Revision

$\dot{x} = f(x)\ ,\ x(0) = x_0$

$f, x \in \mathbb{R}^{n \times 1}$

$x_{EP}\ :\ \dot{x} = 0 \rightarrow$ invariant

Linear $\div$ $-A^{-1} \cdot B \cdot u$

N.L.S. $\rightarrow$ No EPs $\quad \dot{x} = x^2 + 4$

$\rightarrow$ Multiple $\quad \dot{x} = \cos x$

$\rightarrow$ Create/destroy EPs

Local Stability Analysis

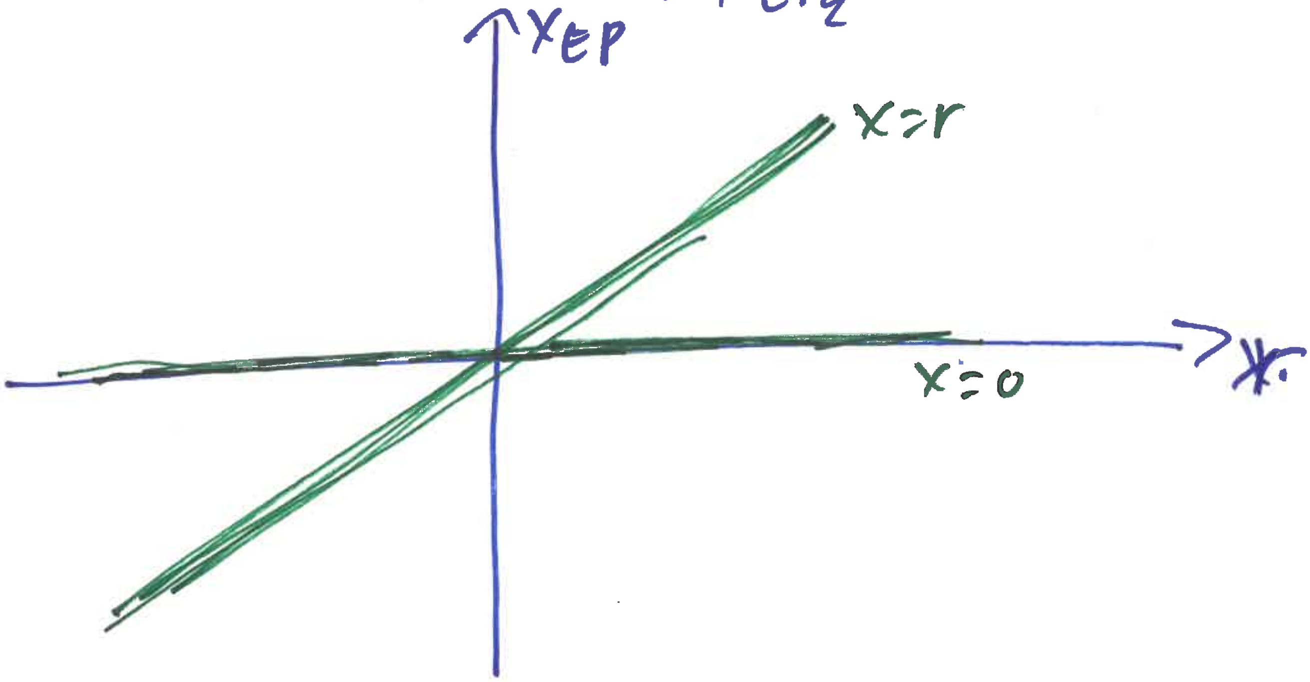
$\Delta \dot{x} = A \cdot \Delta x\ ,\quad \Delta x = x - x_{EP}$

$A = \left. \dfrac{\partial f}{\partial x} \right|_{x = x_{EP}}$

Bifurcations

$$\frac{dx}{dt} = rx - x^2$$

$\dot{x} = 0 \Rightarrow rx - x^2 = 0$ → $x_{EP_1} = 0$, → $x_{EP_2} = r$



$f(x,r) = rx - x^2$ $\quad A = \left.\frac{\partial f}{\partial x}\right|_{x=x_{EP}} =$

