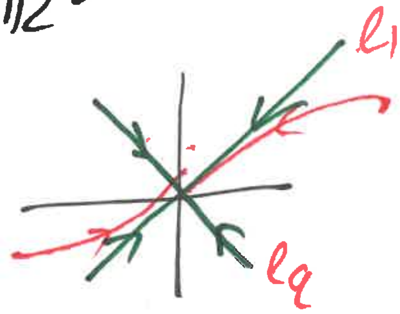
Revision

$\dot{X} = AX, \quad X \in \mathbb{R}^{2\times 1}$

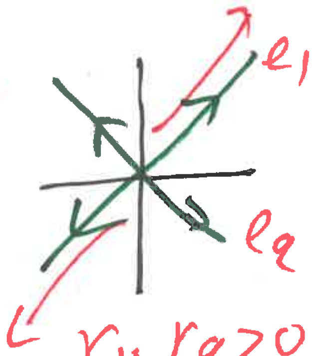
- $r_1 \neq r_2 \in \mathbb{R}.$

$r_1 \downarrow e_1 \quad r_2 \downarrow e_2$



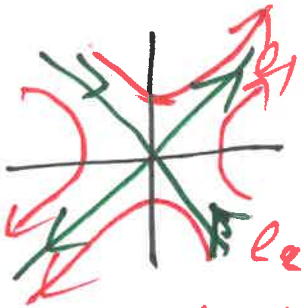
$r_1, r_2 < 0$

STABLE NODE



$r_1, r_2 > 0$

Unstable NODE

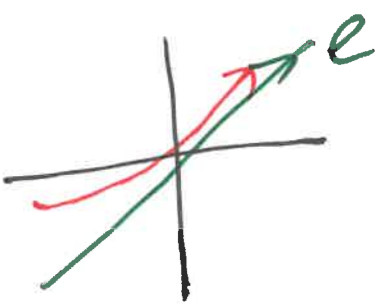


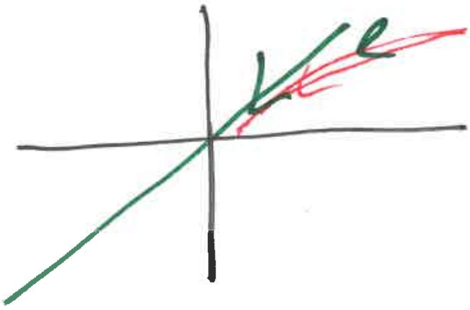
$r_1 > 0, r_2 < 0$

saddle

- $r_1 = r_2$

$\downarrow e \quad \downarrow b$

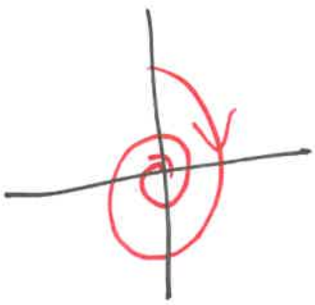




