Revision

$$\dot{x} = Ax + Bu \qquad y = C\cdot x$$

$$\left.\right\} \; x = T\cdot z \;\Rightarrow\; T^{-1}\exists$$

$$\dot{z} = \hat{A}\cdot z + \hat{B}\cdot u \qquad y = \hat{C}\cdot z$$

$$\hat{A} = T^{-1} A\cdot T \Rightarrow A = T\hat{A}T^{-1}$$

$$\hat{B} = T^{-1} B \Rightarrow B = T\cdot\hat{B}$$

$$\hat{C} = C\cdot T \Rightarrow C = \hat{C}\cdot T^{-1}$$

$$I = T\cdot T^{-1}$$

$$G = C\cdot I (sI - A)^{-1} I B$$

$$= \hat{C}\, \underbrace{T^{-1}(sI-A)^{-1} T}_{\downarrow}\, \hat{B}$$

$$(sI - \hat{A})^{-1}$$

$$\underbrace{T^{-1}}_{C^{-1}} \cdot \underbrace{(sI - A)^{-1}}_{B^{-1}} \cdot \underbrace{T}_{A^{-1}} \Rightarrow \begin{cases} C = T \\ B = (sI - A) \\ A = T^{-1} \end{cases}$$

$$C^{-1}\cdot B^{-1}\cdot A^{-1} = (ABC)^{-1}$$

$$\left(T^{-1}(sI - A)\cdot T\right)^{-1}$$

$$(sI - \hat{A})^{-1}$$

$$\dot{x} = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix} \cdot x \,, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

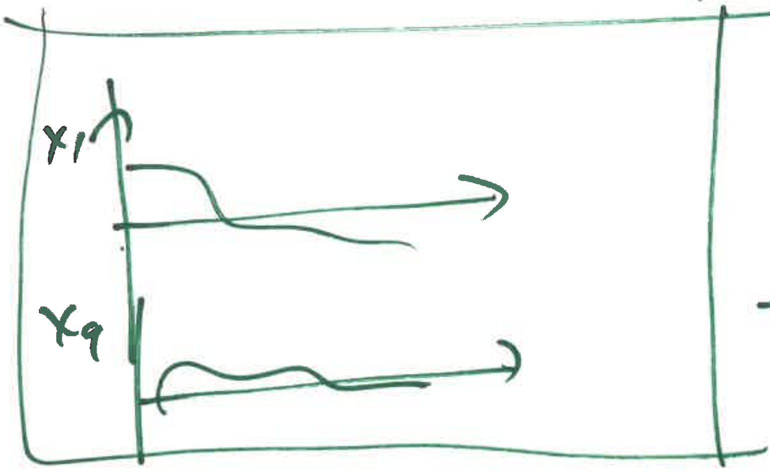
$\lambda_1 = -1 \qquad \lambda_2 = -6$

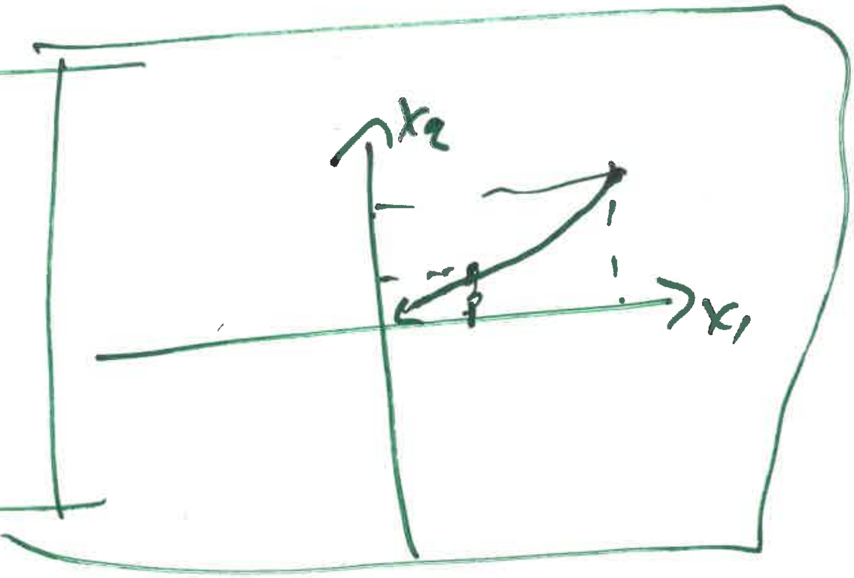
$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$x(t) = \dots$

