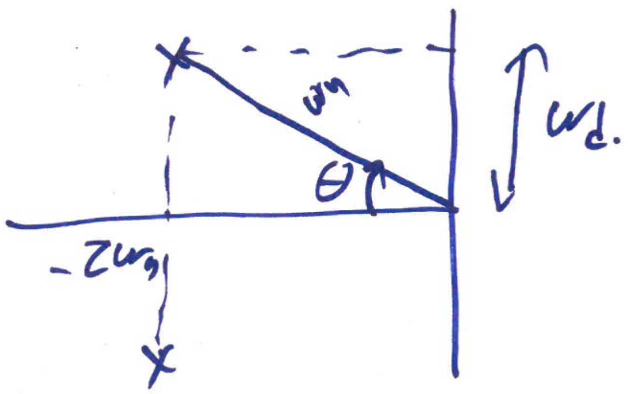




- $\zeta = 1$ $\quad s = -\omega_n$

- $\zeta \in (0,1)$ $\quad s_{1,2} = -\zeta\cdot\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$$s_{1,2} = -\zeta\cdot\omega_n \pm j\omega_d$$



$\omega_d = \omega_n\sqrt{1-\zeta^2}$

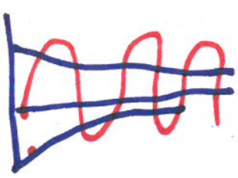
$$d = \sqrt{(\zeta\omega_n)^2 + \omega_d^2}$$

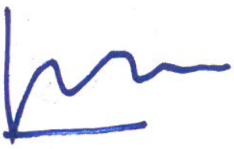
$$= \sqrt{\zeta^2\cdot\omega_n^2 + \omega_n^2(1-\zeta^2)}$$

$$= \sqrt{\cancel{\zeta^2\cdot\omega_n^2} + \omega_n^2 - \cancel{\omega_n^2\zeta^2}}$$

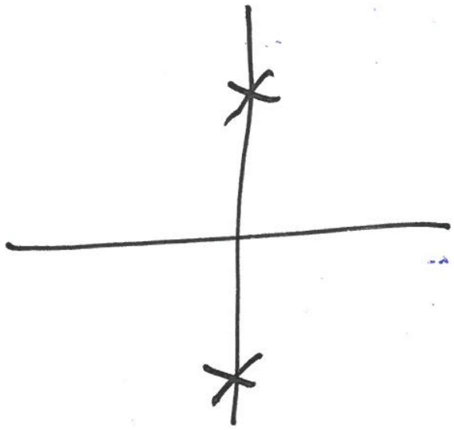
$$= \omega_n$$

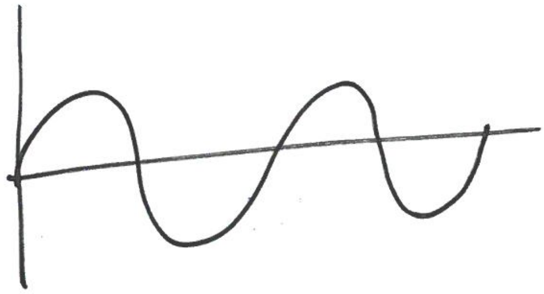
$\cos\theta = \dfrac{\zeta\cdot\omega_n}{\omega_n} = \zeta.$





• $\zeta = 0$ $\qquad s_{1,2} = \pm \omega_n j$





$$G(s) = \frac{3s+1}{s^2+5s+1}$$

C.E. $\qquad s^2 + 5 \cdot s + 1 = 0$

C.E. $\qquad s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2 = 0$

$$5 = 2 \cdot \zeta \cdot \omega_n$$

$$\omega_n^2 = 1 \Rightarrow \omega_n = 1 \text{ rad/s}$$

$$5 = 2 \cdot \zeta \cdot 1 \Rightarrow \zeta = 5/2 = 2.5$$

Over

(24)

$$G(s)=\frac{3s+1}{s^2+s+5}=\frac{Y(s)}{X(s)} \qquad X(s)=1/s$$

C.E. $\quad s^2+s+5=0$

$$2\cdot z\cdot \omega_n = 1$$

$$\omega_n^2=5 \Rightarrow \omega_n=\sqrt{5}\simeq 2.23 \text{ rad/s}$$

$$2\cdot z\cdot 2.23 = 1 \Rightarrow z = 0.223$$

$$t_p=\frac{\pi}{\omega_d} \qquad \omega_d=\omega_n\sqrt{1-z^2}=2.179 \text{ rad/s}$$

$$t_p=\frac{3.14}{2.179}\simeq 1.44 \text{ s}$$

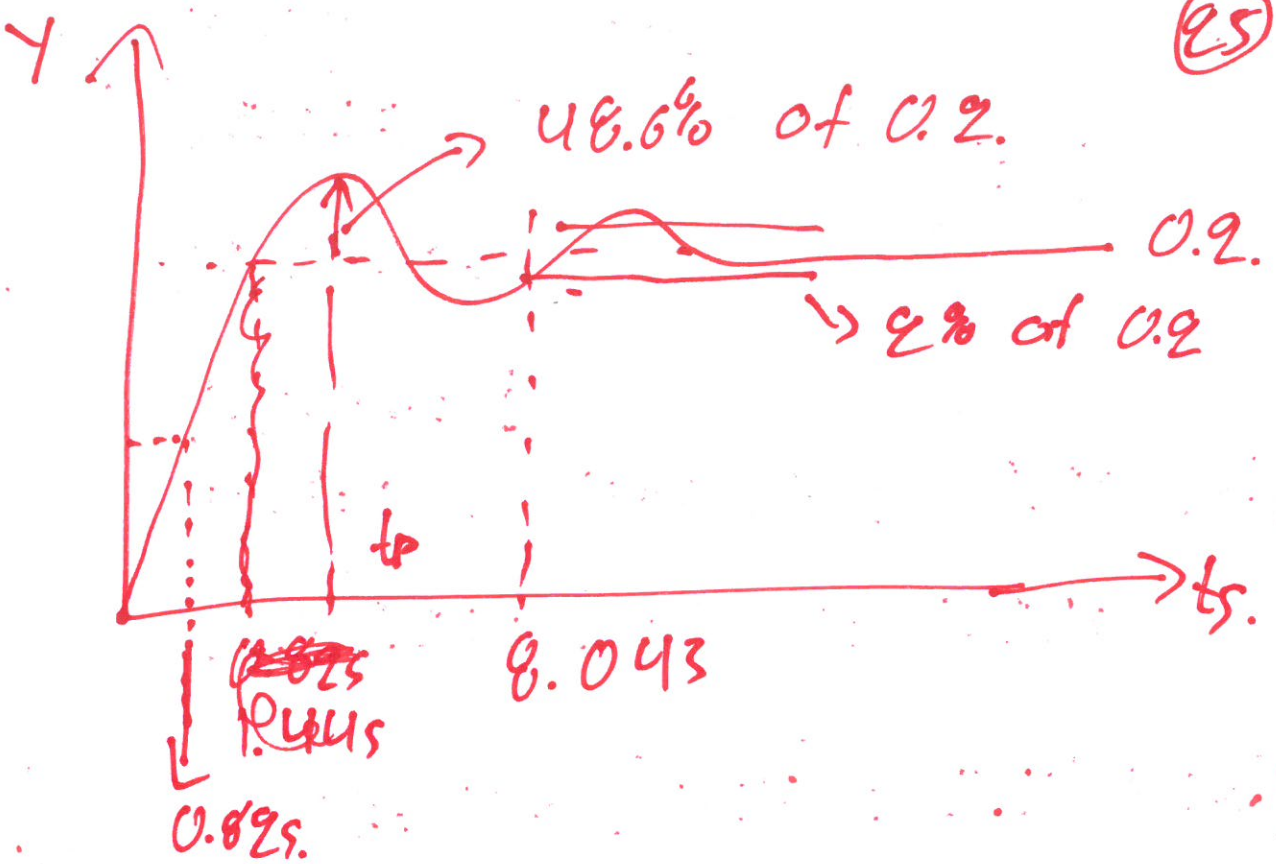
$$t_r=\frac{\pi-\theta}{\omega_d}\simeq 0.82 \text{ s}$$

