Revision

$$x^{(n)} = f(x^{(n-1)}, x^{(n-2)}, \dots, x, t) + g(\quad)\cdot u.$$

$x_d = \dots$

$u = ?\ : \ x \to x_d$

$\tilde{x} = x - x_d$

$u = ?\ : \ \tilde{x} \to 0$

Target Error Dynamics

$$\tilde{x}^{(n)} + h(\tilde{x}^{(n-1)}, \dots, \tilde{x}) = 0$$

$$u = \frac{1}{g}\left(-f + x_d^{(n)} - h(\quad)\right)$$

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e.g. $\ddot{x} + A\dot{x} + Bx = u$ $\quad \left(\begin{array}{l} f = -A\dot{x} - Bx \\ g = 1 \end{array}\right)$

Target

$$\ddot{\tilde{x}} + C\dot{\tilde{x}} + D\tilde{x} = 0 \quad (h = C\dot{\tilde{x}} + D\tilde{x})$$

$$u = \frac{1}{1}\left(+A\dot{x} + Bx + \ddot{x}_d - C\dot{\tilde{x}} - D\tilde{x}\right)$$

Sliding Mode Control

$$\ddot{x} = f(\dot{x}, x, t) + g(\dot{x}, x, t)\cdot u$$

$x_d = \ldots$ $\quad \tilde{x} = x - x_d.$ $\quad \tilde{x} \to 0$

$\dot{\tilde{x}} \to 0$

$u = ?$: $\tilde{x} \to 0$ $\boxed{\text{AND}}$ $\dot{\tilde{x}} \to 0$

$$s = \dot{\tilde{x}} + \lambda \tilde{x}, \quad \lambda > 0$$

$u = ?$: $s \to 0$ $\to$ ODE of s to be stable

$\Downarrow$

$V(s) > 0$

$\dot{V}(s) < 0$

$\ddot{x} = f + g\cdot u$, $u = ?$: $\exists V$: $V(s) > 0$

$\dot{V}(s) < 0$

Choose $V(s) = \frac{1}{2} s^2 > 0$

$u = ?$: $\dot{V}(s) \le 0$

But $V(s) = s\cdot\dot{s}$

$u = ?$: $s\cdot\dot{s} < 0$

Choose $\dot{s} = -s \Rightarrow \dot{V}(s) = -s^2 < 0$

$u = ?$: $\dot{s} = -s$ (also try $\dot{s} = -k \cdot s$, $k > 0$)

$$\left.\begin{array}{l} s = \dot{\tilde{x}} + \lambda \tilde{x} \longrightarrow \dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}} \\ \hookrightarrow -s = -\dot{\tilde{x}} - \lambda \tilde{x} \\ \dot{s} = -s \end{array}\right\} \Rightarrow$$

$$\ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x}$$

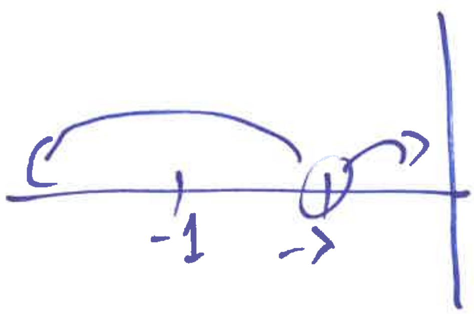
$$\ddot{x} - \ddot{x}_d + \lambda \dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda \tilde{x}$$

$$f + g \cdot u - \ddot{x}_d + \lambda \dot{\tilde{x}} + \dot{\tilde{x}} + \lambda \tilde{x} = 0 \Rightarrow$$

$$u = \frac{1}{g}\left(-f + \ddot{x}_d - (\lambda + 1)\dot{\tilde{x}} - \tilde{x} \cdot \lambda\right).$$

$$\Big\downarrow \ \ddot{x} = f + g \cdot u$$

$$\ddot{\tilde{x}} + (\lambda + 1)\dot{\tilde{x}} + \lambda \tilde{x} = 0 \quad \begin{array}{l} \nearrow r_1 = -1 \\ \searrow r_2 = -\lambda < 0 \end{array}$$