$= r - 2x$ $\xrightarrow{x=0} A = r$, $\xrightarrow{x=r} A = -r$

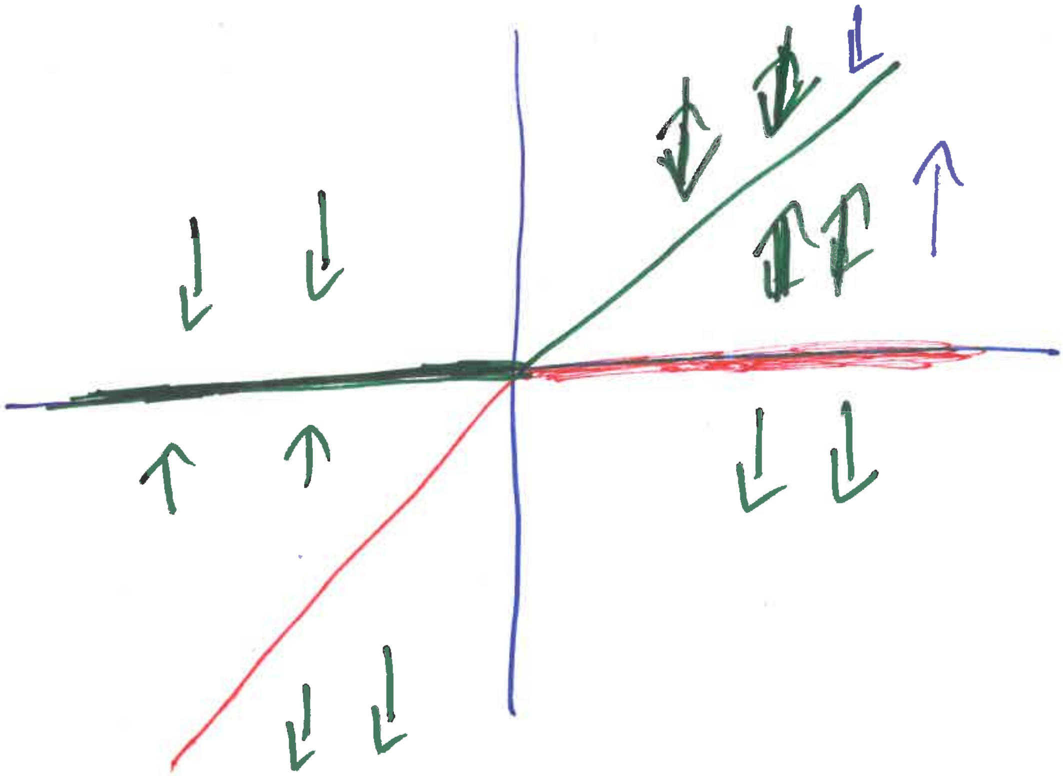
eig

• $r<0$ $\quad A(0) = r < 0$ stable

$\quad A(r) = -r > 0$ unst.