$x_1 = \dots$

$x_2 = \dots$



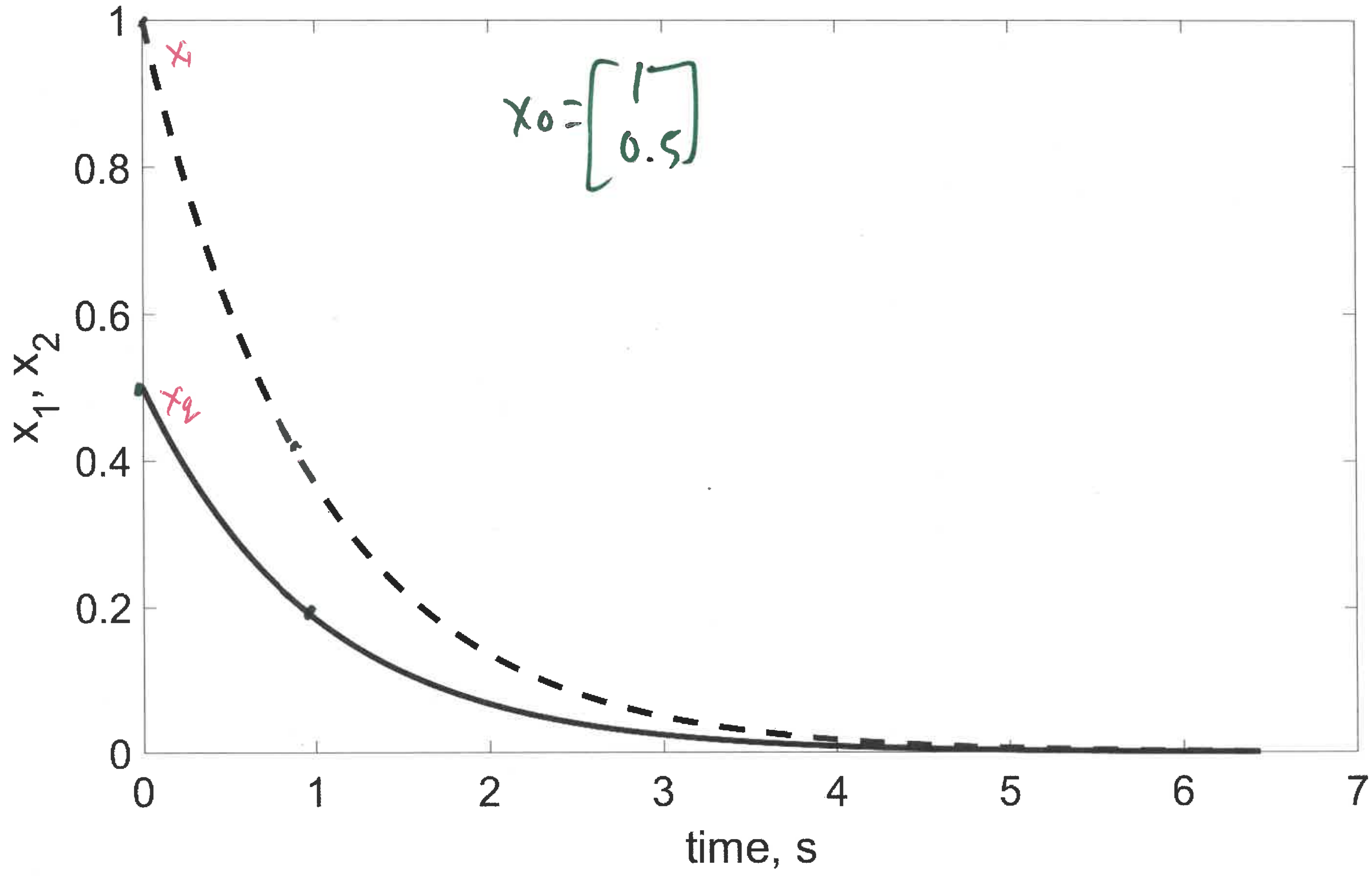


$$x(t) = \underbrace{c_1 e^{-t}}_{\downarrow} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-6t} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$a(t) \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b(t) \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Base $\rightarrow$ eigenbasis vectors

