Revision

$\dot{x} = Ax$

Good

$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \quad \lambda_1 = a, \lambda_2 = b$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{at} & 0 \\ 0 & e^{bt} \end{bmatrix}$$

$$A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} \rightarrow \lambda_1 = \lambda_2 = a$$

$$e_1 = [1 \quad 0]^T$$

$$e^{At} = \begin{bmatrix} e^{at} & te^{at} \\ 0 & e^{at} \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \rightarrow \lambda_1 = \lambda_2 = a.$$

$$e_1 = [1 \quad 0]^T, \quad e_2 = [0 \quad 1]^T$$

$$e^{At} = \begin{bmatrix} e^{at} & 0 \\ 0 & e^{at} \end{bmatrix}$$

Gave me one extra L. I. eigv.

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \rightarrow \lambda = a \pm bi$$

$$e_1 = [1 \quad j]^T, \quad e_2 = [1 \quad -j]^T$$

$$e^{At} = e^{at} \cdot \begin{bmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$ 3 L.I. eigv.

$$A = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$ 2 L.I. eigv.

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$$ 2 L.I. eigv.

$$A = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$$ 1 L.I. eigv.

$$A = \begin{bmatrix} a & b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}$$

$$e_1 = \begin{bmatrix} i \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \bar{e}_1 = \begin{bmatrix} -i \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ i \\ -1 \end{bmatrix}, \quad e_4 = \bar{e}_3$$

$$A = \begin{bmatrix} a & b & 1 & 0 \\ -b & a & 0 & 1 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix} = \begin{bmatrix} B & I \\ 0 & B \end{bmatrix}$$

$$e_1 = \begin{bmatrix} i \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} -i \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} -i \\ 1 \\ 0 \\ 0 \end{bmatrix}. \quad e_4 = \overline{e_3}$$

$$P_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \qquad P_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 5 \\ 0 & 0 & -5 & -3 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 5 & 0 & 0 \\ 0 & 0 & -5 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 5 \\ 0 & 0 & 0 & 0 & -5 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left.\begin{array}{l}\dot{x} = Ax + Bu \\ y = C\cdot x + Du.\end{array}\right\} \Rightarrow \text{Better}$$

$\cancel{B \in \mathbb{R}^{p\times 1}}$

$u \in \mathbb{R}^{p\times 1}$

$B \in \mathbb{R}^{n\times p}$

$$\underset{n\times 1}{x} = \overset{n\times n}{T}\cdot \underset{n\times 1}{z} \rightarrow \dot{x} = T\cdot \dot{z}$$

$$z = \underbrace{T^{-1}}_{\text{inv}}\cdot x \rightarrow \dot{z} = T^{-1}\cdot \dot{x}$$

$$T\cdot \dot{z} = A\,T\cdot z + B\cdot u$$

$$\dot{z} = \underbrace{T^{-1}\cdot A\cdot T}_{\substack{n\times n \\ \hat{A}}}\cdot z + \underbrace{T^{-1}\cdot B}_{\substack{n\times p \\ \hat{B}}}\,u.$$

$$\boxed{\dot{z} = \hat{A}z + \hat{B}\cdot u}$$

$$y = C\cdot x + D\cdot u$$

$$= \underbrace{C\cdot T}_{\hat{C}}\cdot z + \underbrace{D}_{\hat{D}}\cdot u$$

$$\boxed{y = \hat{C}\cdot z + \hat{D}\cdot u}$$

New S.S. model

$$e^{At} \rightarrow ?\ e^{\hat{A}t}$$

$\hat{A} = T^{-1} \cdot A \cdot T$ $\qquad$ $\hat{B} = T^{-1} \cdot B$ $\qquad$ $\hat{C} = C \cdot T$

$T \cdot \hat{A} \cdot T^{-1} = A$ $\qquad$ $T \cdot \hat{B} = B$ $\qquad$ $\hat{C} \cdot T^{-1} = C$