• $r>0$ ..... unstable

stable



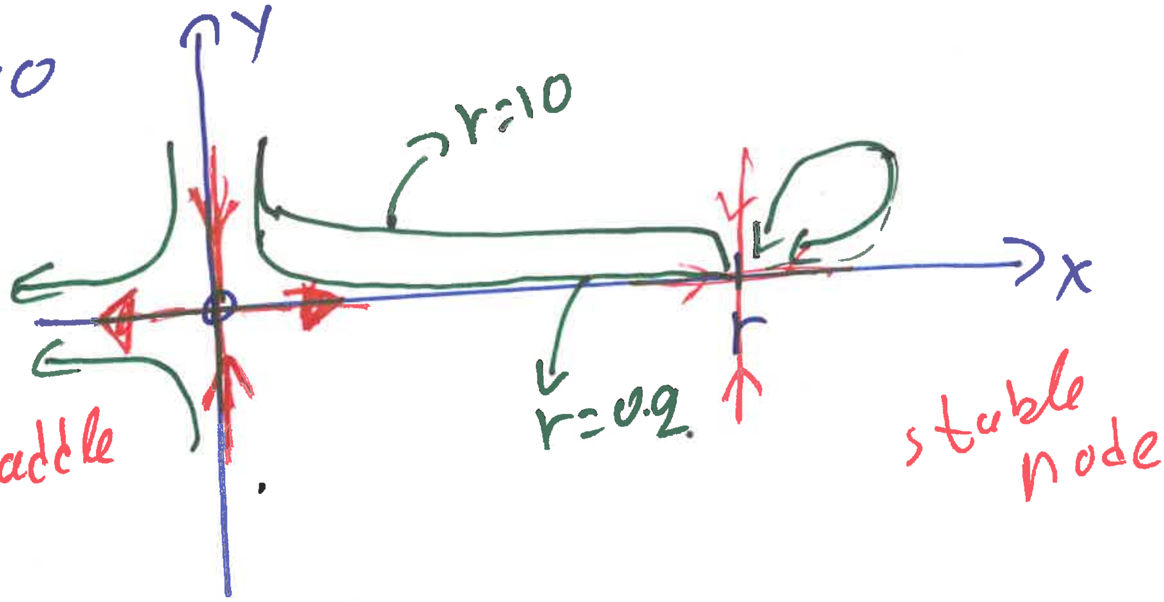
Transcritical Bif.

2) $\dot{x} = rx - x^2$, $\dot{y} = -y$ $\Big\} \Rightarrow A = \begin{bmatrix} r-2x & 0 \\ 0 & -1 \end{bmatrix}$

$A(0,0) = \begin{bmatrix} r & 0 \\ 0 & -1 \end{bmatrix}$ $\begin{cases} \lambda_1 = r \to e_1 = [1 \ \ 0]^T \\ \lambda_2 = -1 \to e_2 = [0 \ \ 1]^T \end{cases}$

$A(r,0) = \begin{bmatrix} -r & 0 \\ 0 & -1 \end{bmatrix}$ $\begin{cases} \lambda_1 = -r \to e_1 = [1 \ \ 0]^T \\ \lambda_2 = -1 \to e_2 = [0 \ \ 1]^T \end{cases}$