40b



$$X = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-6t}$$

$$X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot 1 + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} \Rightarrow \begin{matrix} c_1 = \ldots \\ c_2 = \ldots \end{matrix}$$

$$X_0 = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \Rightarrow \ldots\ldots \quad \begin{matrix} c_1 = 2 \\ c_2 = 0 \end{matrix}$$

$$x(t) = 2e^{-t} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

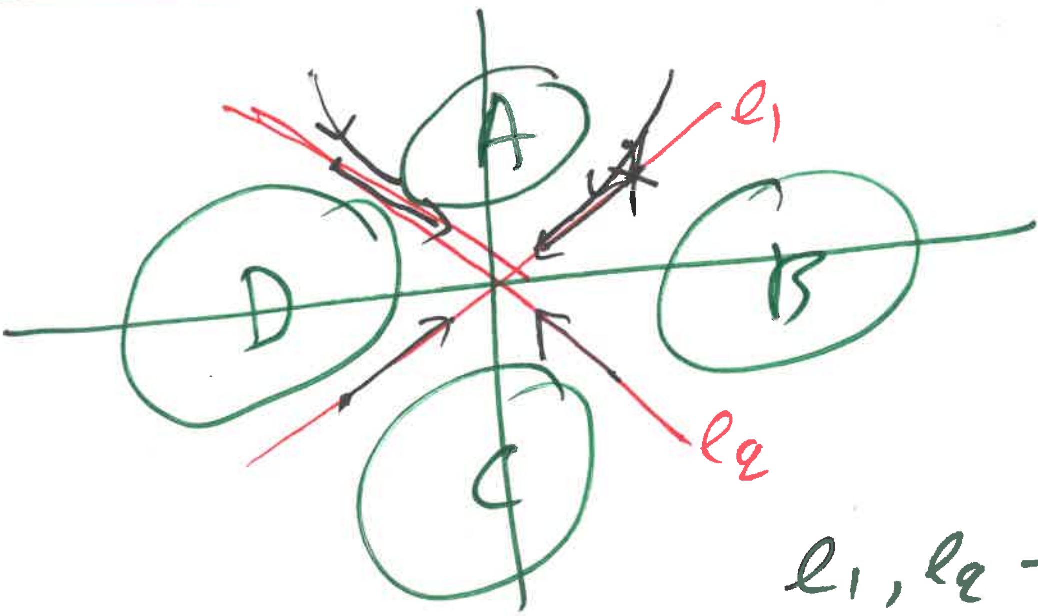
$\Downarrow$

$\forall t$

If at $t = t_1$ $x(t_1) = K \cdot e_1$ or $K e_2$

remain on e_1 or e_2

$\forall t$, $t \in (-\infty, +\infty)$



$e_1, e_2 \to$ invariant under Time

$$A = \begin{bmatrix} -5.5 & 4.5 \\ 4.5 & -5.5 \end{bmatrix}$$

$\lambda_1 = -10$ $\qquad\qquad\qquad$ $\lambda_2 = -1$

