Revision

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L.I. $a, b \in \mathbb{R}^{n\times 1}$ $\nexists$ $k \in \mathbb{R}$

$: \quad a = b \cdot k$

L.I. $a, b, c \in \mathbb{R}^{n\times 1}$ $\nexists$ $k_1, k_2 \in \mathbb{R}$

$: \quad c = k_1 \cdot a + k_2 \cdot b$

$$\left.\begin{array}{l} \dot{x} = A \cdot x \\ x = e^{rt} e \end{array}\right\} \Rightarrow \left.\begin{array}{l} (A - rI)\cdot e = 0 \\ |A - \lambda I| = 0 \end{array}\right\} \left.\begin{array}{l} e = \dots \\ r = \dots \end{array}\right\} x = e^{rt} e$$

if multiple eigenvalues

$$x = t \cdot e \cdot e^{rt} + e^{rt} \cdot b$$

$$e = (A - rI) \cdot b$$

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$$A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$a \neq b$

$\lambda_1 = a, \ \lambda_2 = b$

2 L.I.

$e_1 = [1 \ \ 0]^T$

$e_2 = [0 \ \ 1]^T$

$$e^{At} = \begin{bmatrix} e^{at} & 0 \\ 0 & e^{bt} \end{bmatrix}$$

$$A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$$

$\lambda_1 = a = \lambda_2$

1. L.I.

$e = [1 \ \ 0]^T$

$b = \dots$

$$e^{At} = \begin{bmatrix} e^{at} & te^{at} \\ 0 & e^{at} \end{bmatrix}$$

$$A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$$

$\lambda_1 = \lambda_2 = a$

2 L.I.

$e_1 = [1 \ \ 0]^T$

$e_2 = [0 \ \ 1]^T$

$$e^{At} = \begin{bmatrix} e^{at} & 0 \\ 0 & e^{at} \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$\lambda_1 = a + bi \rightarrow e_1 = [1 \ \ i]^T$

$\lambda_2 = a - bi \rightarrow e_2 = [1 \ \ -i]^T$

$$e^{At} = e^{at} \cdot \begin{bmatrix} \cos bt & \sin bt \\ -\sin bt & \cos bt \end{bmatrix}$$

$$A = \begin{bmatrix} a_1 & 0 \cdots & 0 \\ 0 & a_2 \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & a_n \end{bmatrix}$$

$\lambda_1 = a_1, \ \lambda_2 = a_2 \ \cdots$

n L.I.

$e_1 = [1 \ 0 \ 0 \ \cdots \ 0]^T$

$e_2 = [0 \ 1 \ 0 \ \cdots \ 0]^T$

$\vdots$

$e_n = [0 \ 0 \ 0 \ \cdots \ 1]^T$

$$e^{At} = \begin{bmatrix} e^{a_1 t} & 0 \cdots & 0 \\ 0 & e^{a_2 t} \cdots & 0 \\ \cdots & & e^{a_n t} \end{bmatrix}$$

$$A = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$$

$\lambda_1 = \lambda_2 = \lambda_3 = a$

$(A - aI) \cdot e = 0$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$0 \cdot V_1 + 1 \cdot \cancel{V_2}^{\to 0} + 0 \cdot V_3 = 0$

$0 \cdot V_1 + 0 \cdot V_2 + 1 \cdot \cancel{V_3}^{\to 0} = 0$

1 L.I.

$V_1 = \text{any} = 1$

$e = [1 \ 0 \ 0]^T$

$$e^{At} = \begin{bmatrix} e^{at} & te^{at} & \frac{1}{2} t^2 \cdot e^{at} \\ 0 & e^{at} & te^{at} \\ 0 & 0 & e^{at} \end{bmatrix}$$

$$A = \begin{bmatrix} \alpha & 1 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}$$

$\lambda_1 = \lambda_2 = \lambda_3 = \alpha$

$0 \cdot V_1 + \cancelto{0}{V_2} + 0 \cdot V_3 = 0$

$V_1 = \text{any}$

$V_3 = \text{any}$

2 L.I.

$e_1 = [1 \quad 0 \quad 0]^T \qquad e_2 = [0 \quad 0 \quad 1]^T$

$e_3 = [5 \quad 0 \quad 6]^T$ Not a 3rd

$e_3 = 5 \cdot e_1 + 6 \cdot e_2$ L.I.

$$A = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 1 \\ 0 & 0 & \alpha \end{bmatrix}$$

$\lambda_1 = \lambda_2 = \lambda_3 = \alpha$

$0 \cdot V_1 + 0 \cdot V_2 + \cancelto{0}{V_3} = 0$

$V_1 = \text{any}, \quad V_2 = \text{any}$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2 L.I.

$$A = \begin{bmatrix} \alpha & 1 & 0 & 0 \\ 0 & \alpha & 1 & 0 \\ 0 & 0 & \alpha & 1 \\ 0 & 0 & 0 & \alpha \end{bmatrix}$$

