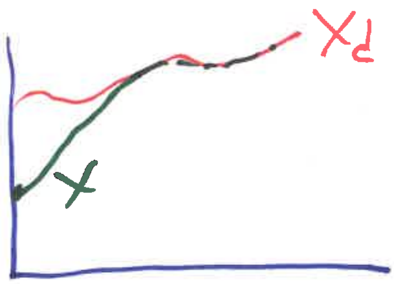
Revision

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \ldots, t) + g(\qquad)\cdot u.$$

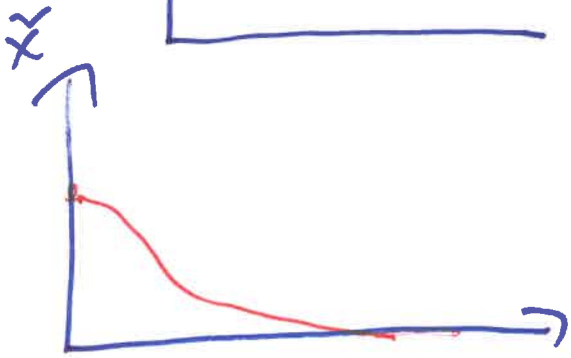
x_d = Given, eg. $x_d = e^{-t}\cdot \cos 3t$



$u = ?$: $x \to x_d$

$\tilde{x} = x - x_d$

$u = ?$: $\tilde{x} \to 0$



ODE of $\tilde{x}$ is stable

Target

Specific Dynamics

error

$\Rightarrow$ Eigs of the ODE of error.

$\lambda_1 = ?$ $\lambda_2 = ?$ .... $\lambda_n = ?$

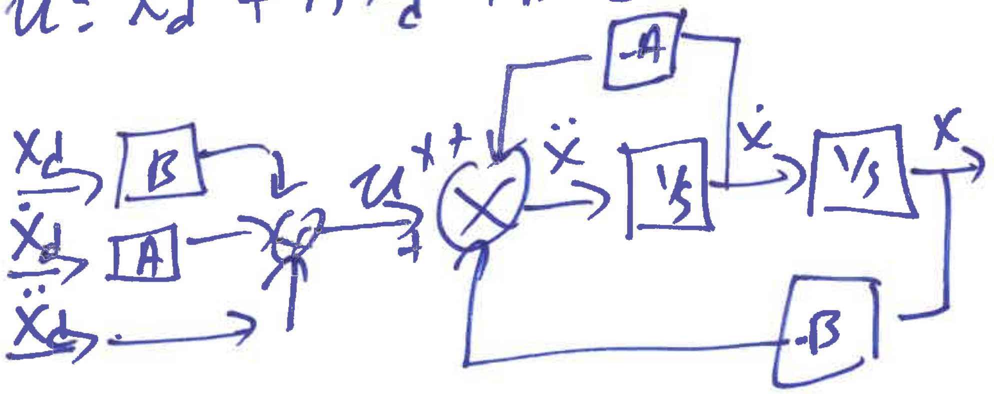
$\ddot{x} + A\dot{x} + Bx = u.$ A, B are known.

$r^2 + Ar + B = 0$ stable eigs.

① Target

$\ddot{\tilde{x}} + A\dot{\tilde{x}} + B\tilde{x} = 0$

$u = \ddot{x}_d + A\dot{x}_d + Bx_d$



② if r_1 or $r_2 > 0$ (or slow)

$\longrightarrow$ Target λ_1, λ_2

$(r-\lambda_1)(r-\lambda_2) = 0$

$r^2 - (\lambda_1 + \lambda_2) r + \lambda_1 \lambda_2 = 0$

$\ddot{\tilde{x}} + C\dot{\tilde{x}} + D\tilde{x} = 0$

$u = ?$: $\ddot{\tilde{x}} + C\dot{\tilde{x}} + D\tilde{x} = 0$

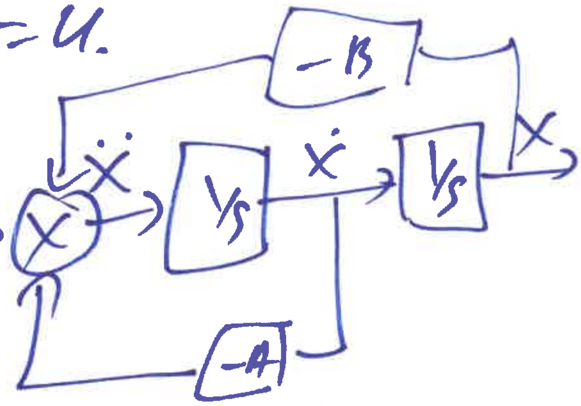
I start from $\ddot{x} + A\dot{x} + Bx = u$