$M_p = 0.486$ or 48.6%

$$t_{s_{2\%}} = 8.043 \text{ s}$$

$$Y(s)=\frac{1}{s}\cdot\frac{3s+1}{s^2+s+5}$$

$$Y_{ss}=\lim_{s\to 0}\cancel{s}\,\cancel{\frac{1}{s}}\,\frac{3s+1}{s^2+s+5}=\frac{1}{5}=0.2$$



25
Y
48.6% of 0.2.
0.2.
2% of 0.2
tp
ts.
8.043
1.444s
0.82s.

Newcastle University

Supplementary Answer Paper

Student Number..

Number of questions only to be written in this margin

26

$$G(S) = \frac{3S+1}{S^2+S+1} = \frac{Y}{X}$$

$$\left.\begin{aligned} &C.E. \quad S^2+S+1=0 \\ &S^2+2 \cdot Z \cdot w_n S+1=0 \end{aligned}\right\} \Rightarrow$$

$$w_n = 1 \text{ rad/s}$$

$$Z = 0.5$$

$$t_p = \pi / w_d \qquad w_d = w_n \sqrt{1-Z^2}$$

$$= 1 \cdot \sqrt{1-0.5^2}$$

$$= 0.86 \text{ rad/s.}$$

$$t_p = \frac{3.14}{0.86} \simeq 3.63 \text{ s.}$$

$$M_p = \exp\left(-\frac{Z \cdot \pi}{\sqrt{1-Z^2}}\right)$$

$$= \exp\left(\frac{-0.5 \cdot 3.14}{\sqrt{1-0.5^2}}\right) = \ldots = 16.4\%$$

Newcastle University

Supplementary Answer Paper

Student Number.......................................

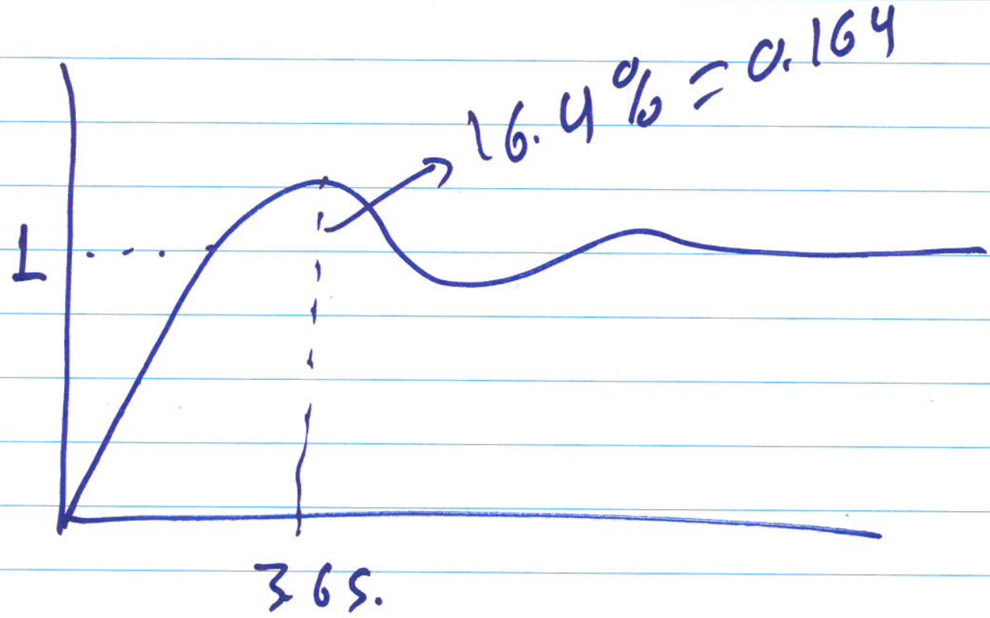
Number of questions only to be written in this margin

Q7

$$Y(s) = X(s)\,\frac{3s+1}{s^2+s+1}$$

Given $X(s) = 1/s$

$$Y_{ss} = \lim_{s\to 0} \cancel{s}\,\frac{1}{\cancel{s}}\,\frac{3s+1}{s^2+s+1} = 1$$