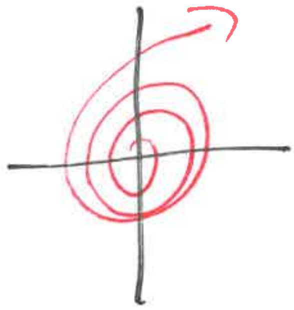
eigenvectors = invariant

Focus

- $r = a + bi$

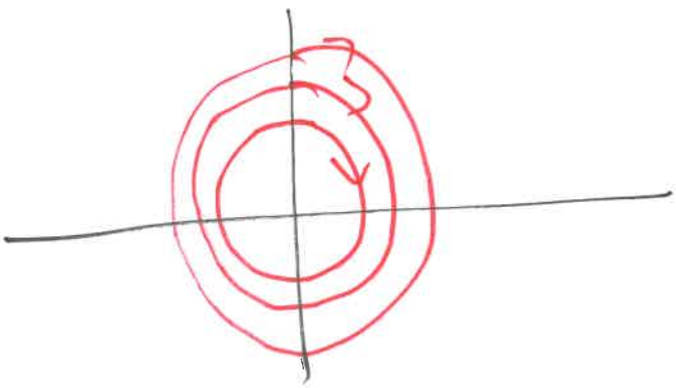


$a < 0$



$a > 0$

- $r = bi$



E.P., F.P., S.P. → invariant

$\dot{x}=0 \rightarrow Ax+Bu=0 \Rightarrow x_{EP}=-A^{-1}\cdot Bu$

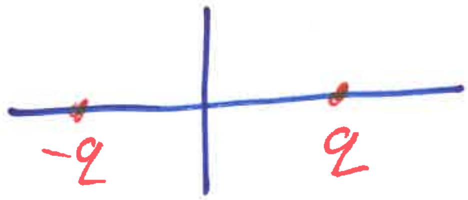
$\xrightarrow{u=0} x_{EP}=0$

Ch. 4 $\qquad \dot{x}=f(x,u)$

1) if x_1 is a soln of $\cancel{\Rightarrow}$ $x_2=k\cdot x_1$ a soln

2) F.P.

Multiple



$\dot{x}=x^2-4 \Rightarrow x_{EP_1}=2, \quad x_{EP_2}=-2$

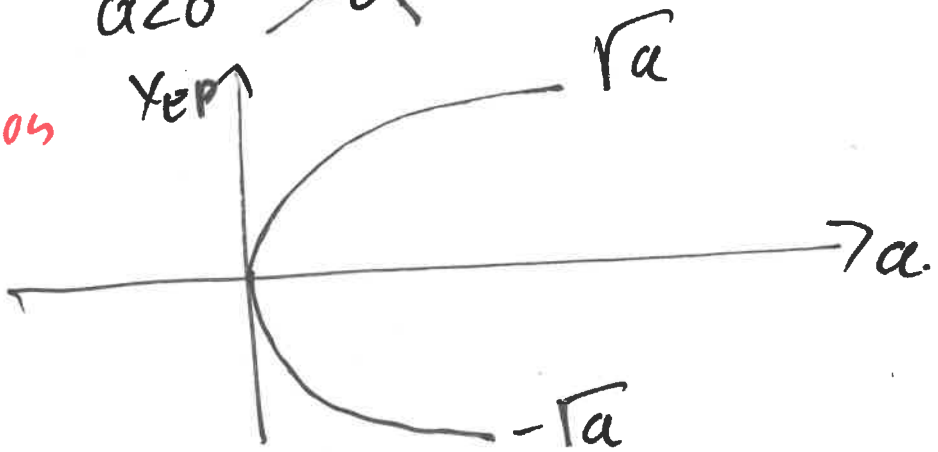
☐ No F.P.