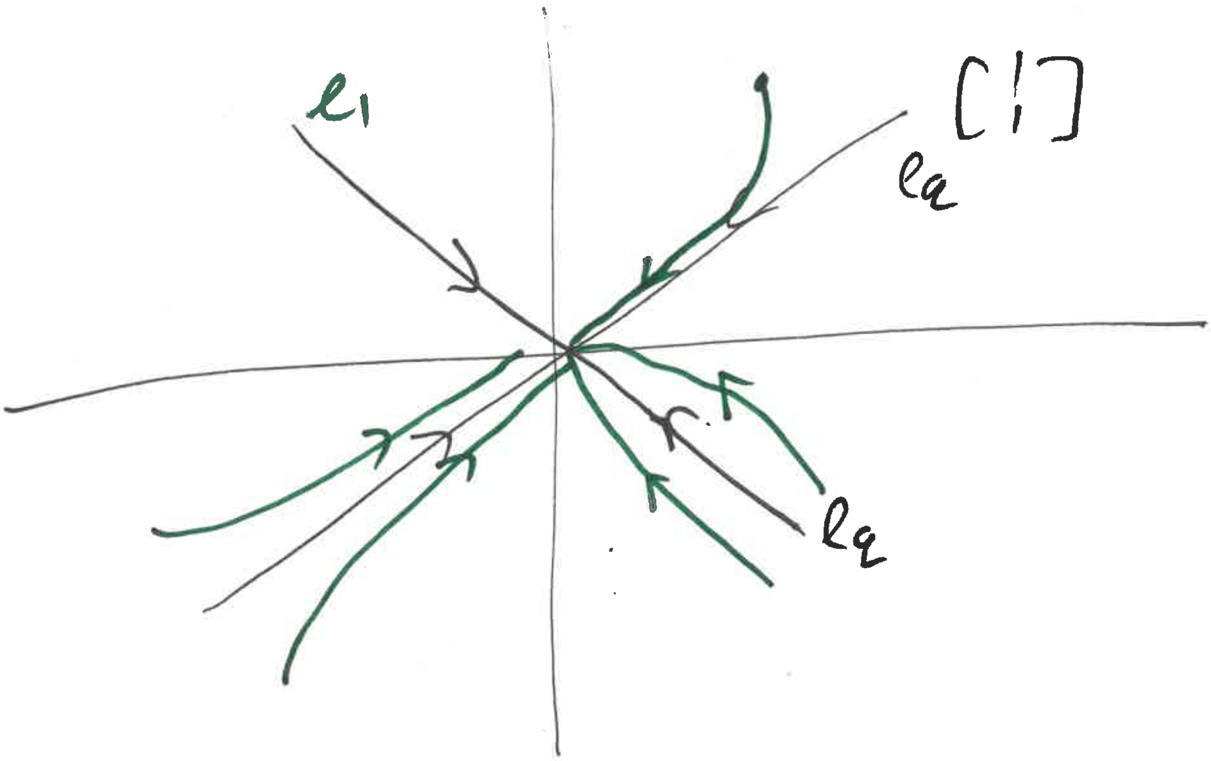
$\downarrow$

$e_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\qquad\qquad\qquad$ $e_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\downarrow$ $\qquad\qquad\qquad\qquad\qquad$ $\downarrow$

$c_1 e_1 e^{-10t}$ $\qquad\qquad\qquad$ $c_2 e_2 e^{-t}$

$$x = \underbrace{c_1 e_1 e^{-10t}}_{\to 0} + \underbrace{c_2 \cdot e_2 e^{-t}}_{\text{Dom.}}$$



$A \to \ldots \quad \lambda_1 = \lambda_2 = \lambda \to e$

$b =$ Gen.

$$x = c_1 \cdot (et + b) e^{\lambda t} + c_2 e^{\lambda t} \cdot e$$

• $t = 0 \quad x_0 = e \cdot k$

$$c_1 \cdot (e \cdot 0 + b) \cdot 1 + c_2 \cdot 1 \cdot e = k \cdot e$$

$$c_1 \cdot b + c_2 \cdot e = k \cdot e$$

$$c_1 \cdot b + (c_2 - k) \cdot e = 0$$

$$c_1 = 0, \quad c_2 = k.$$

$$x = 0 + k e^{\lambda t} \cdot e$$

$e \to$ inv.