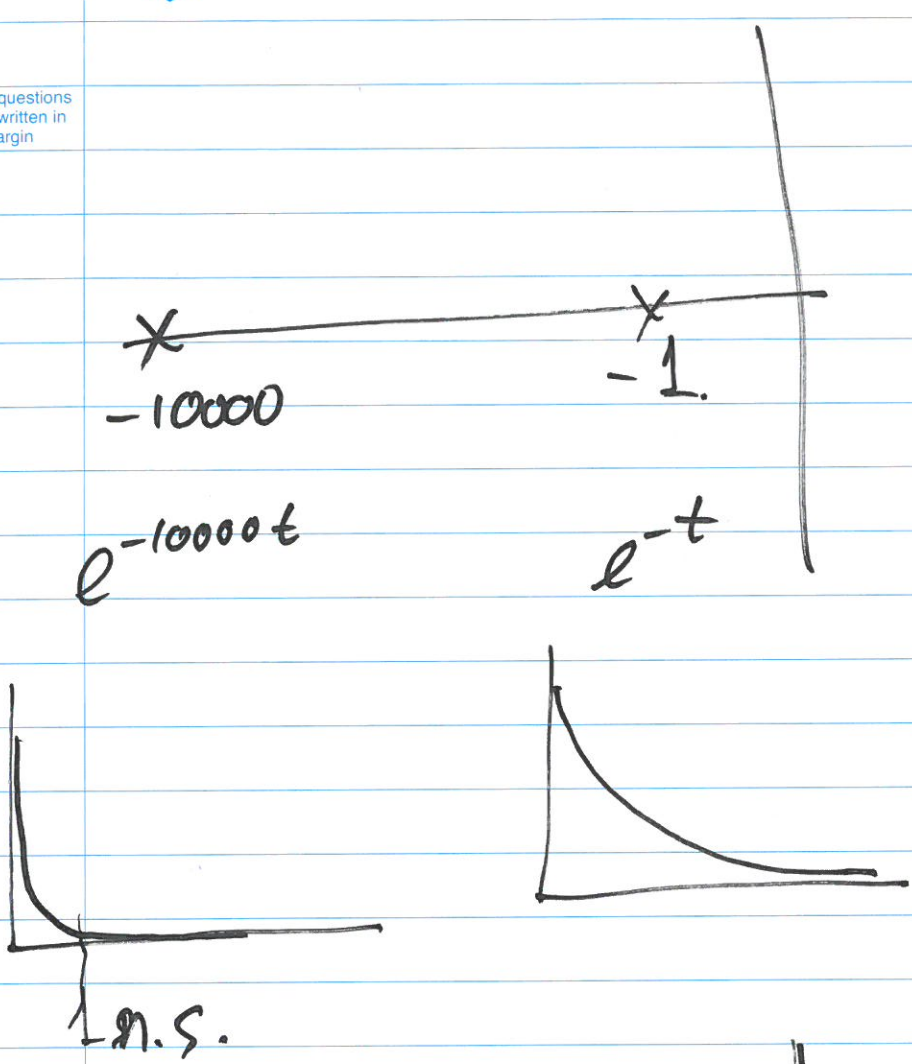
Newcastle University

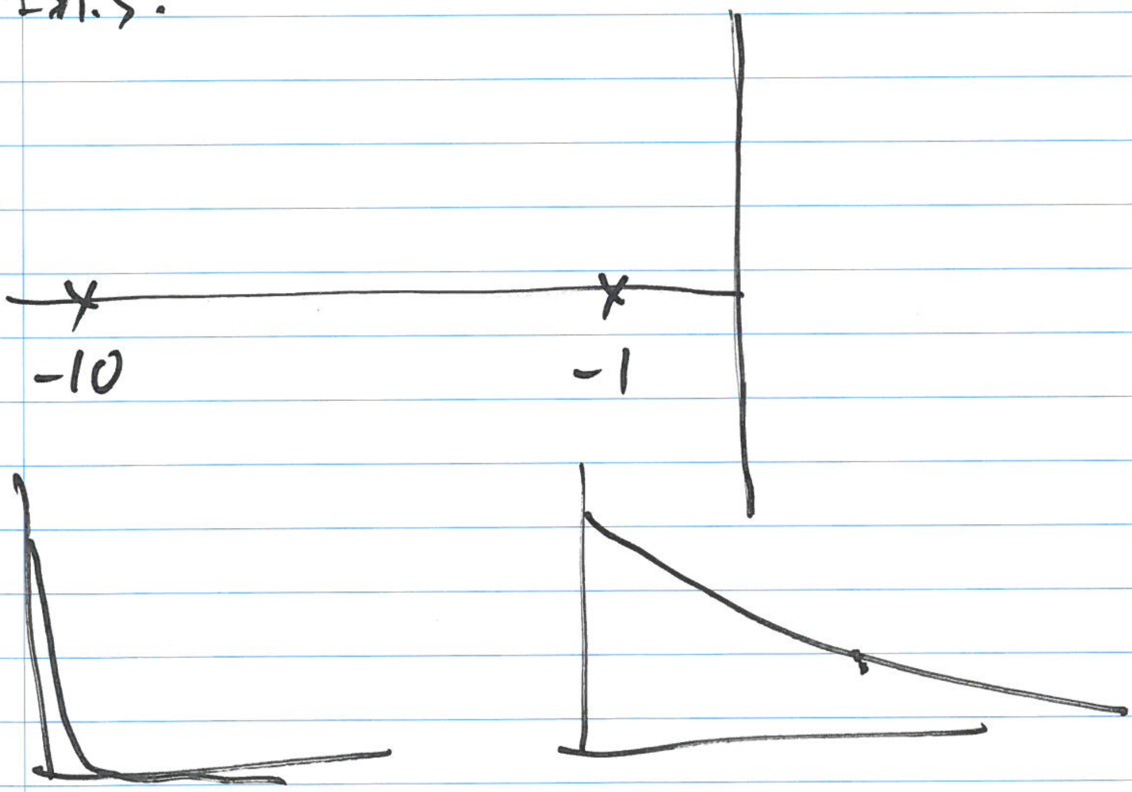
Supplementary Answer Paper

Student Number....................................

28

Number of questions only to be written in this margin





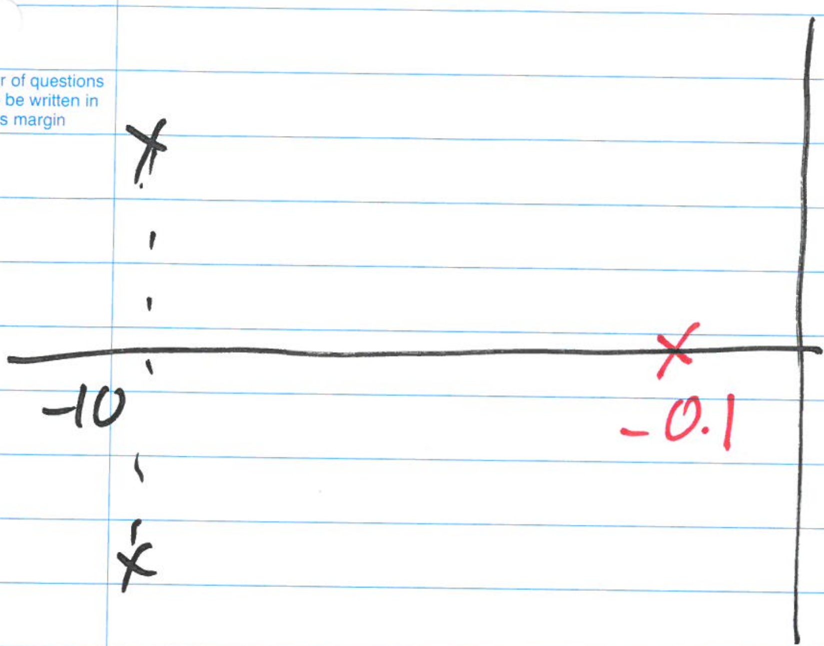
Newcastle University

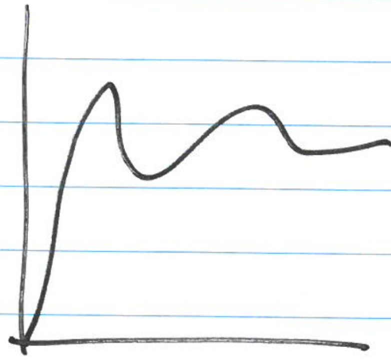
Supplementary Answer Paper

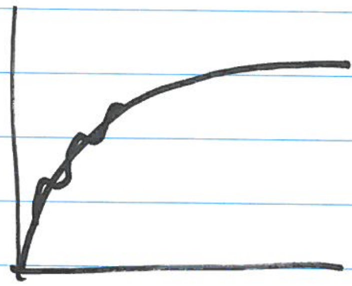
Student Number..