$\dot{x}=x^2+4 \qquad \nexists\, x: \dot{x}=0$

☐ $\dot{x}=x^2-a.$ $\quad \xrightarrow{a\geq 0} x_{EP}=\pm\sqrt{a}$

$\xrightarrow{a<0} \cancel{x_{EP}}$

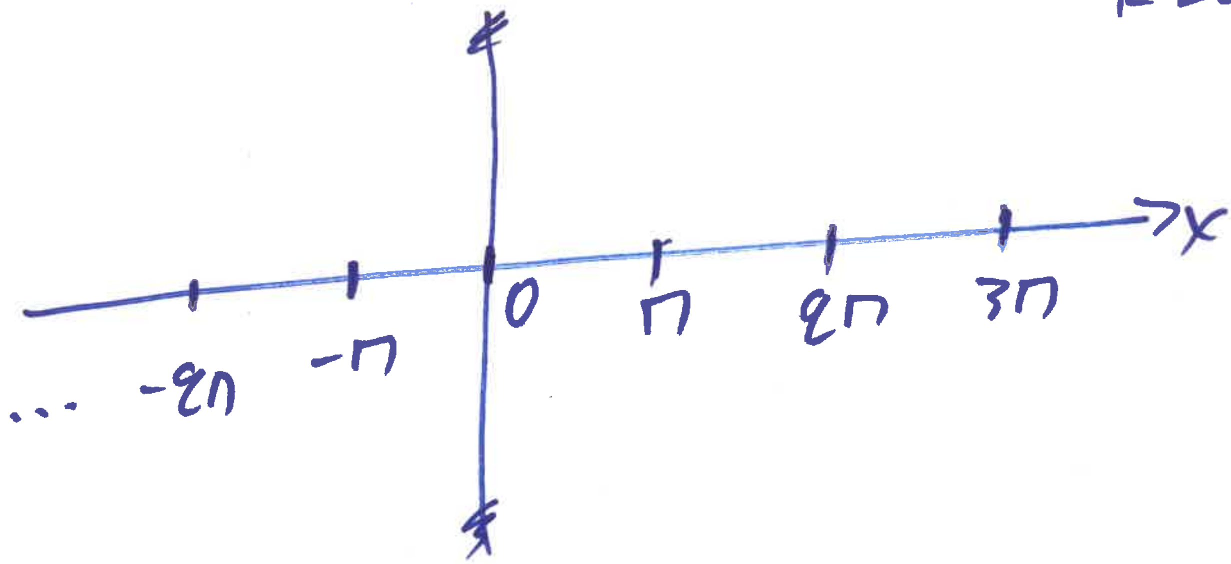
Bifurcation



$\dot{x} = \sin(x)$

$\dot{x} = 0 \Rightarrow \sin(x) = 0 \Rightarrow x_{EP} = \pi \pm k\pi$

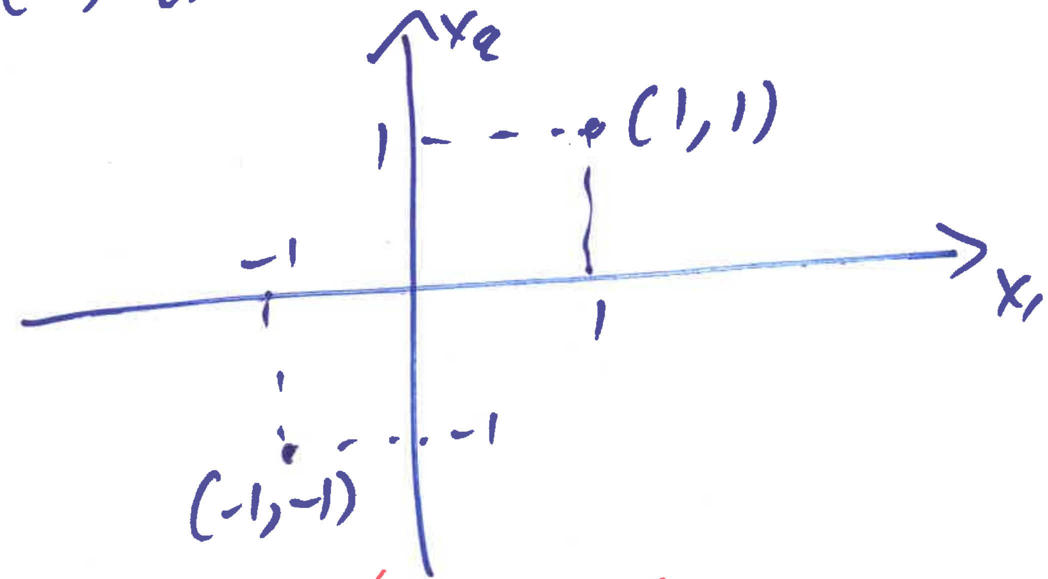
$k = 0, 1, 2, \ldots$



$$\left.\begin{aligned}\dot{x}_1 &= x_1 - x_2\\ \dot{x}_2 &= x_1^2 + x_2^2 - 2\end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_1 - x_2 &= 0\\ x_1^2 + x_2^2 - 2 &= 0\end{aligned}\right\} \Rightarrow$$