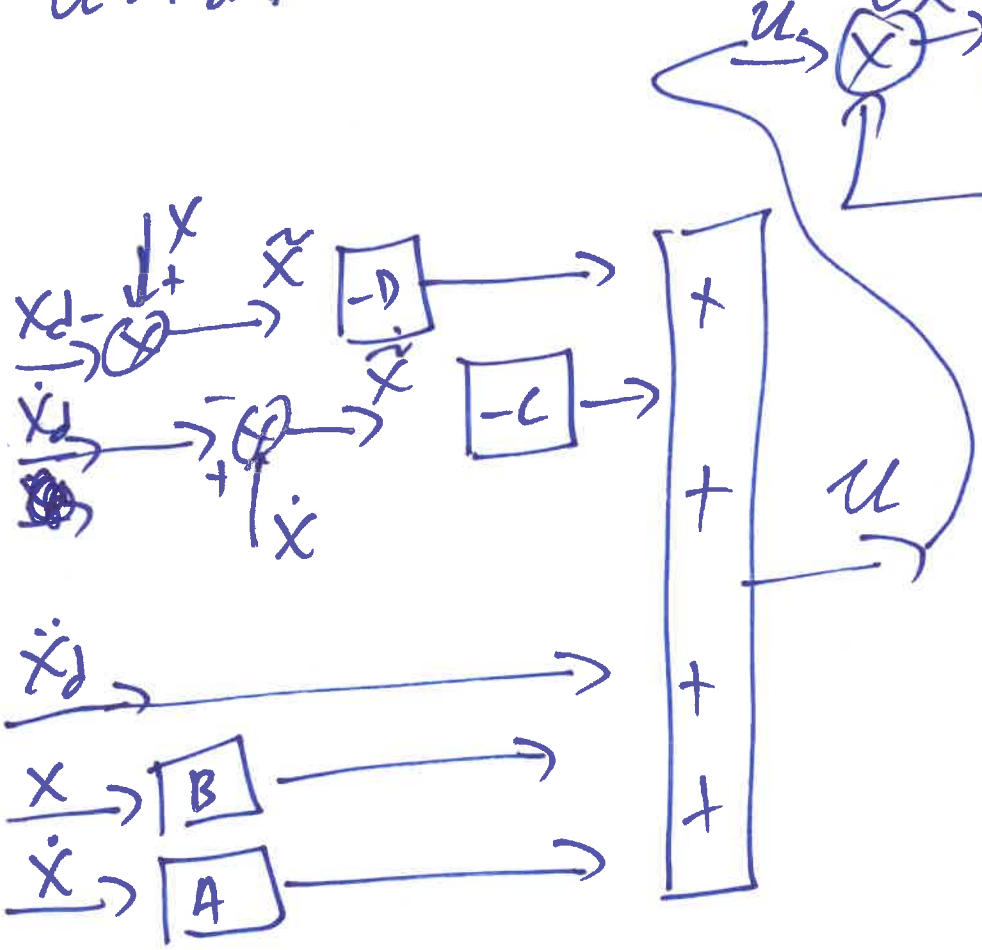
$u = \ddot{x}_d + A\dot{x} + Bx - C\dot{\tilde{x}} - D\tilde{x}$

$$\ddot{x} + \cancel{A\dot{x}} + \cancel{Bx} = \ddot{x}_d + \cancel{A\dot{x}} + \cancel{Bx} - C\dot{\tilde{x}} - D\tilde{x}$$

ODE $\quad \ddot{x} + A\dot{x} + Bx = u.$

$$u = \ddot{x}_d + A\dot{x} + Bx - C\cdot\dot{\tilde{x}} - D\tilde{x}$$





(13)

$$\ddot{x} + A\dot{x} + Bx = u.$$

$$\tilde{x}''' + C\cdot\tilde{x}'' + D\tilde{x}' + E\tilde{x} = 0$$

$$u = A\dot{x} + Bx + C\tilde{x}'' - D\tilde{x}' - E\tilde{x} - \dddot{\tilde{x}} + \ddot{x}$$