Number of questions only to be written in this margin

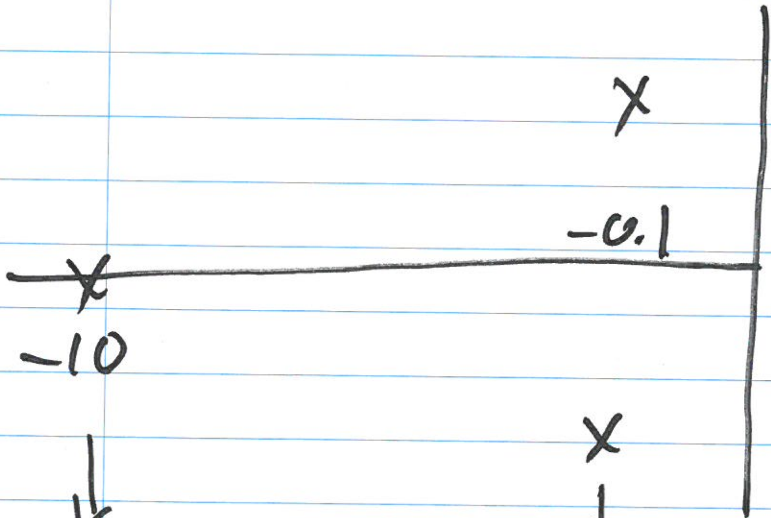
29







$e^{-10t}\cdot\cos(\ )$ $e^{-0.1t}$



e^{-10t} $e^{-0.1t}\cdot\cos(\ )$

Newcastle University

Supplementary Answer Paper

Student Number.......................................

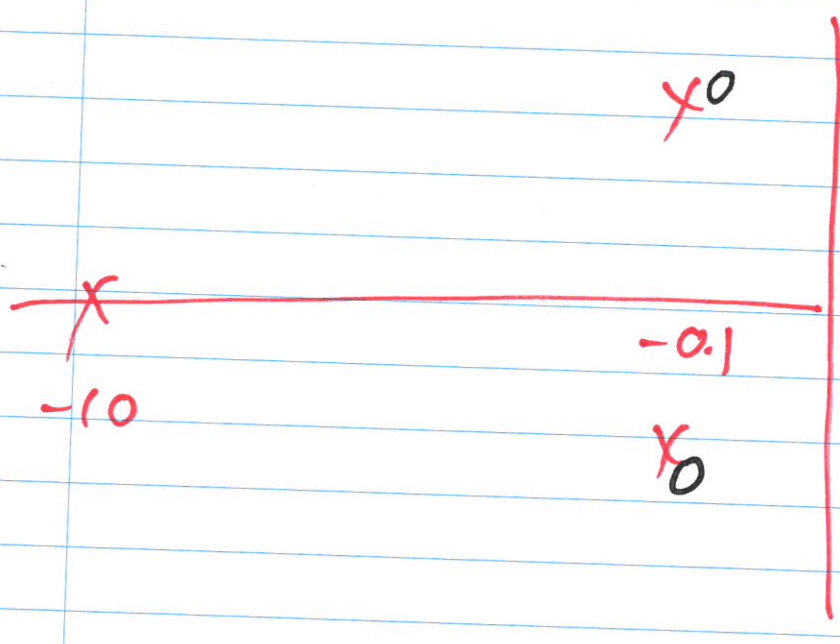
Number of questions only to be written in this margin

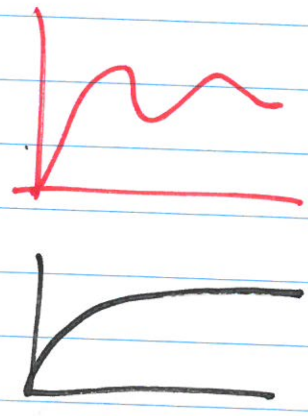
30

$$G_1(s) = \frac{1}{s^2+s+1}$$

$$G_2(s) = \frac{\cancel{(s+1)}}{(\cancel{s+1}.0001)(s^2+s+1)}$$

0000
0000





$$G_1(s) = \frac{1}{s+10}$$

$$G_2 = \frac{s^2+s+1.0001}{(s^2+s+1)(s+10)}$$

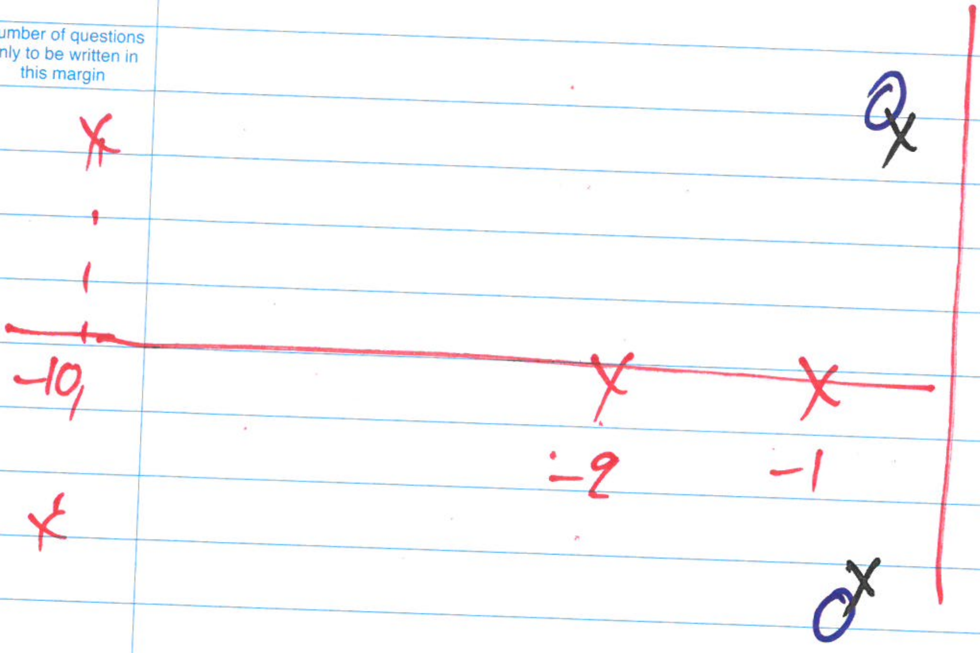
Newcastle University

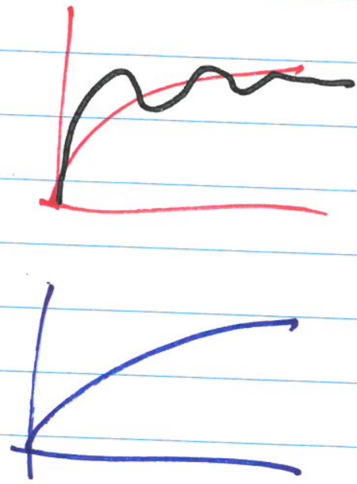
Supplementary Answer Paper

Student Number...