$(x_1, x_2) = (1, 1)$

$(x_1, x_2) = (-1, -1)$

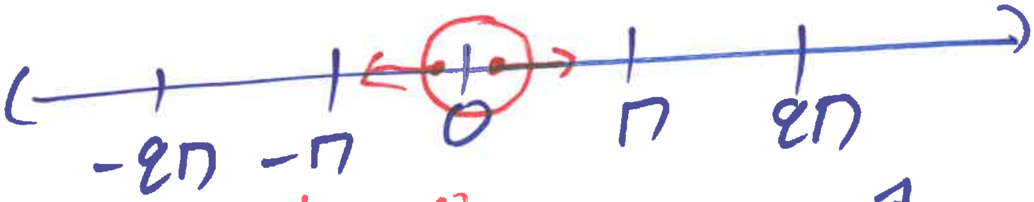


L.S → stable/unstable sys

NLS → stable/unstable F.P.s

$\dot{x} = \sin(x)$

$\sin(x) = 0 \Rightarrow x_{EP} = \pi \pm k\pi$



at $x = 0$

Taylor of $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\sin(x) \boxed{\simeq} x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ $\quad x \in [-2, 2]$

$\sin(x) \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!}$ $\quad x \in (-1.8, 1.8)$

$\sin(x) \simeq x - \frac{x^3}{3!}$ $\quad x \in [-1, 1]$

$\sin(x) \simeq x$ $\quad x \in [-0.5, 0.5]$

$\dot{x} = x \Rightarrow x = e^t \cdot c$

$\dot{x} = f(x)$
$x = x_0$

$$f(x) \simeq f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0) + \left.\frac{\partial^2 f}{\partial^2 x}\right|_{x=x_0} (x - x_0)^2$$

$$\dot{x} = f(x_0) + \frac{\partial f}{\partial x}\bigg|_{x=x_0}(x-x_0) + \cancel{\cdots}$$

$$\dot{x} = f(x_0) + \frac{\partial f}{\partial x}\bigg|_{x=x_0}(x-x_0)$$

For ANY x_0

$x_0 = x_{EP}$

$$\dot{x} = \cancelto{0}{f(x_{EP})} + \frac{\partial f}{\partial x}\bigg|_{x=x_{EP}}(x - x_{EP})$$

$$\dot{x} = A \cdot \underbrace{(x - x_{EP})}_{\Delta x}$$

$$\Delta x = x - x_{EP}$$

$$\dot{x} = A \cdot \Delta x$$

$$\Delta \dot{x} = \dot{x} - 0$$

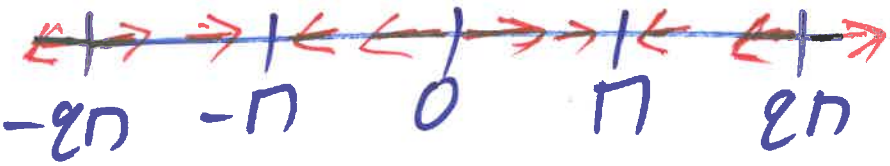
$$\boxed{\Delta \dot{x} = A \cdot \Delta x} \rightarrow$$ Models the perturbations