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$$

$$= I + T \cdot \hat{A} T^{-1} \cdot t + \frac{(T\hat{A} \cdot T^{-1})^2}{2!} t^2 + \dots$$

$$(T\hat{A}T^{-1})^2 = T \cdot \hat{A} \cdot T^{-1} \cdot T \cdot \hat{A} \cdot T^{-1} = T \cdot \hat{A}^2 \cdot T^{-1}$$

$$\vdots$$

$$(T\hat{A}T^{-1})^k = T \cdot \hat{A}^k \cdot T^{-1}$$

$$e^{At} = I + T \cdot \hat{A} \cdot T^{-1} \cdot t + \frac{T\hat{A}^2 \cdot T^{-1}}{2!} t^2 + \dots$$

$\downarrow$

$T \cdot I \cdot T^{-1}$

$$= T \cdot I \cdot T^{-1} + T \cdot \hat{A} \cdot t\, T^{-1} + T\left(\frac{1}{2!}\hat{A}^2 \cdot t^2\right) T^{-1} + \dots$$

$$= T \cdot \left(I + \hat{A} t + \frac{1}{2!}\hat{A}^2 \cdot t^2 + \frac{1}{3!}\hat{A}^3 \cdot t^3 + \dots\right) T^{-1}$$

$$e^{At} = T \cdot e^{\hat{A}t} \cdot T^{-1}$$

$$e^{\hat{A}t} = T^{-1} \cdot e^{At} \cdot T$$

$$G_A = C \cdot (sI - A)^{-1} \cdot B$$

$$G_{\bar{A}} = \bar{C} \cdot (sI - \bar{A})^{-1} \cdot \bar{B}$$

$$G_A = C \cdot I \cdot (sI - A)^{-1} \cdot I \cdot B$$

$$= \underbrace{C \cdot T}_{\bar{C}} \cdot \underbrace{T^{-1} (sI - A)^{-1} \cdot T}_{} \cdot \underbrace{T^{-1} B}_{\bar{B}}$$

If

$$(sI - \bar{A})^{-1}$$

$$(ABC)^{-1} = C^{-1} \cdot B^{-1} \cdot A^{-1}$$

$$(T \cdot C \;) T^{-1})^{-1} = T^{-1} (\;)^{-1} T$$

$$(T (sI - A) T^{-1})^{-1} = T^{-1} \cdot (sI - A)^{-1} \cdot T$$

$$(T \cdot sI - T \cdot A) T^{-1}$$

$$(s \underbrace{T \cdot T^{-1}}_{I} - T A T^{-1})$$

$$(sI - \bar{A})$$

$$A \xrightarrow{\lambda_1 \neq \lambda_2} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$A \xrightarrow[e_1,\ e_2]{\lambda = \lambda_1 = \lambda_2} \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

$$A \xrightarrow[e_1]{\lambda = \lambda_1 = \lambda_2} \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$$

$$A \xrightarrow[\lambda = a \pm bi]{} \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$X = T \cdot Z$$

$T = ?$

$$(A - \lambda I) \cdot e = 0$$

$$Ae = \lambda e$$

- $\lambda_1 = a$
  $\lambda_2 = b$
  $a \neq b$

$\downarrow$

e_1, e_2 are LI

$T = [e_1 \ \ e_2]$

is inv.

$$\left.\begin{aligned} &\cdot\ A e_1 = \lambda_1 e_1 \\ &\cdot\ A e_2 = \lambda_2 \cdot e_2 \end{aligned}\right\} A[e_1 \ \ e_2] =$$

$$= [e_1 \ \ e_2] \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$A \cdot T = T \cdot \Lambda$$

$$\boxed{A = T \cdot \Lambda \cdot T^{-1}}$$

$\hat{A}$

$A \to \lambda_1 = \lambda_2 = \lambda \to e_1$

$$\downarrow$$

$$\hat{A} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \qquad Ae = \lambda \cdot e$$

$$T = [e_1 \quad \cancel{e_2}] \quad \ldots\ldots \quad \cancel{T^{-1}}$$

$$e = (A - \lambda I) \cdot b$$

$$e = A \cdot b - \lambda b$$

$$Ab = e + \lambda b.$$

$$\boxed{\begin{aligned} Ae &= \lambda e \\ Ab &= e + \lambda b \end{aligned}}$$

$$A[e \quad b] = [\lambda e \quad e + \lambda b]$$

$$[\lambda e + 0 \cdot b \quad 1 \cdot e + \lambda b]$$

$$T = [e \quad b] \qquad = [e \quad b] \cdot \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