31

Number of questions only to be written in this margin



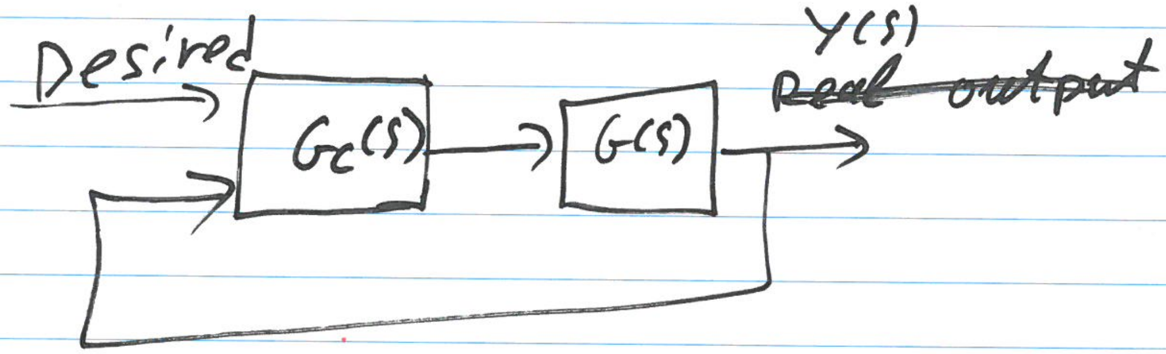


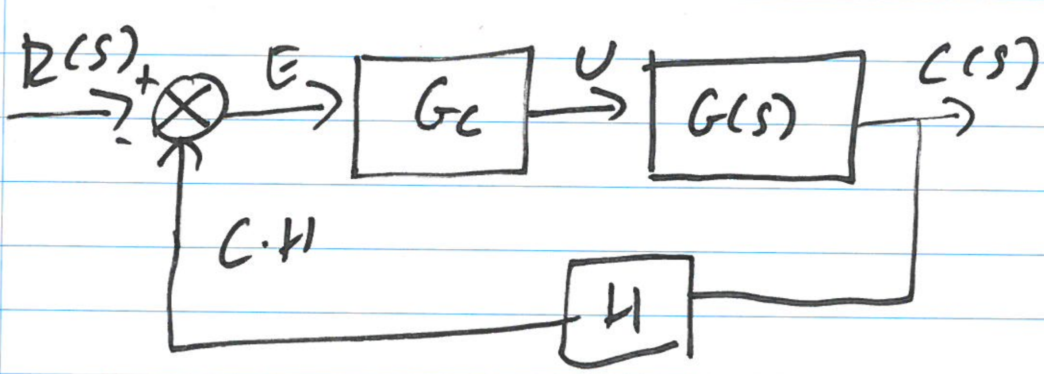
Newcastle University

Supplementary Answer Paper

Student Number: 39

Number of questions only to be written in this margin





$$E = R - C \cdot H$$

$$U = E \cdot G_c \qquad \frac{C(s)}{R(s)} = ???$$

$$C = G \cdot U \Rightarrow C = G \cdot E \cdot G_c.$$

$$C = G \cdot G_c \left(R - CH\right)$$

$$C = G \cdot G_c \cdot R - G \cdot G_c \cdot C \cdot H$$

$$C \cdot \left(1 + G \cdot G_c \cdot H\right) = G \cdot G_c \cdot R.$$

$$\frac{C}{R} = \frac{G \cdot G_c}{1 + G \cdot G_c \cdot H}$$

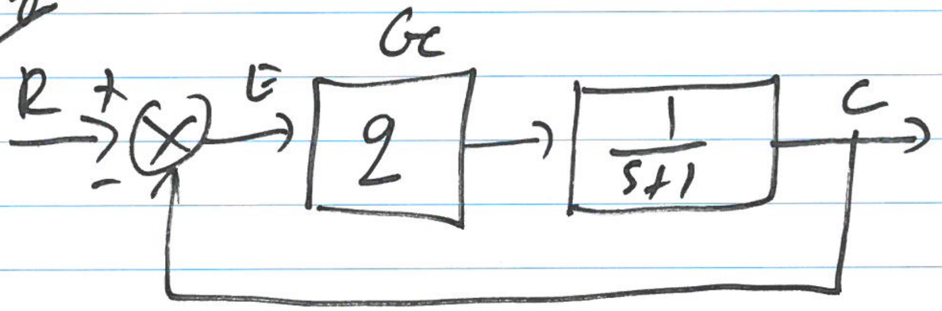
C.L.T.F.

$G(s) = \frac{1}{s+1}$ O.L.C.E. $s+1=0$

$s=-1$

stable

e.g.



e^{-t}

$$C/R = \frac{G \cdot G_c}{1+G \cdot G_c} = \frac{2 \cdot \frac{1}{s+1}}{1+2\frac{1}{s+1}} = \frac{2}{s+1+2}$$

$$= \frac{2}{s+3}$$

C.L.C.E. $s+3=0$

$s=-3$

stable

e^{-3t} 3 times faster

Newcastle University

Supplementary Answer Paper

Student Number: 35

Number of questions only to be written in this margin

$$C = R \cdot G_{CL}(s)$$

$$= \frac{1}{s} \frac{K}{s+2+K}$$

$$C_{ss} = \lim_{s \to 0} \cancel{s} \frac{1}{\cancel{s}} \frac{K}{s+2+K} = \frac{K}{2+K}$$

~~• K = 0: $C_{ss} = 1/2$~~

• $K = 1$ $\quad C_{ss} = \frac{1}{2+1} = 1/3$

• $K = 10$ $\quad C_{ss} = \frac{10}{12}$

• $K = 1000$ $\quad C_{ss} = \frac{1000}{1002} \simeq 1$

Newcastle University

Supplementary Answer Paper

Student Number 34

Number of questions only to be written in this margin

$$G_{OL}(s) = \frac{1}{s+2}.$$

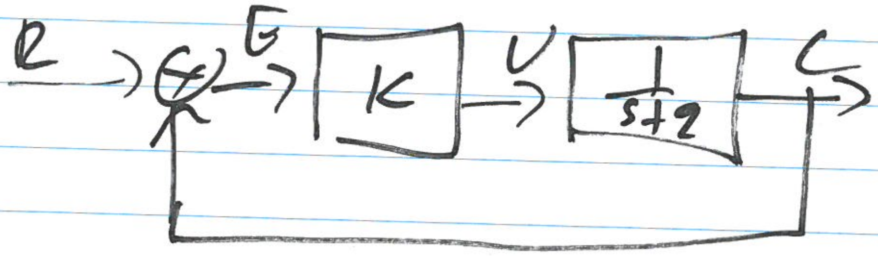
$$r(t) = 1$$
$$R(s) = 1/s$$

$$\frac{Out}{In} = \frac{1}{s+2}$$

$$Out = In \cdot \frac{1}{s+2}$$

$$= \frac{1}{s} \frac{1}{s+2}.$$

$$Out_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \frac{1}{s+2} = 1/2 = 0.5$$