$$A = \frac{\partial f}{\partial x}\bigg|_{x=x_{EP}}$$ Jacobian of f at x_{EP}

Linearisation around x_{EP}

$\dot{x} = \sin(x)$

$x_{EP} = \pi \pm k\pi$



$f(x) = \sin(x)$

$$A = \frac{df}{dx}\Big|_{x=x_{EP}} = \cos(x)\Big|_{x=x_{EP}}$$

$$\dot{x} \simeq \sin(x_{EP}) + \cos(x_{EP})\cdot(x-x_{EP})$$

$$\Delta\dot{x} \simeq 0 + \cos(x_{EP})\cdot(x-x_{EP})$$

• $x_{EP}=0$ $\quad \Delta\dot{x} = 1\cdot(x-x_{EP}) = \Delta x$

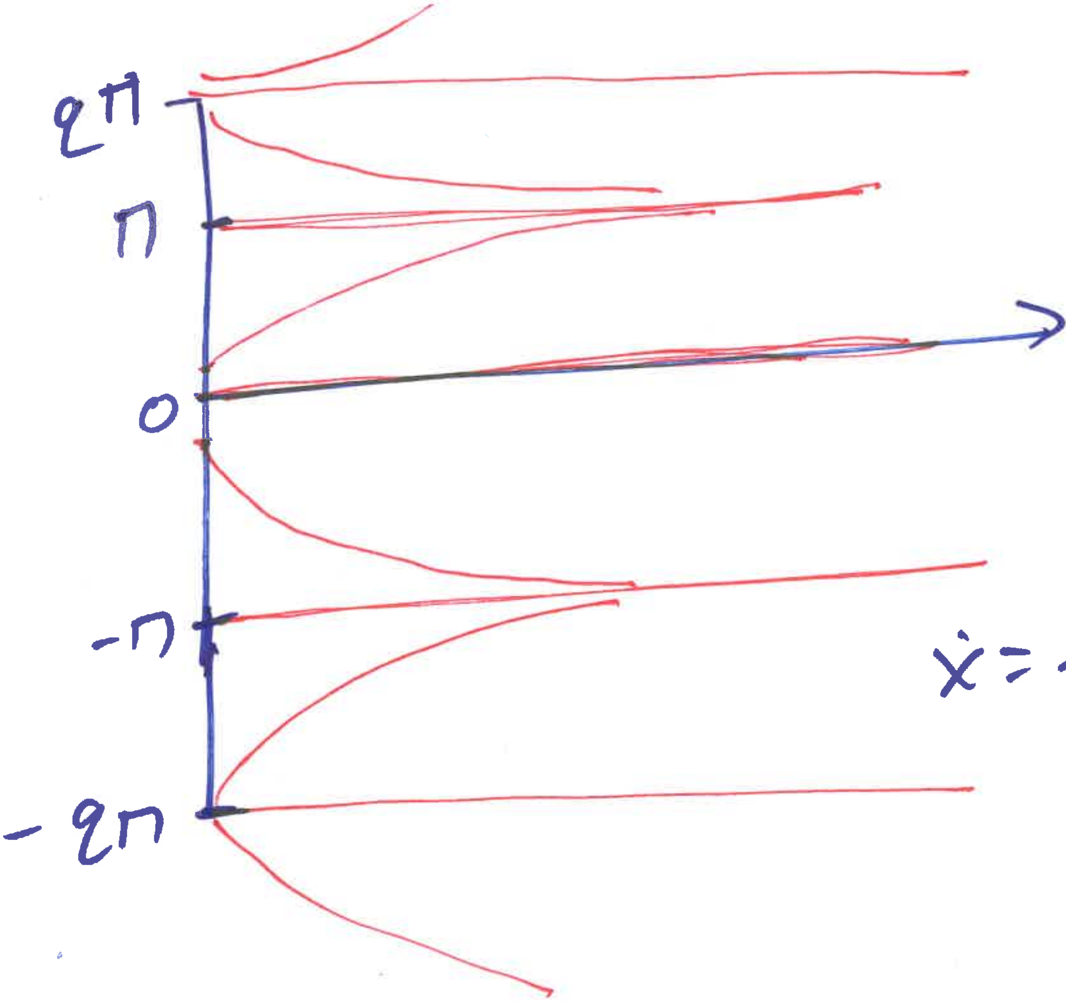
$$\Delta x = e^{t}\cdot c.$$

unstable.

• $x_{EP}=\pi$ $\quad \Delta\dot{x} = \overset{-1}{\cos(\pi)}\cdot\Delta x$

$$\Delta\dot{x} = -\Delta x \Rightarrow \Delta x = e^{-t}\cdot c.$$

• $x_{EP}=-\pi$ $\quad \Rightarrow \dots \quad \Delta x = e^{-t}\cdot c.$



$\dot{x} = \sin x$

$\dot{x} = x - y$

$\dot{y} = x + y - 2x \cdot y$

1) FPs
2) Stability
3) Diagram State Space

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} x - y \\ x + y - 2xy \end{bmatrix}$$

- FPs.

$$\left.\begin{aligned} x - y &= 0 \\ x + y - 2xy &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} &x = y \\ &x + x - 2x^2 = 0 \end{aligned}$$