$r>0$



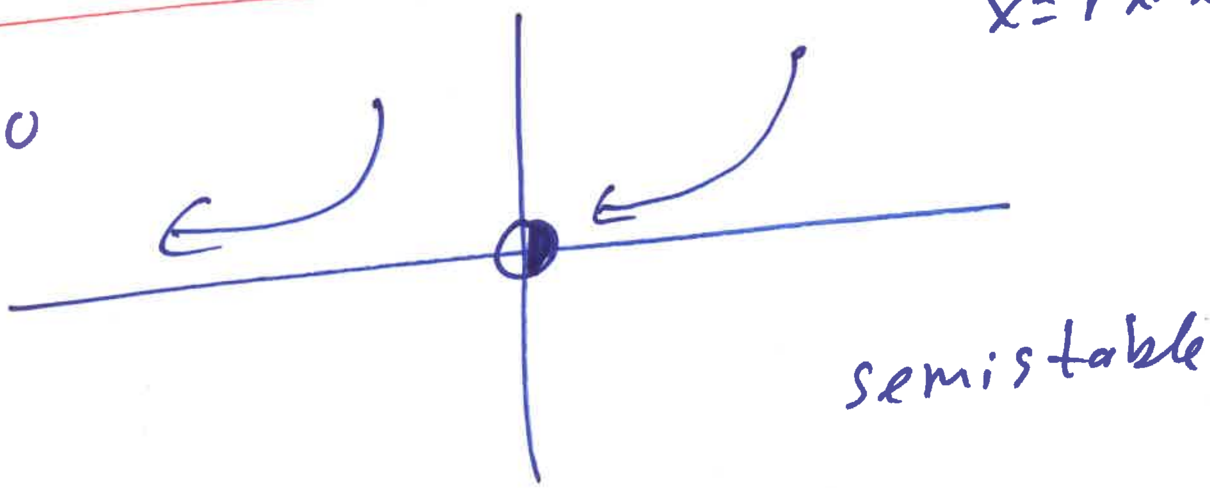
$r=10$

$r=0.2$

Homework: $r<0$

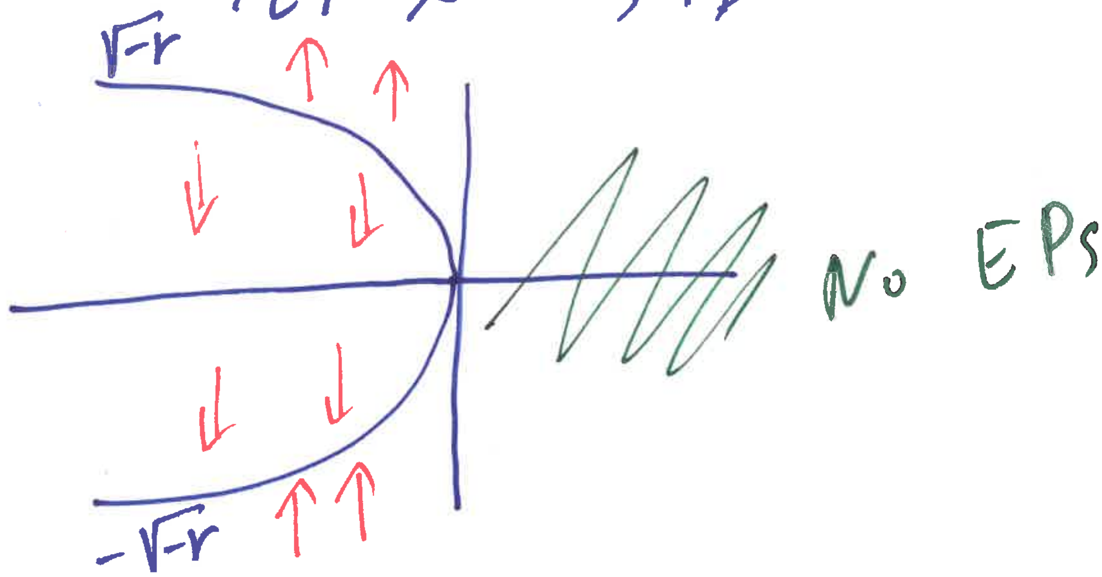
$r=0$

$\dot{x}=rx-x^2=-x^2$



$\dot{x} = r + x^2 \Rightarrow x_{EP} = \pm\sqrt{-r}\ ,\quad r<0$ (64)