• $t = 0 \quad x_0 = k \cdot b$

$$c_1 \cdot b + c_2 \cdot e = k \cdot b$$

$$c_2 = 0$$

$$c_1 = k$$

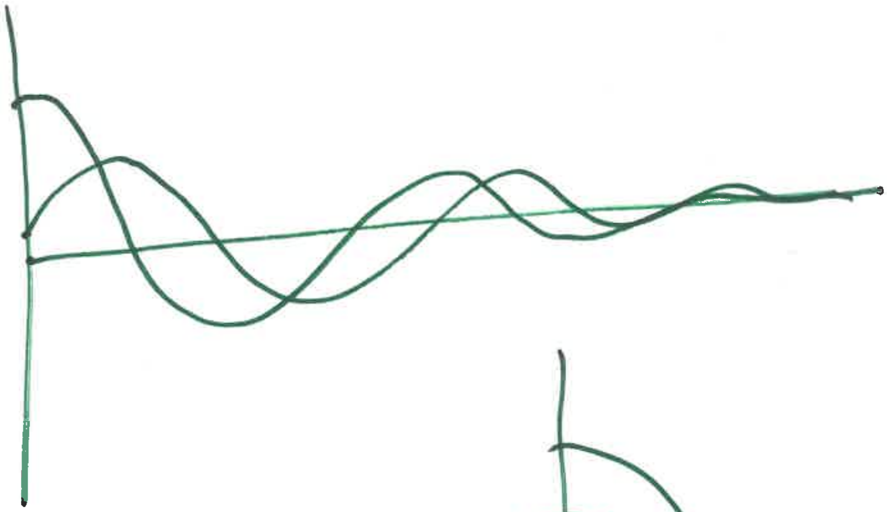
$$x = k(e \cdot t + b) + 0$$

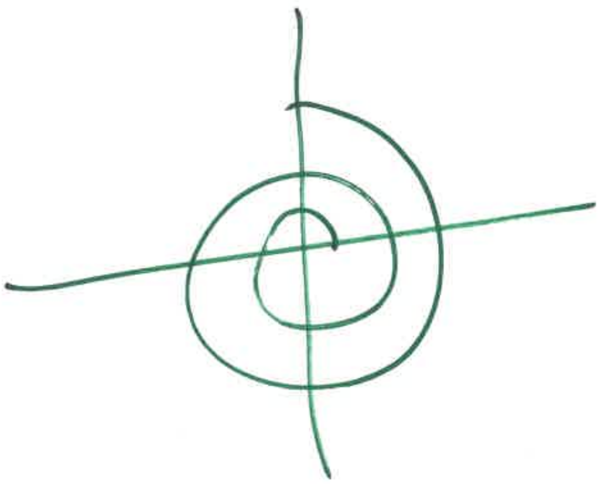
$b \to$ is not inv.

$\lambda = a \pm bi$ $\quad\quad$ $e = [v \pm wi]$

$$x = c_1 e\, e^{\lambda t} + c_2 \bar{e} \cdot e^{\bar{\lambda} t}$$

$$e^{\lambda t} = e^{at} \cdot e^{bit}$$





Chapter 4

$\dot{x} = f(x,u)$, $f, x \in \mathbb{R}^{n\times 1}$
$u \in \mathbb{R}^{p\times 1}$

$\dot{x} = -x^2$

$x(t) = \frac{1}{t+c}$

$\dot{x} = -\frac{1}{(t+c)^2}(t+c)' = -\frac{1}{(t+c)^2} = -x^2$

$x = 3\cdot\frac{1}{t+c} \Rightarrow \dot{x} = -3\frac{1}{(t+c)^2}$

$-x^2 = -9\frac{1}{(t+c)^2}$

$\dot{x} \neq -x^2$

$\dot{x} = Ax + B\cdot u$ if stable $0 = Ax_{ss} + B\cdot u$

$\dot{x} = Ax$ if stable $x \to 0$

$x_{ss} = -A^{-1}\cdot B\cdot u$

Fixed point
St. point
Eq. point

$\dot{x} = Ax \qquad x_{EP} = 0$

unst.