$$\boxed{AT = T \cdot \hat{A}}$$

$$A \to \lambda = a \pm bi$$

$$\downarrow$$

$$\hat{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$\lambda_1 = a + bi \qquad \lambda_2 = a - bi$$

$$\downarrow$$

$$\underbrace{e_1}_{2\times 1} = [v + wi], \quad e_2 = [v - wi]$$

$$A e_1 = \lambda e_1$$

$$A \cdot [v + wi] = (a + bi) \cdot (v + wi)$$

$$= av + a \cdot wi + bvi - bw$$

$$Av + Awi = (av - bw) + (aw + bv) \cdot i$$

$$A \cdot v = a \cdot v - b \cdot w \qquad Aw = a \cdot w + bv.$$

$$[Av \quad Aw] = [av - bw \quad aw + bv]$$

$$A \cdot \underbrace{[v \quad w]}_{T} = [v \quad w] \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$AT = T \cdot \hat{A}$$

$\boxed{T = \text{iuv} \; ???}$

$\dot{x} = Ax + Bu \qquad x = T \cdot z.$

$\downarrow$

$\bar{A} \qquad e^{At} = T \cdot e^{\bar{A}t} \cdot T^{-1}$

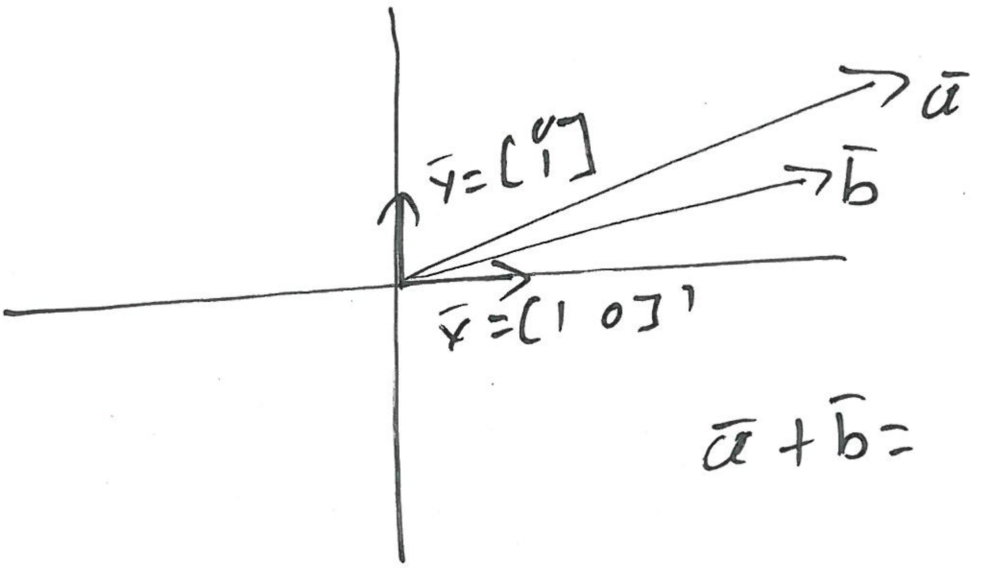
- $\lambda_1 \neq \lambda_2 \rightarrow T = [e_1 \quad e_2]$

$$\bar{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$e^{\bar{A}t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

$$e^{At} = T \cdot e^{\bar{A}t} \cdot T^{-1}$$

Vector Space



$\bar{a} = k_1 \cdot \bar{x} + k_2 \cdot \bar{y}$

$\bar{a} + \bar{b} =$

~~Def~~.

A set of vectors → vector space

$\bar{a} \cdot 1 = \bar{a}$

$\bar{a} + 0 = \bar{a}$

$\bar{a} + \bar{b} =$ vector.