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots) + g(\quad)\cdot u.$$

~~Target~~

$$u = \frac{1}{g(\,)}\cdot\left(\; -f(\,) + x_d^{(n)} - h(\tilde{x}, \dot{\tilde{x}}, \ddot{\tilde{x}}\dots) \;\right)$$

$$x^{(n)} = f(\,) + \cancel{g(\,)}\cdot\frac{1}{\cancel{g(\,)}}\cdot\left(-f + x_d^{(n)} - h(\quad)\right)$$

$$= \cancel{f} - \cancel{f} + x_d^{(n)} - h(\tilde{x}, \dot{\tilde{x}}, \ddot{\tilde{x}}, \dots)$$

$$\underbrace{x^{(n)} - x_d^{(n)}}_{\tilde{x}^{(n)}} + h(\tilde{x}, \dot{\tilde{x}}, \dots).$$

$\ddot{x} = f(\dot{x}, x, t) + g(\dot{x}, x, t)\cdot u.$

$x_d = \ldots \qquad \tilde{x} = x - x_d$

Target

ODE for error stable

$u = ?$

$\tilde{x} \to 0$

$\dot{\tilde{x}} \to 0$

Define $s = \dot{\tilde{x}} + \lambda \tilde{x}, \quad \lambda \in \mathbb{R}^+$

$u = ?$: $s \to 0$ or ODE of s is stable

Assume $s = 0$

$\dot{\tilde{x}} = -\lambda \tilde{x}$

$\tilde{x} = e^{-\lambda t} \cdot \tilde{x}(0) \to 0$

$u = ?$: $\exists\, V(s)$ $\begin{cases} V(s) > 0 \\ \dot{V}(s) < 0 \end{cases}$ $\longrightarrow$ ODE of s stable

$\downarrow$

$s \to 0$

$\dot{\tilde{x}} \to 0 \qquad \tilde{x} \to 0$

I choose $V(s) = \frac{1}{2} s^2 > 0$

$u = ?$: $V(s) = \frac{1}{2} s^2 > 0$

$\dot{V}(s) = \frac{1}{2} 2 \cdot s \cdot \dot{s} = s \cdot \dot{s}$

$u = ?$: $V(s) = \frac{1}{2} s^2$

<u>AND</u>

$\dot{V} = s \cdot \dot{s} < 0$

$u = ?$: $s \cdot \dot{s} < 0$

If $s \cdot \dot{s} = -s^2$

If $s = -\dot{s}$

$u = ?$ $s = -\dot{s}$ or $\dot{s} = -s$

$s = \dot{\tilde{x}} + \lambda \tilde{x} \longrightarrow -s = -\dot{\tilde{x}} - \lambda \tilde{x}$

$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}}$

$\tilde{x} = x - x_d$

$\ddot{\tilde{x}} = \ddot{x} - \ddot{x}_d$

$\dot{s} = \underbrace{\ddot{x}}_{f(\,) + g(\,) \cdot u} - \ddot{x}_d + \lambda \dot{\tilde{x}}$

$u = ?$:

$$f(\,) + g(\,)\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda\tilde{x}$$

$$u = \frac{1}{g(\,)}\cdot\left(-f(\,) + \ddot{x}_d - \lambda\dot{\tilde{x}} - \dot{\tilde{x}} - \lambda\tilde{x}\right)$$

$$\dot{x} = f + g\cdot u$$

$$= f + \cancel{g}\cdot\frac{1}{\cancel{g}}\left(-f + \ddot{x}_d - (\lambda+1)\dot{\tilde{x}} - \tilde{x}\lambda\right)$$

$$\ddot{x} = \cancel{f} - \cancel{f} + \ddot{x}_d - (\lambda+1)\cdot\dot{\tilde{x}} - \lambda\tilde{x}$$