$\cancel{(x, y)}$
$\cancel{(0, 0)}$

$2x - 2x^2 = 0$

$x(1 - x) = 0$

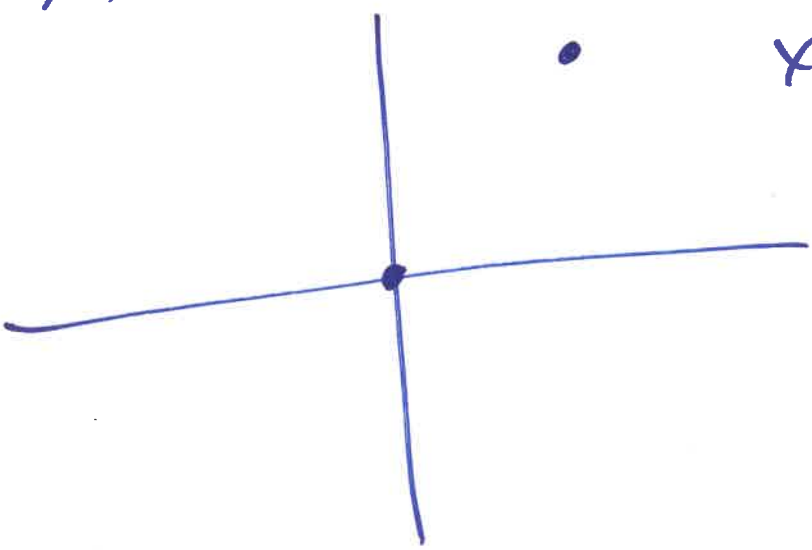
$x = 0$

or

$x = 1$

$(0, 0)$

$(1, 1)$



$\begin{cases} x_1 = x \\ x_2 = y \end{cases}$ 57

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$f_1 = x - y = f_1(x, y)$$

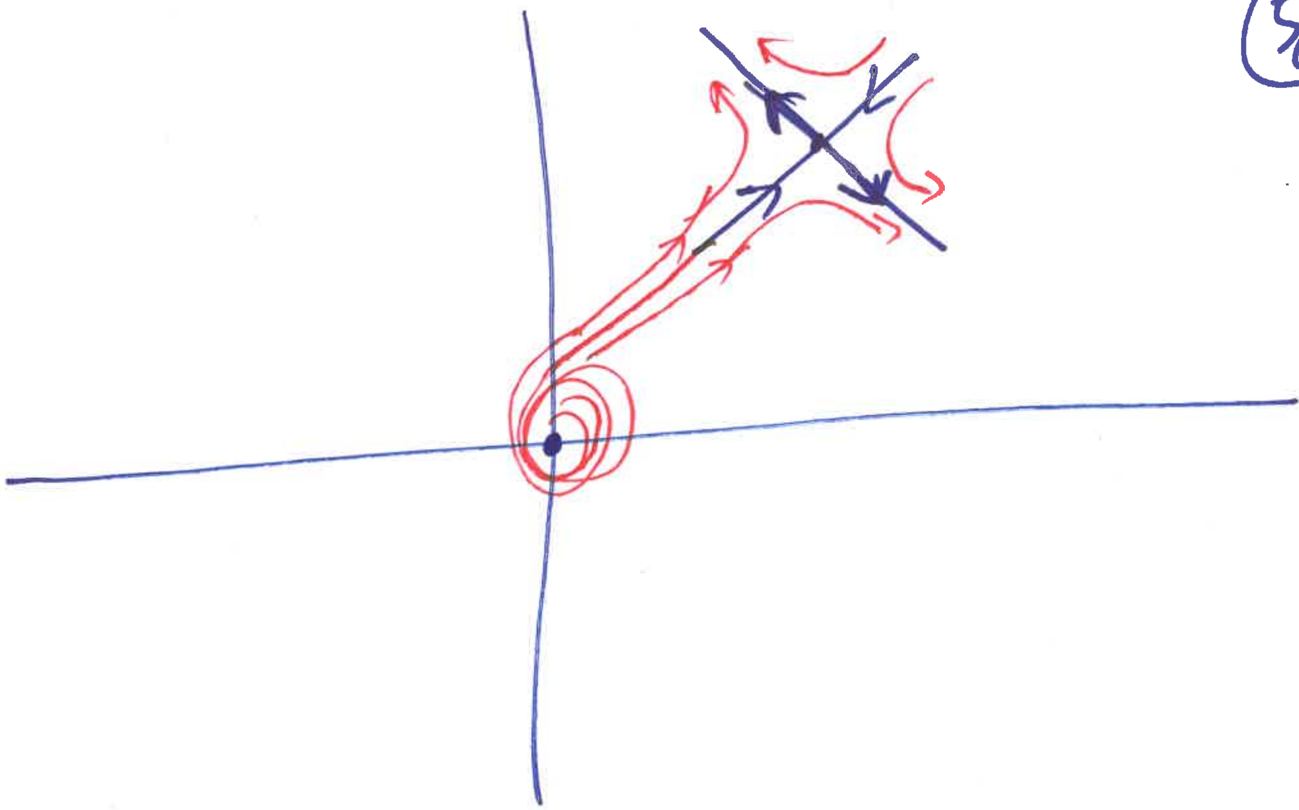
$$f_2 = x + y - 2xy.$$

$$A = \begin{bmatrix} 1 & -1 \\ 1-2y & 1-2x \end{bmatrix}$$

$$A(0,0) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \to \lambda = 1 \pm 2i$$ unstable Focus

$$A(1,1) = \to \lambda_1 = 1.41 \to e_1 = \begin{bmatrix} -2.41 \\ 1 \end{bmatrix}$$

$$\to \lambda_2 = -1.41 \to e_2 = \begin{bmatrix} -1 \\ -2.41 \end{bmatrix}$$