$\dot{x} = x^2 + r \qquad x_{EP}\ \nexists\ ,\quad r>0$



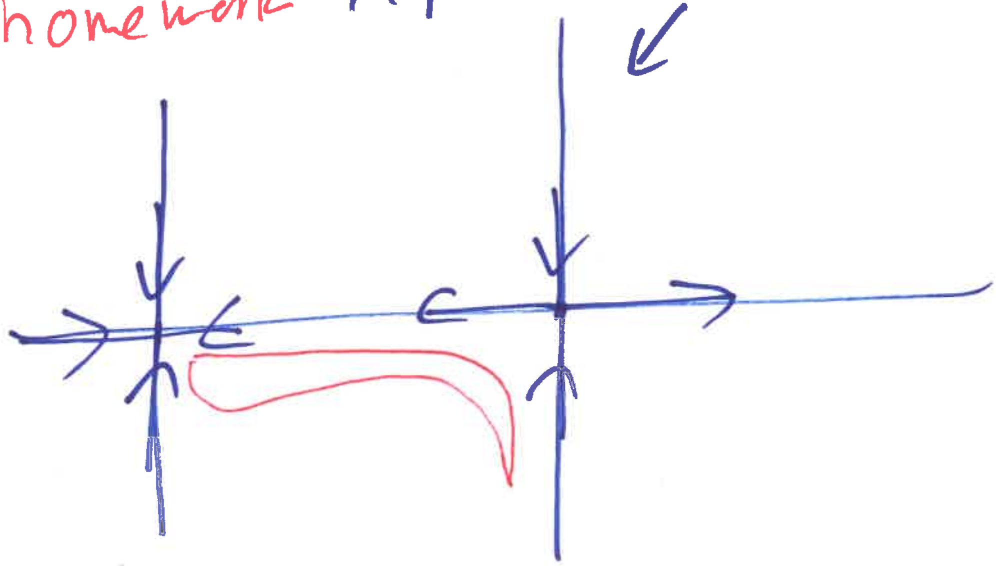
$$A = 2x \begin{cases} \xrightarrow{\sqrt{-r}} 2\sqrt{-r} > 0 & \text{unstable} \\ \xrightarrow{-\sqrt{-r}} -2\sqrt{-r} < 0 & \text{stable} \end{cases}$$

$\dot{x} = \alpha + x^2$
$\dot{y} = -y$

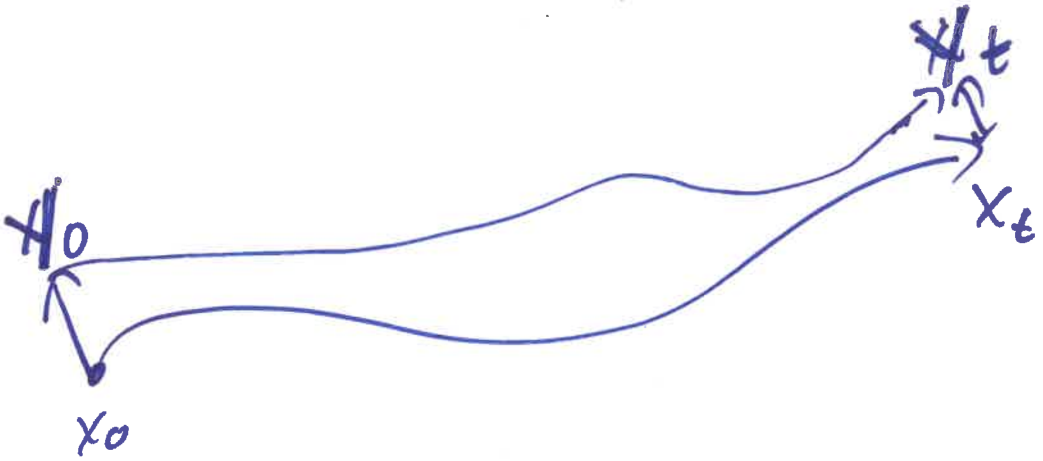
$$A = \begin{bmatrix} 2x & 0 \\ 0 & -1 \end{bmatrix}$$