$$\ddot{x} = \ddot{x}_d - (\lambda+1)\dot{\tilde{x}} - \lambda\tilde{x}$$

$$\ddot{x} - \ddot{x}_d + (\lambda+1)\dot{\tilde{x}} + \lambda\tilde{x} = 0$$

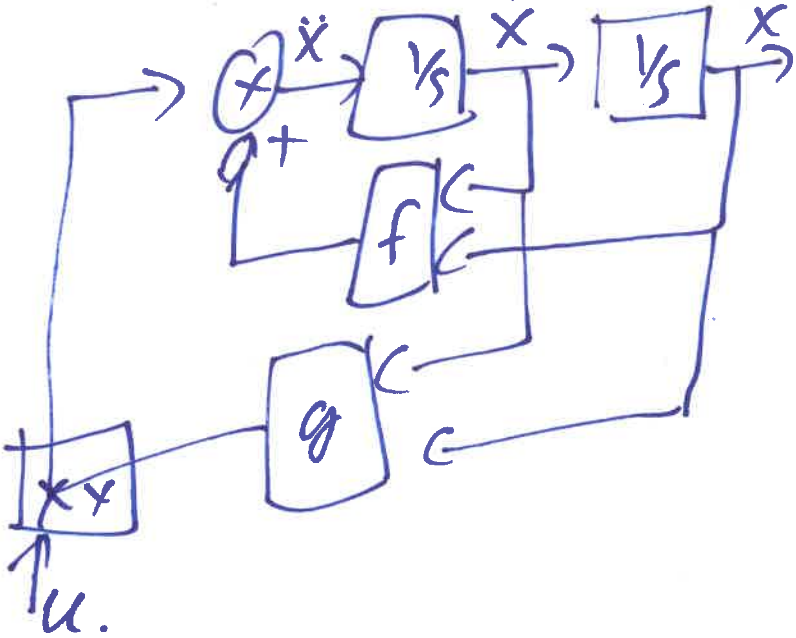
$$\ddot{\tilde{x}} + (\lambda+1)\cdot\dot{\tilde{x}} + \lambda\tilde{x} = 0$$

$$r^2 + (\lambda+1)\cdot r + \lambda = 0 \qquad \Delta = (\lambda+1)^2 - 4\lambda = \lambda^2 + 2\lambda + 1 - 4\lambda = (\lambda-1)^2$$

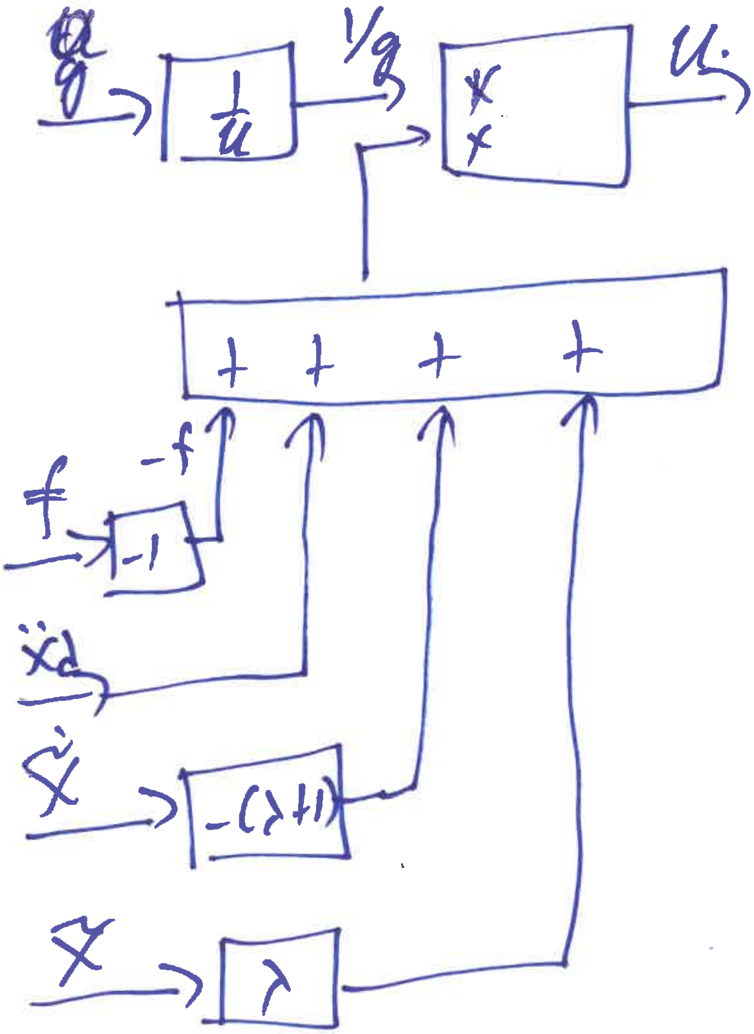
$$r_{1,2} = \frac{-(\lambda+1) \pm (\lambda-1)}{2}$$

$$\cancel{k\in\emptyset}\quad r_1 = -1 \qquad r_2 = -\lambda < 0$$

77



$\ddot{x}$
1/s
$\dot{x}$
1/s
x
+
+
f
g
x
u.



g
$\frac{1}{u}$
1/g
x
x
u
+ + + +
-f
f
-1
$\ddot{x}_d$
$\dot{\tilde{x}}$
$-(\lambda+1)$
$\tilde{x}$
λ

10

$\ddot{x} + 3\dot{x} + 2x = u.$

$\ddot{x} = -3\dot{x} - 2x + u.$

$f = -3\dot{x} - 2x$

$g = 1.$