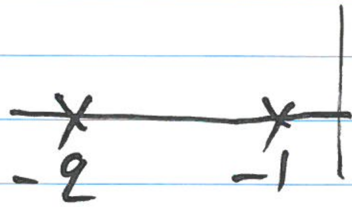
$$G_{CL}(s) = \frac{G \cdot G_c}{1 + G G_c} = \frac{K \; 1/(s+2)}{1 + K \frac{1}{s+2}} =$$

$$= \frac{K}{s+2+K}.$$

C.L.C.E $\quad s + 2 + k = 0$

$$s = -2 - k$$

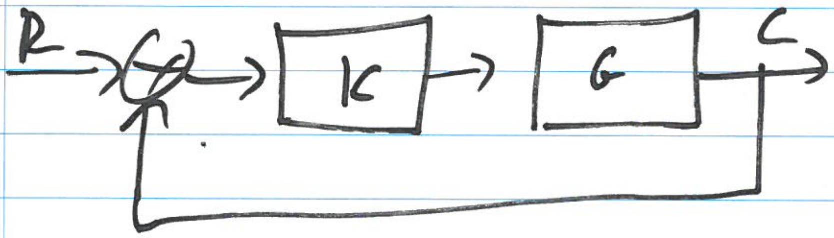
$$G(s) = \frac{1}{(s+1)(s+2)}$$



O.L.C.E. $(s+1)(s+2) = 0$

$s_1 = -1 \quad , \quad s_2 = -2$

$e^{-t} \qquad e^{-2t}$



$$G_{OL} = \frac{K}{(s+1)(s+2)}$$

C.L.T.F. $G_c(s) = \dfrac{K}{(s+1)(s+2)+K}$

C.L.C.E. $s^2 + 3s + 2 + K = 0$

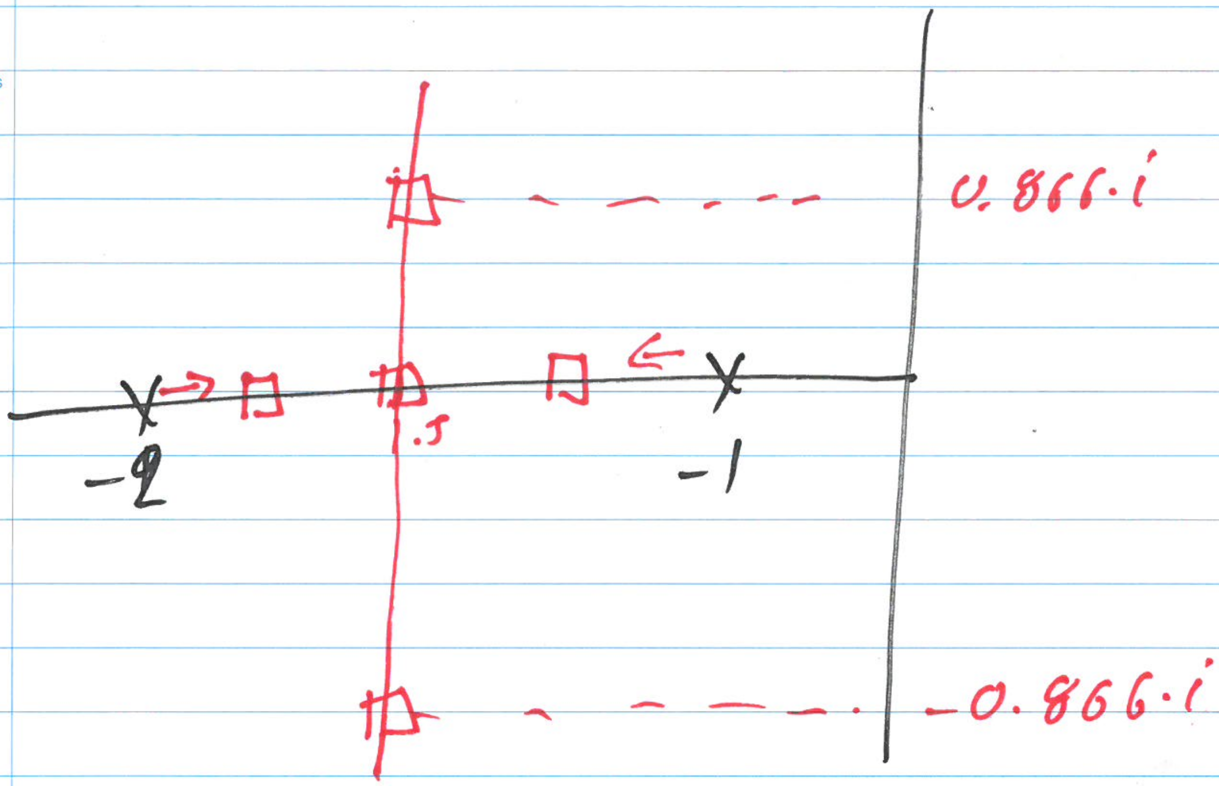
- $K = 0$ $\quad s^2 + 3s + 2 = 0 \rightarrow s_1 = -1$, $s_2 = -2$
- $K = 0.1$ $\quad s^2 + 3s + 2 + 0.1 = 0 \rightarrow s_1 = -1.11$, $s_2 = -1.88$
- $K = 0.25$ $\quad s^2 + 3s + 2 + 0.25 = 0$ $\quad s_1 = s_2 = -1.5$
- $K = 1$ $\quad s^2 + 3s + 2 + 1 = 0 \rightarrow s = -1.5 \pm 0.86 i$

Newcastle University

Supplementary Answer Paper

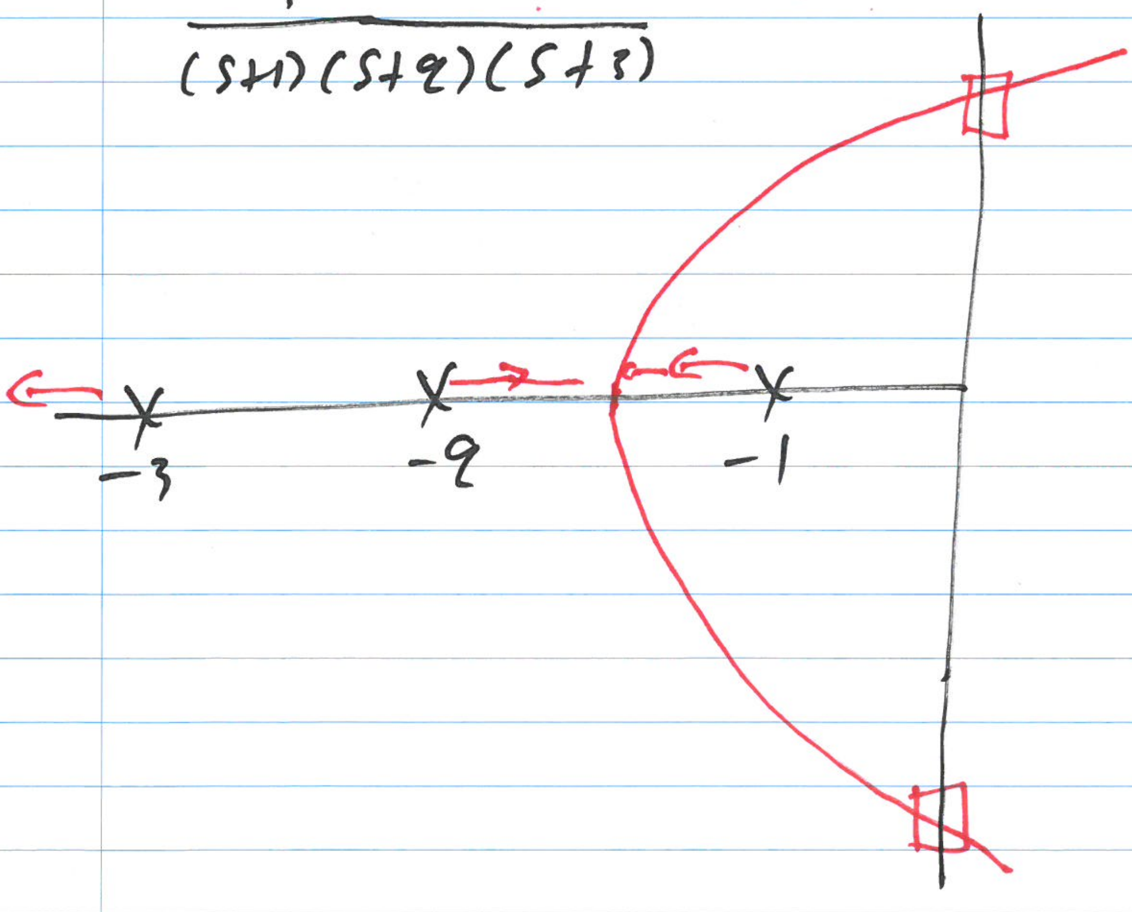
Student Number 36

Number of questions only to be written in this margin



$e^{-\zeta \cdot \omega_n t}$

$$\frac{1}{(s+1)(s+2)(s+3)}$$



Newcastle University

Supplementary Answer Paper

Student Number: 37.