homework reproduce this

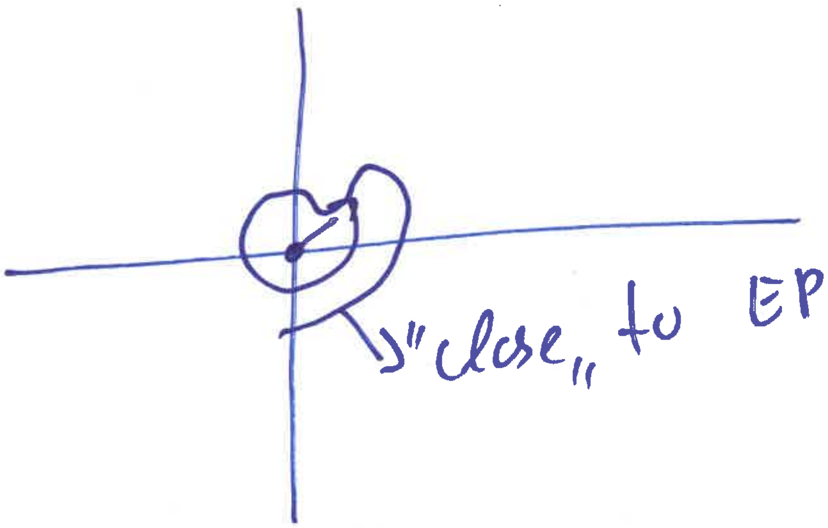
$r<0$



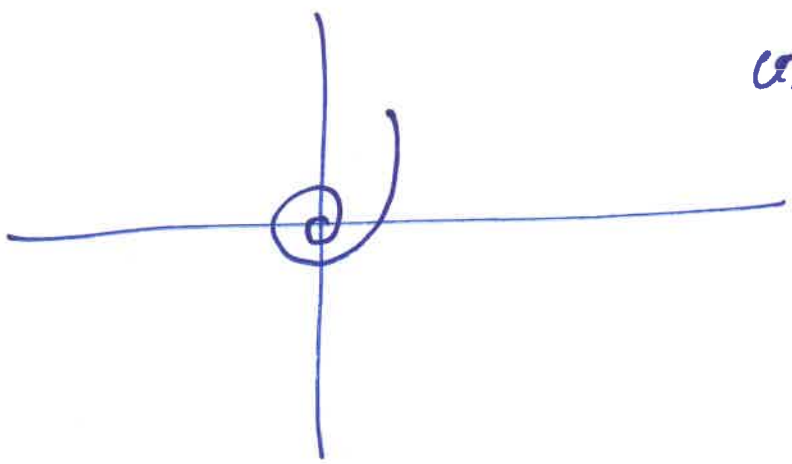
Lyapunov



$\phi \; Y_t - X_t \to$ small



asy. stable



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$\dot{x} = f(x,u)$, $x \in \mathbb{R}^{n\times 1}$

$$V(x) \begin{cases} \to V(0) = 0 \\ \to V(x) > 0 \\ \to \dot{V}(x) < 0 \end{cases} \Bigg\} \ \dot{x} = f(\ ) \text{ is stable}$$

$\dot{x} = -x + y - xy^2$

$\dot{y} = -2x - y - x^2 y$

$V(x,y) = x^2 + y^2$

$V(0) = 0$

$V(x,y) > 0$

$\boxed{\dot{V} < 0}$ → I want.

$$\dot{V} = \frac{\partial V}{\partial x} \cdot \dot{x} + \frac{\partial V}{\partial y} \cdot \dot{y}$$

$$2x \cdot \dot{x} + 2y \cdot \dot{y}$$

$$2 \cdot x \cdot (-x + y - x \cdot y^2) + 2y \cdot (-2x - y - x^2 y)$$

$$= -2x^2 + 2xy - 2x^2y^2 - 4x \cdot y - 2y^2 - \boxed{2x^2y}$$

$\downarrow$

$2x^2y^2$

$$= \dots = -2(x+y)^2 - 4x^2y^2 < 0$$

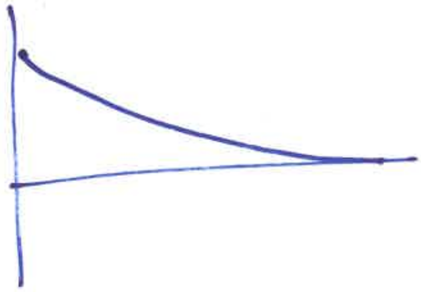
Chapter 5

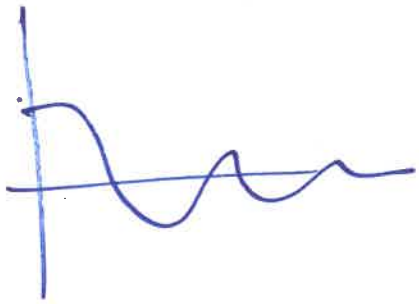
$\ddot{x} + A\dot{x} + Bx = u$, $A, B \to$ known