$x = c_1 e_1 e^{\lambda_1 t} + c_2 e_2 e^{\lambda_2 t}$

$x_0 = c_1 e_1 + c_2 e_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

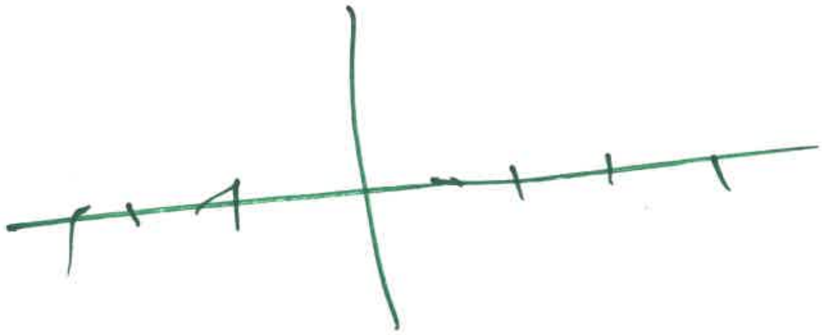
$\Rightarrow c_1 = c_2 = 0$

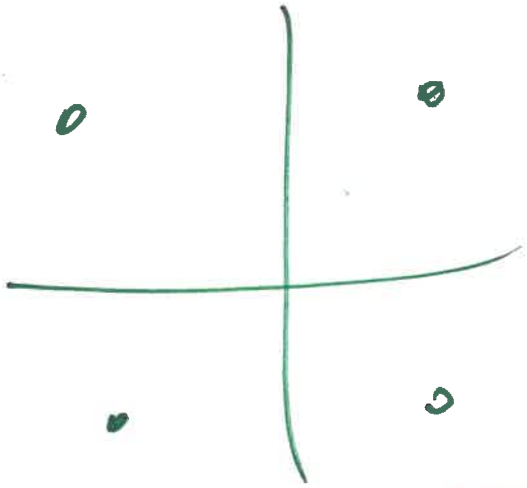
$\dot{x} = x^2 - 4$

$x_{EP}^2 - 4 = 0$

$x_{EP} = \pm 2$

$\dot{x} = \cos \omega t$





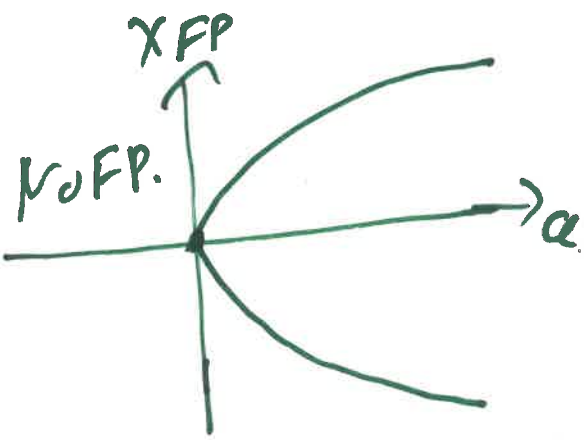
$\dot{x} = x^2 - a$

$x^2 = a \Rightarrow x_{FP} = \pm\sqrt{a}$

2 F.P. if $a > 0$

0 F.P. if $a < 0$

1 FP if $a = 0$



$\dot{x} = Ax + Bu \Rightarrow x_{EP} = -A^{-1}\cdot B\cdot u$

$A \to A(a)$ F.P will chang.

But Not the # of FPs

$\dot{x} = 3x + 5$

$3x_{FP} = -5 \Rightarrow x_{FP} = -5/3$

$\dot{x} = a\cdot x + 5 \qquad x_{FP} = -5/a$

$\dot{x}_1 = x_1 - x_2$

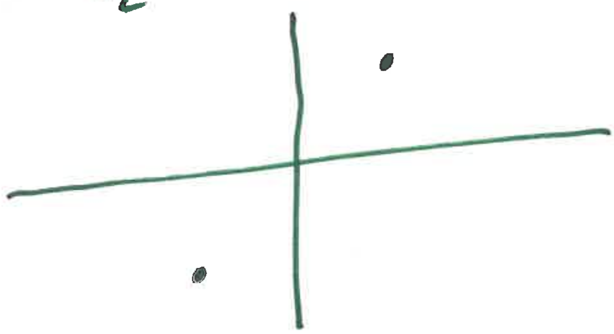
$\dot{x}_2 = x_1^2 + x_2^2 - 2$

$\left.\begin{matrix}\dot{x}_1 = 0\\ \dot{x}_2 = 0\end{matrix}\right\} \Rightarrow \begin{bmatrix} x_1 - x_2 \\ x_1^2 + x_2^2 - 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$

$x_1 = x_2 \qquad x_1^2 + x_1^2 = 2 \Rightarrow x_1^2 = 1 \Rightarrow x_1 = \pm 1$

$x_2 = \pm 1$

$x_{EP1} = (1, 1) \quad , \quad x_{EP2} = (-1, -1)$



$$\left.\begin{aligned} \dot{x}_1 &= x_1^2 \cdot x_2 + 3 \cdot x_1 x_2 - 10 \cdot x_2 \\ \dot{x}_2 &= x_1^2 \cdot x_2 - 4 \cdot x_1 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} x_1^2 \cdot x_2 + 3 x_1 x_2 - 10 \cdot x_2 &= 0 \\ x_1^2 \cdot x_2 - 4 \cdot x_1 &= 0 \end{aligned}\right\} \Rightarrow$$

$$x_2 \cdot (x_1^2 + 3x_1 - 10) = 0$$

$$x_1 \, (x_1 x_2 - 4) = 0$$

$x_2 = 0$ or $x_1^2 + 3x_1 - 10 = 0$

$$x = \frac{-3 \pm \sqrt{9-(-40)}}{2}$$

$$= \frac{-3 \pm \sqrt{49}}{2}$$

$$= \frac{-3 \pm 7}{2}$$

$x_{1A} = 2$

$\boxed{x_A = 2}$

$\boxed{x_B = -5}$

$x_{1B} = -5$

If

- $x_2 = 0 \Rightarrow -4 \cdot x_1 = 0 \Rightarrow (0,0)$
- $x_1 = 2 \Rightarrow 2 \cdot (2 \cdot x_2 - 4) = 0 \Rightarrow x_2 = 2$

  $(2, 2)$
- $x_1 = -5$ $\quad -5(-5 \cdot x_2 - 4) = 0 \Rightarrow (5x_2 + 4) = 0$

  $x_2 = -4/5$

  $(-5, -4/5)$