Number of questions only to be written in this margin

$$G_{OL}(s) = \frac{1}{(s+1)(s+2)}$$

$z > 1$

$\omega_n = \sqrt{12}$

~~$z = 0.433$~~

$$G_{CL}(s) = \frac{k}{(s+1)(s+2)+k.}$$

C.L.T.E $\quad s^2 + 3s + (2+k) = 0$

G.C.E $\quad s^2 + 2 \cdot 2 \cdot \omega_n \cdot s + \omega_n^2$

$\omega_n^2 = 2+k$

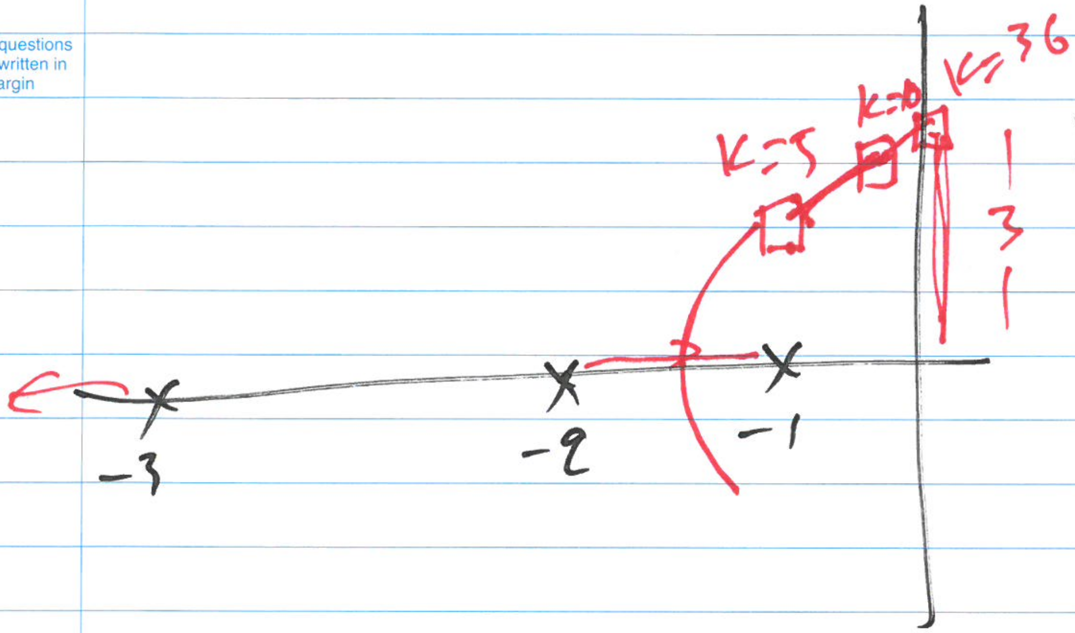
$12 = 2+k \Rightarrow \boxed{k=10}$

Newcastle University

Supplementary Answer Paper

Student Number 38

Number of questions only to be written in this margin



$K=0 \quad S_1 = -1$

$S_2 = -2$

$S_3 = -3$

$S_1 = Re_1 + i \cdot Im_2$

$S_2 = Re_1 - i\, Im_2$

$S_3 = Re_3$

5

$S_1 = Re_2 + Im_2 \cdot i$

$S_3 = \ldots\ldots$

10

$S_1 = a_1 + i\, w_1$

C.E: $(s+1)(s+9)(s+3)+k=0$

$s^3+5.s^2+9.s+9+k=0$

At M.S. case $s=j\omega$

$s^3=-j\omega^3$ $\quad s^2=-\omega^2$

$-j\omega^3+5(-\omega^2)+9.j\omega+9+k=0$

$=0+0j$

*

$-5.\omega^2+9+k=0$

$-\omega^3+9.\omega=0$

*

$5(-\omega^2)+9+k+i(-\omega^3+9.\omega)=0+0i$

$-\omega^3+9\cdot\omega=0$

$\cdot\ \omega=0 \qquad \cdot\ \omega\neq 0 \quad -\omega^2+9=0$

$\omega^2=9$

$\omega=3$ rad/s.