$\bar{x}, \bar{y}$ are subset of → Basis

$\bar{a} = k_1 \cdot \bar{x} + k_2 \cdot \bar{y}$

$$\bar{a} = \begin{bmatrix} 10 \\ 5 \end{bmatrix} \Rightarrow k_1 = 10, \quad k_2 = 5 \qquad \bar{x} = [1\ \ 0]^T, \quad y = [0\ \ 1]^T$$

$$\bar{x} = [1\ \ 0]^T \qquad \bar{z} = [0\ \ -3]$$

$$\begin{bmatrix} 10 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

② $p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $q = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

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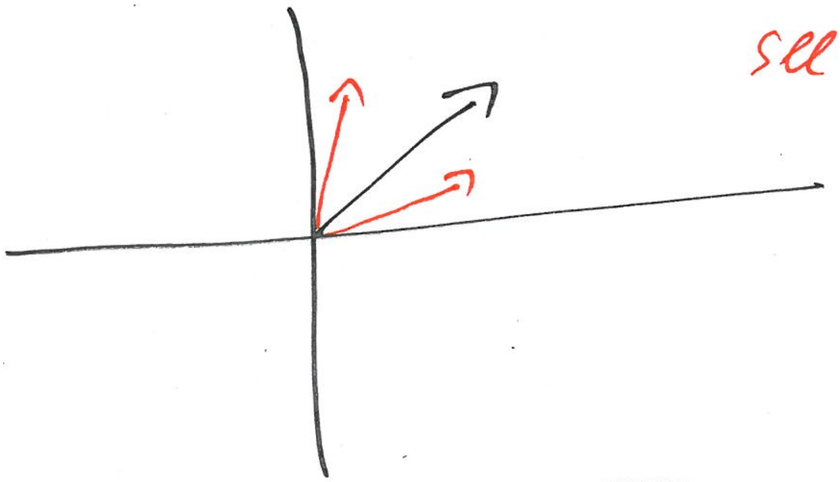
$$\begin{bmatrix} 10 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + k_2 \cdot \begin{bmatrix} 5 \\ -3 \end{bmatrix} \Rightarrow k_1 = \dots \quad k_2 = \dots$$

$p = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $q = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 10 \\ 5 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + k_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$



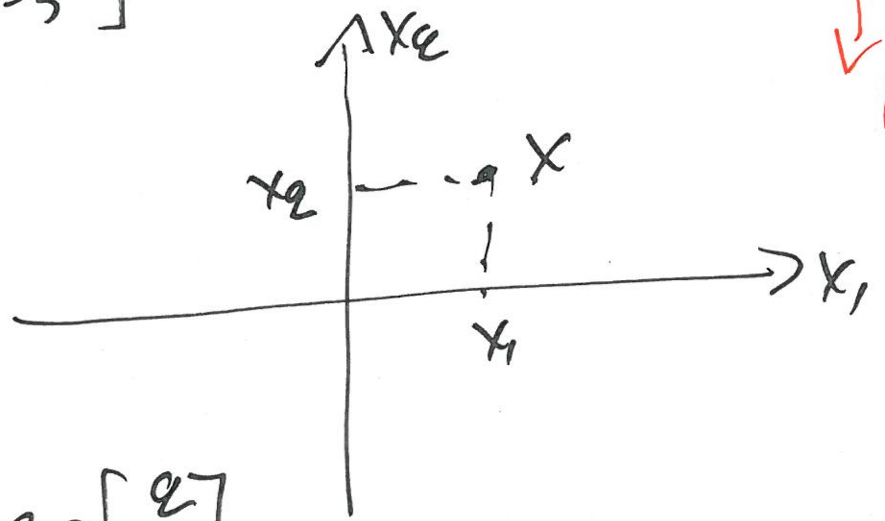
see next week

$V = \{x^2, 3x^2, 3.1x^2 \dots \}$

all 2nd order poln. with zero $\cdot x$ + zero

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} x. \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Basis



$$\lambda_1 = -1 \Rightarrow e_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda_2 = -6 \Rightarrow e_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x(t) = \underbrace{c_1\, e^{-t}}_{a(t)} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \underbrace{c_2 \cdot e^{-6t}}_{b(t)} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x(t) = a(t) \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b(t) \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$