$u = 0$ $\downarrow$

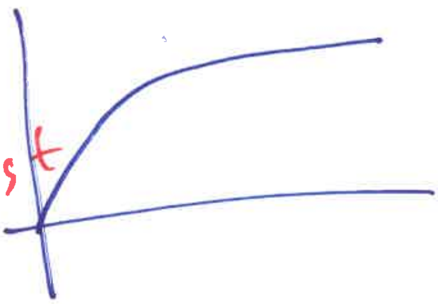
$r^2 + Ar + B = 0$ $\to r_1 < 0$, $r_2 < 0$ stable

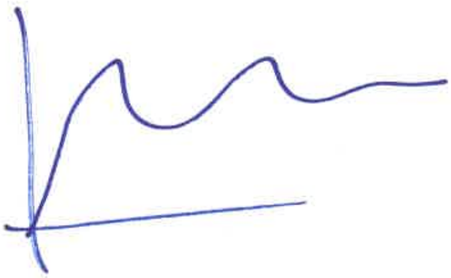
$x_0 \neq 0$
$u = 0$





$x_0 \neq 0$
$u = \text{const.}$





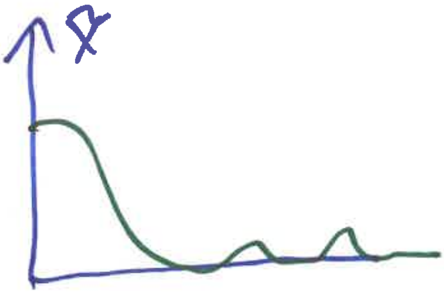
e.g. $x_d = e^{-t} \cos t$

$x_d \to$ Desired trajectory

$u = ?$ $x \to x_d$

$u = ?$ $\tilde{x} \to 0$

$\tilde{x} = x - x_d$.



$\ddot{x} + A\dot{x} + Bx = u$ control

$A, B =$ known

$r_1, r_2 < 0$

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$x_d = \dots$

$x \to x_d.$ $\tilde{x} \to 0$

if somehow

$\tilde{x} = x - x_d$

$\dot{\tilde{x}} = \dot{x} - \dot{x}_d$

$\ddot{\tilde{x}} = \ddot{x} - \ddot{x}_d$

$\ddot{\tilde{x}} + A\dot{\tilde{x}} + B\tilde{x} = 0$

since $r_1, r_2 < 0$

$\tilde{x}(0)$

$\tilde{x} \to 0 \Rightarrow x \to x_d$

$u = ?$:

$$\ddot{\tilde{x}} + A\dot{\tilde{x}} + B\tilde{x} = 0$$

$$(\ddot{x} - \ddot{x}_d) + A(\dot{x} - \dot{x}_d) + B(x - x_d) = 0$$

$$\ddot{x} + A\dot{x} + Bx - \ddot{x}_d - A\dot{x}_d - Bx_d = 0$$

$$\ddot{x} + A\dot{x} + Bx = \ddot{x}_d + A\dot{x}_d + Bx_d$$

So if $\boxed{u = \ddot{x}_d + A\dot{x}_d + Bx_d}$

$\tilde{x} \to 0$ or $x \to x_d.$

This method will fail if r_1 or $r_2 > 0$ or if sys is slow.

$\ddot{x} + A\dot{x} + Bx = u.$ A, B known

$r_1, r_2, r_3 > 0$

$u = ?$: $x \to x_d$ or $\tilde{x} \to 0$

$u = ?$: ODE of $\tilde{x}$ which stable + fast.

Next week.