$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \alpha.$

$0 V_1 + \cancelto{0}{V_2} + 0 V_3 + 0 \cdot V_4 = 0$

$0 V_1 + 0 V_2 + \cancelto{0}{V_3} + 0 \cdot V_4 = 0$

$0 V_1 + 0 V_2 + 0 V_3 + \cancelto{0}{V_4} = 0$

$V_1 = \text{any} = 1 \qquad e = [1 \quad 0 \quad 0 \quad 0]^T$

1 L.I.

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 1 & 0 \\ 0 & 0 & a & 1 \\ 0 & 0 & 0 & a \end{bmatrix} \quad \cdots \quad \begin{aligned} v_3 &= v_4 = 0 \\ v_1 &= \text{any} \\ v_2 &= \text{any} \end{aligned}$$

$e_1 = [1 \;\; 0 \;\; 0 \;\; 0]^T$

$e_2 = [0 \;\; 1 \;\; 0 \;\; 0]^T$

2 L I.

$$A = \begin{bmatrix} a & 1 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 1 \\ 0 & 0 & 0 & a \end{bmatrix} \quad \cdots$$

2 LI

$e_1 = [1 \;\; 0 \;\; 0 \;\; 0]^T$

$e_2 = [0 \;\; 0 \;\; 1 \;\; 0]^T$

$$A = \begin{bmatrix} a & 1 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

3 L.I

$e_1 = [1 \;\; 0 \;\; 0 \;\; 0]^T$

$e_2 = [0 \;\; 0 \;\; 1 \;\; 0]^T$

$e_3 = [0 \;\; 0 \;\; 0 \;\; 1]^T$

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

4 L.I.

$$A = \begin{bmatrix} a & & & & \\ & a & & & \\ & & a & & \\ & & & \ddots & \\ & & & & a \end{bmatrix} \to 1 \text{ or } 0$$

# of zeros = how many +1 L.I.

$$A_1 = \begin{bmatrix} a & b & 0 & 0 \\ -b & a & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix}, \quad A_1 = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}, \quad B = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

4 L.I

$$A_2 = \begin{bmatrix} a & b & 1 & 0 \\ -b & a & 0 & 1 \\ 0 & 0 & a & b \\ 0 & 0 & -b & a \end{bmatrix}, \quad A_2 = \begin{bmatrix} B & I \\ 0 & B \end{bmatrix}$$

2 L.I.

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1. $e_1 = [1 \; i \; 0 \; 0]$, $e_2 = [1 \; -i \; 0 \; 0]$

   $e_3 = [0 \; 0 \; 1 \; i]$, $e_4 = [0 \; 0 \; 1 \; -i]$

2. $e_1 = [1 \; i \; 0 \; 0]$, $e_2 = [1 \; -i \; 0 \; 0]$

   $e_3 = e_1$ $\qquad$ $e_4 = e_1$

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$$A_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

4 L.I

$$A_2 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

4 L.I.

$$A_3 = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3 L.I.

$$A_4 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

4 L.I

$$A_5 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

3. L.I.

$$A=\begin{bmatrix}-1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & -4\end{bmatrix}$$

4 L.I

$$A=\begin{bmatrix}-1 & 1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & -1 & 1\\ 0 & 0 & -1 & 1\end{bmatrix}$$

4 L.I.

$$A=\begin{bmatrix}-1 & 1 & 1 & 0\\ -1 & 1 & 0 & 1\\ 0 & 0 & -1 & 1\\ 0 & 0 & -1 & 1\end{bmatrix}$$

2. L.I

$$A=\begin{bmatrix}-1 & 1 & 3 & 0\\ -1 & 1 & 0 & 3\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & -4\end{bmatrix}$$

$$\dot{x} = A x + B\cdot u$$
$$y = C\cdot x + D\cdot u$$

$\Rightarrow$ "Better" S.S. model?

$x \in \mathbb{R}^{n\times 1} \rightarrow A \in \mathbb{R}^{n\times n}$

$u \in \mathbb{R}^{p\times 1} \rightarrow B \in \mathbb{R}^{n\times p}$

$y \in \mathbb{R}^{q\times 1} \rightarrow C \in \mathbb{R}^{q\times n}$

Similarity transformation

$$\underset{n\times 1}{x} = \underset{n\times n}{T}\cdot \underset{n\times 1}{z} \xRightarrow{T=\text{const.}} \dot{x} = T\cdot \dot{z}$$