$-5\cdot9+9+k=0 \Rightarrow$

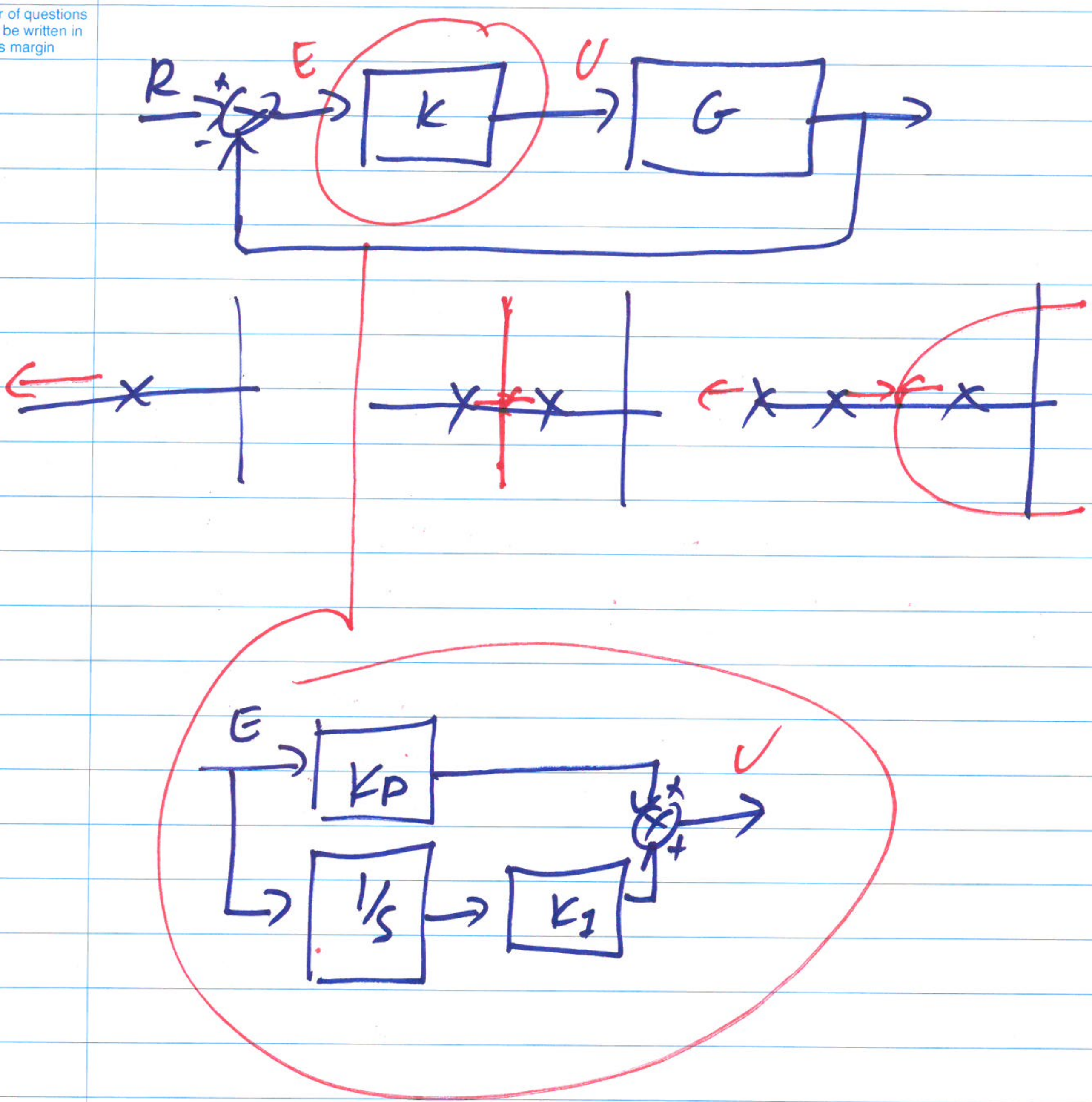
$\boxed{k=36}$

Newcastle University

Supplementary Answer Paper

Student Number: 40

Number of questions only to be written in this margin



Newcastle University

Supplementary Answer Paper

Student Number 41

$$G_c(s) = K_p$$

$$G_c(s) = K_p + K_I \frac{1}{s}$$

$$G(s) = \frac{1}{s+1}$$

$$G_{OL} = \left(K_p + K_I/s\right) \cdot \frac{1}{s+1}$$

$$= \frac{K_p s + K_I}{s} \frac{1}{s+1} = \frac{K_p s + K_I}{s(s+1)}$$

$$\frac{C}{R} = G_{CL}(s) = \frac{G_{OL}}{1+G_{OL}} = \frac{K_p s + K_I}{s(s+1) + K_p s + K_I}.$$

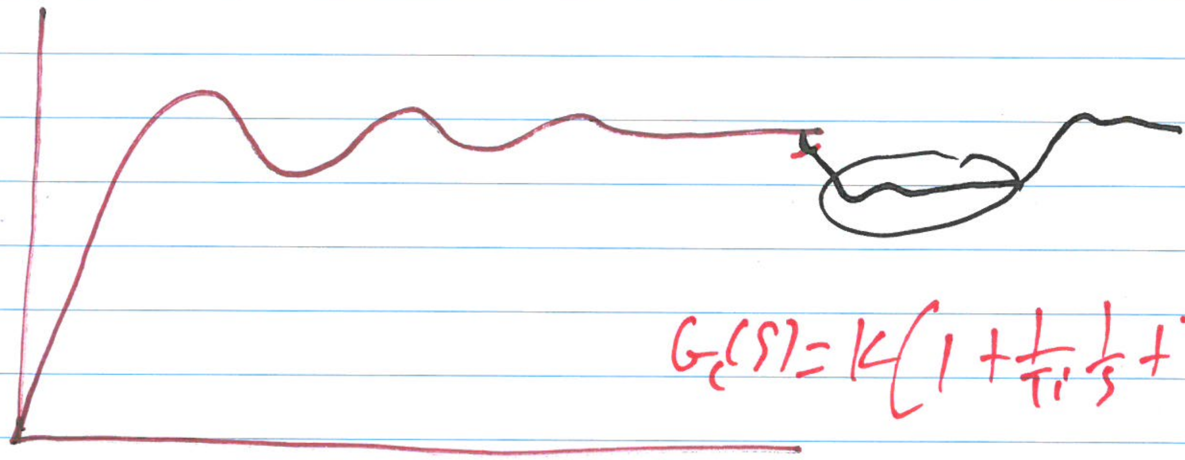
$$R(s) = 1/s$$

$$C_{ss} = \lim_{s \to 0} s \, R(s) \frac{K_p s + K_I}{s(s+1) + K_p s + K_I}$$

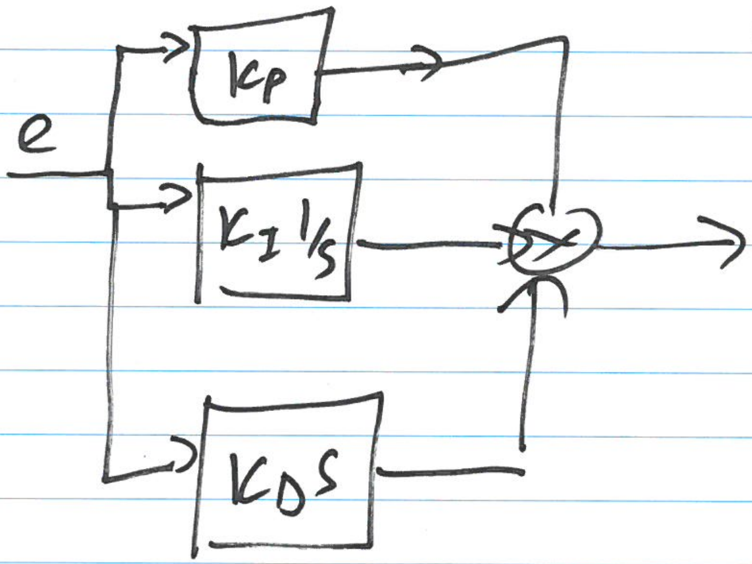
$$1/s$$

$$= \frac{0 + K_I}{0 + 0 + K_I} = \frac{K_I}{K_I} = 1.$$

Number of questions only to be written in this margin



$G_c(s) = K\left(1 + \frac{1}{T_i}\frac{1}{s} + T_d\right.$



$G_c = K_p + K_I \frac{1}{s} + K_D s$

$= K_p\left(1 + \frac{K_I}{K_p}\frac{1}{s} + \frac{K_D}{K_p} s\right)$

$\frac{1}{T_i}$ T_D

$G(s) = \frac{1}{(s+1)(s+9)(s+3)}$ $G_c(s) = ?$

~~G_{CLOOP}~~ =

$K_{cr} = 36$

$w = 3\,rad/s$

$P_{cr} = ?$

$K_p = 0.6\,K_{cr}$
$= 0.6 \cdot 36$
$= 21.6$

$w = \frac{2\pi}{T}$

$\boxed{P_{cr} = \frac{2\pi}{3} = \ldots}$

Newcastle University

Supplementary Answer Paper

Student Number........43........

Number of questions only to be written in this margin