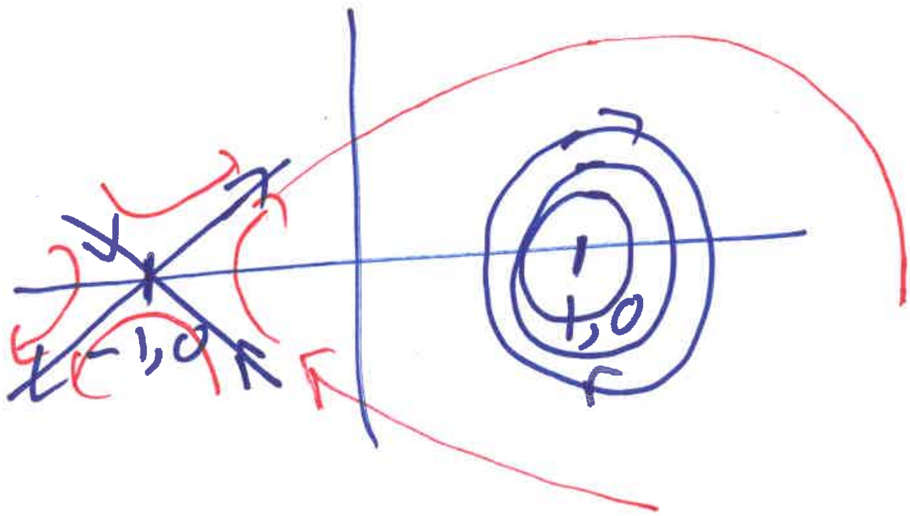


$\dot{x} = y \cdot e^{y} = f_1$

$\dot{y} = 1 - x^2 = f_2$

$f_1 = 0 \quad y\cancel{e^{y}} = 0 \Rightarrow y = 0$

$f_2 = 0 \quad 1 - x^2 = 0 \Rightarrow x = \pm 1$



(59)

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_1} \end{bmatrix}$$

$f_1 = ye^y \qquad f_2 = 1 - x^2$

$$A = \begin{bmatrix} 0 & e^y + ye^y \\ -2x & 0 \end{bmatrix}$$

$$A(1,0) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \Rightarrow \lambda = \pm 1.411 i$$

$$A(-1,0) = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$\rightarrow \lambda_1 = -1.4 \quad e_1 = \begin{bmatrix} -0.5 \\ 0.8 \end{bmatrix}$

$\rightarrow \lambda_2 = 1.4$

$e_2 = [0.5 \quad 0.8]^T$