when $s=0 \Rightarrow \dot{\tilde{x}} + \lambda \tilde{x} = 0$

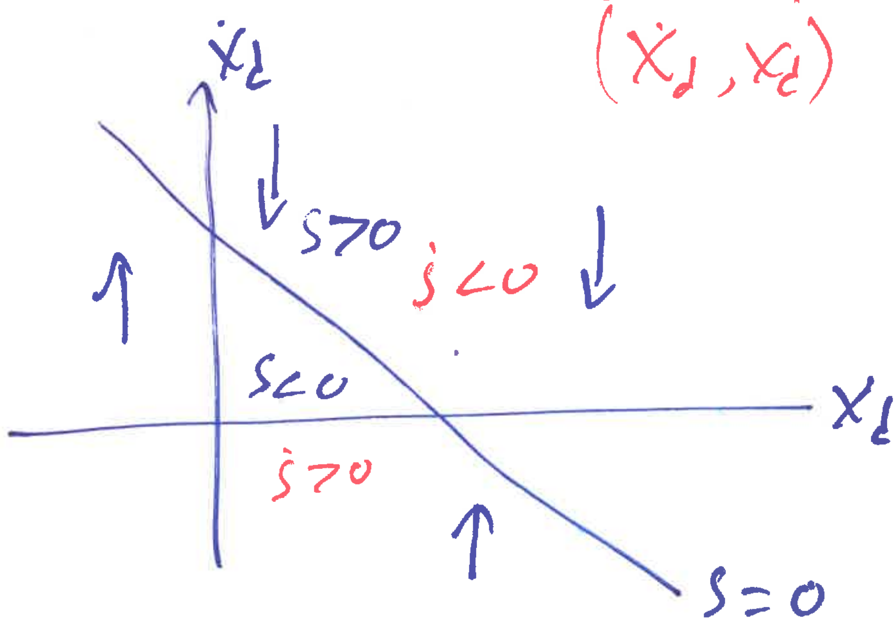
$$\Downarrow$$

$$(\dot{x}-\dot{x}_d) + \lambda (x - x_d) = 0$$

$$\dot{x}-\dot{x}_d = -\lambda (x - x_d)$$

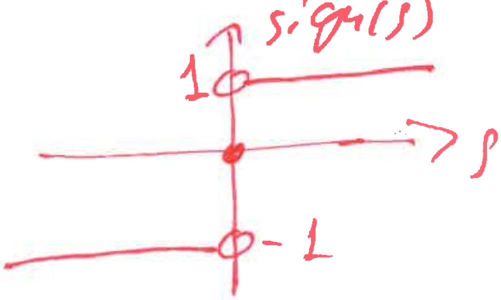
$$\boxed{\begin{aligned} y - y_0 &= \lambda (x - x_0) \\ y &= \lambda x + b \end{aligned}}$$

a family of equs of straight lines, with slope $-\lambda$ that passes through $(\dot{x}_d, x_d)$



$\dot{s} \neq -s$ e.g. $\dot{s} = -0.9 \cdot s$

if $\cancel{\dot{s} \neq s}$ but $\dot{s} = -k \cdot \underbrace{sign(s)}$



Let's find $\dot{V} = s \cdot \dot{s}$

$$= s\,(-k \cdot sign(s))$$

$$= -k \cdot s \cdot sign(s)$$

if $s > 0$ $\quad s \cdot sign(s) = s$

if $s < 0$ $\quad s \cdot sign(s) = -s$

$$f(s) \begin{cases} s, & s > 0 \\ -s, & s < 0 \end{cases}$$

$$|s| \begin{cases} s, & s > 0 \\ -s, & s < 0 \end{cases}$$

So $\dot{V} = -k|s|$

So ~~if~~ $u = ?$: $\dot{s} = -k \cdot sign(s)$

$u = ?$: $\dot{s} = -k \cdot sign(s)$ $\boxed{\dot{s} = -s}$

$$\ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = -k \cdot sign(s)$$

$$f + g \cdot u \overset{-\ddot{x}_d}{+} \lambda \dot{\tilde{x}} = -k\, sign(s)$$

$\vdots$

$$u = \frac{1}{g} \cdot (-f + \ddot{x}_d - \dot{\tilde{x}} \lambda - k \cdot sign(s))$$

$\ddot{x} = f + g \cdot u$ $\qquad \dot{V} = -k|s| < 0$

$= \ldots\ldots\ldots$

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home ex. $1/\Delta T = 3.8\ Hz$ $\qquad k = +0.15$

$\Delta Y = 6.6 \cdot 10^{-4}$

$1/\Delta T = 25\ Hz.$ $\qquad k = +1$

$\Delta Y = 9.9 \cdot 10^{-5}$

$k = 10$

$1/\Delta T = 250\ Hz$

$\Delta Y = 9.8 \cdot 10^{-6}$

$$\boxed{\ddot{x} = f + g \cdot u} \rightarrow \text{Real}$$

$$\ddot{x} = \hat{f} + \hat{g}\, u \rightarrow \text{Estimated / Assumed}$$

$$|\hat{f} - f| \leq F \qquad |\hat{g} - g| \leq G$$

Simplify $g = \hat{g} = 1$

$$u = \frac{1}{1}\left(-\hat{f} + \ddot{x}_d - \lambda \dot{\tilde{x}} - k\,\mathrm{sign}(s)\right)$$

Impact ↑ on $\dot{s}$

of V

on $\dot{V}$

$$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}} = f + \cancel{g} \cdot u - \ddot{x}_d + \lambda \dot{\tilde{x}}$$

$$= f + 1\left(-\hat{f} + \cancel{\ddot{x}_d} - \cancel{\lambda \dot{\tilde{x}}} - k\,\mathrm{sign}(s)\right) - \cancel{\ddot{x}_d} + \cancel{\lambda \dot{\tilde{x}}}$$

$$= f - \hat{f} - k\,\mathrm{sign}(s)$$

if $f = \hat{f} \Rightarrow \dot{s} = -k \cdot \mathrm{sign}(s) \Rightarrow \dot{V} = -k|s| < 0$

86

if $f > \hat{f}$

$$\dot{V} = S \cdot \dot{S} = S \cdot \left(f - \hat{f} - K\,\mathrm{sign}(S) \right)$$

$$|f - \hat{f}| = f - \hat{f}$$

$< F$ $\qquad$ $< F$

$$\dot{V}(S) = S \cdot \left(f - \hat{f} - K\,\mathrm{sign}(S) \right)$$

I want to see how/when $\dot{V} < 0$

$\cancel{S > 0}$ $\qquad \dot{V}(S) = S \cdot \left(f - \hat{f} - K\,\mathrm{sign}(S) \right)$

$$\dot{V}(S) < 0$$

$$S \cdot \left(f - \hat{f} - K\,\mathrm{sign}(S) \right) < 0$$

- $S > 0$ $\qquad (f - \hat{f} - K \cdot 1) < 0$

$$K > f - \hat{f} > F$$