T inv $\quad z = T^{-1}\cdot x \Rightarrow \dot{z} = T^{-1}\cdot \dot{x}$

$$T\cdot \dot{z} = A\cdot T\cdot z + B\cdot u$$
$$\dot{z} = \underbrace{T^{-1} A\cdot T}_{\substack{n\times n \\ \hat{A}}}\cdot z + \underbrace{T^{-1} B}_{\substack{n\times p \\ \hat{B}}}\cdot u$$

$$\boxed{\dot{z} = \hat{A}\cdot z + \hat{B}\cdot u}$$

$\hat{A} = T^{-1} A T$

$\hat{B} = T^{-1} B$

$\hat{C} = C T$

$C = \hat{C}\cdot T^{-1}$

$B = T\cdot \hat{B}$

$A = T\cdot \hat{A}\cdot T^{-1}$

$$y = C\cdot x + D\cdot u$$
$$= \underbrace{C\cdot T}_{\hat{C}}\cdot z + \underbrace{D}_{\hat{D}}\cdot u$$

$$\boxed{y = \hat{C}\cdot z + \hat{D}\cdot u}$$

$$|A-\lambda I|=0$$

$$|\hat{A}-\lambda I|=0$$

$$|T^{-1}A\cdot T-\lambda T^{-1}T|=0$$

$$|T^{-1}A\cdot T-\lambda\cdot T^{-1}I\cdot T|=0$$

$$|T^{-1}(A-\lambda I)\cdot T|=0$$

$$\cancel{|T^{-1}|}\cdot|A-\lambda I|\cdot\cancel{|T|}=0$$

$$|A-\lambda I|=0$$

$$G_x=C\cdot(sI-A)^{-1}B$$

$$\Downarrow$$

$$G_z=\hat{C}\cdot(sI-\hat{A})^{-1}\hat{B}$$

$$G_x=C\cdot I\cdot(sI-A)^{-1}I\cdot B$$

$$=\underbrace{C\cdot T}\cdot T^{-1}\cdot(sI-A)^{-1}\cdot T\cdot\underbrace{T^{-1}\cdot B}$$

$$\hat{C}\cdot T^{-1}(sI-A)^{-1}\cdot T\ \hat{B}$$

I want $(sI-\hat{A})^{-1}=T^{-1}(sI-A)^{-1}T$

$$T^{-1}\cdot(sI-A)^{-1}\cdot T=(sI-\hat{A})^{-1}$$

$$(s\,T^{-1}I\cdot T-T^{-1}AT)$$

$$(T^{-1}(sI-A)\cdot T)^{-1}$$

$$T^{-1}(sI-A)^{-1}\cdot T$$

$$\boxed{A=T\cdot\hat{A}\cdot T^{-1}}$$

$$e^{At} \quad \overset{?}{} \quad e^{\hat{A}t}$$

$$e^{At}=I+At+\frac{(At)^2}{2!}+\frac{(At)^3}{3!}+\dots$$

$$=I+T\cdot\hat{A}\cdot T^{-1}t+\frac{(T\cdot\hat{A}\cdot T^{-1})^2\cdot t^2}{2!}+\dots$$

$$(T\cdot\hat{A}\cdot T^{-1})^2=(T\cdot\hat{A}\cdot T^{-1})\cdot(T\cdot\hat{A}\cdot T^{-1})$$

$$=T\cdot\hat{A}^2\cdot T^{-1}$$

$$I=T\cdot I\cdot T^{-1}$$

$$e^{At}=T\cdot I\cdot T^{-1}+T\hat{A}T^{-1}\cdot t+\frac{T\cdot\hat{A}^2\cdot T^{-1}\cdot t^2}{2!}+\dots$$

$$=T\cdot\left(I+\hat{A}t+\frac{\hat{A}^2t^2}{2!}+\dots\right)T^{-1}$$

$$=T\,e^{\hat{A}\cdot t}\cdot T^{-1}$$

$$\dot{x} = Ax + Bu$$
$$y = C \cdot x$$
$$x = T \cdot z$$
$$\boxed{T = ?}$$

$$\hat{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}, \quad A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

$$\dot{z} = \hat{A} z + \hat{B} u$$
$$y = \hat{C} \cdot x$$
$$e^{\hat{A} t}$$
$$e^{At} = T \cdot e^{\hat{A} t} \cdot T^{-1}$$

$$T = ?$$

$$A \begin{cases} \to a \Rightarrow \lambda_1 \\ \to b \Rightarrow \lambda_2 \end{cases} \quad a \neq 0$$

$$(A - \lambda I)\, e_1 = 0$$

$$A \cdot e_1 = \lambda_1 e_1$$

$$\underbrace{A}_{n\times n} \cdot \underbrace{e_2}_{n\times 1} = \underbrace{\lambda_2}_{1\times 1} \cdot \underbrace{e_2}_{n\times 1}$$

$$\underbrace{A}_{n\times n} \cdot \underbrace{[\underbrace{e_1}_{n\times 1} \quad \underbrace{e_2}_{n\times 1}]}_{n\times 2} = \underbrace{\overset{\text{inv.}}{[e_1 \quad e_2]}}_{n\times 2} \cdot \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{\times\ 2\times 2}$$

$$A \cdot T = T \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad T = [e_1 \quad e_2]$$

$$AT = T \cdot \Lambda$$

$$AT = T \cdot \hat{A}$$

$$A = T \cdot \hat{A} \cdot T^{-1}$$