$$\frac{d^{(n)}x}{dt^{(n)}} = f(x^{(n-1)}, x^{(n-2)}, \ldots \ddot{x}, \dot{x}, t)$$ 11

$\downarrow$ Linear

$$x^{(n)} + p_{n-1}x^{(n-1)} + \ldots + \cancel{p_0}\cancel{x} = 0$$

$x_1, x_2, x_3 \ldots x_k$ are solns

$\varphi = \sum_{i=1}^{k} x_i \cdot c_i$ is also a soln

$x_1, x_2 \ldots x_n$ L.I. if $|W| \neq 0$

$$W = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \\ \vdots & & & \\ x_1^{n-1} & x_2^{n-1} & \ldots & x_n^{n-1} \end{bmatrix}$$

A pos soln is $x = e^{rt}$

$r^n + p_{n-1}r^{n-1} + \ldots + p_0 = 0$ ← C.E.

- $r_1 \neq r_2 \ldots$ $x_1 = e^{r_1 t}$, $x_2 = e^{r_2 t}$ .... $r_1, r_2 \in \mathbb{R}$.
- $r = a \pm bi$, $x_1 = e^{(a+bi)t}$, $x_2 = e^{(a-bi)t}$
- $r = r_1 = r_2 = \ldots r_k$ $x_1 = e^{rt}$, $x_2 = te^{rt}$ .... $x_k = t^{k-1} \cdot e^{rt}$

$\downarrow$
$r \in \mathbb{R}$

$$x = \sum_{i=1}^{n} c_i \cdot x_i$$

$\dot{X} = A \cdot X$, $X = [x_1 \ x_2 \dots x_n]^T$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ & & & \\ a_{n1} & \dots & \dots & a_{nn} \end{bmatrix}$$

k solns $X^{(1)}, X^{(2)} \dots \dots X^{(k)}$

$\varphi = \sum_{i=1}^{k} c_i X^{(i)}$ is also a soln

n L.I. solns

$|W| \neq 0$, $W = [X^{(1)} \ X^{(2)} \ X^{(3)} \dots X^{(n)}]$

any soln $X = \sum_{i=1}^{n} c_i \cdot X^{(i)}$

Try $\underbrace{X}_{\text{vector}} = e^{rt} \cdot \underbrace{e}_{\text{vector}}$

$r \in \mathbb{R}$ or $r \in \mathbb{C}$

$(A - rI) \cdot e = 0 \Rightarrow |A - rI| = 0$

$\hookrightarrow r_1 = \dots$
$r_2 = \dots$
$r_3 = \dots$

$\left.\right\} \begin{matrix} e_1 & e^{(1)} \\ e_2 & e^{(2)} \\ e_3 & e^{(3)} \end{matrix}$

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- $r_1 \neq r_2 \neq r_3 \dots \quad X = \sum_{i=1}^{n} e^{(i)} \cdot C_i \cdot e^{r_i t}$

- $r = r_1 = r_2$

If $X^{(1)} = e^{rt} \cdot e^{(1)}$

If I try $\underbrace{X^{(2)} = t \cdot e^{rt} \cdot e^{(1)}}$

$$\dot{X} = A \cdot X$$

$$e^{(1)} \cancel{e^{rt}} = 0 \Rightarrow e^{(1)} = 0$$

If I try $X^{(2)} = e^{rt} \cdot e^{(1)} t + e^{rt} \cdot e^{(2)}$

$$= e^{rt} (t e^{(1)} + e^{(2)})$$

$$\dot{X}^{(2)} = r e^{rt} (t e^{(1)} + e^{(2)}) + e^{rt} e^{(1)}$$

$$A \cdot X^{(2)} = A \cdot (e^{rt} \cdot (t e^{(1)} + e^{(2)}))$$

$$= r e^{rt} \cdot t e^{(1)} + r e^{rt} \cdot e^{(2)} + e^{rt} e^{(1)}$$

$$= r \cdot t \cdot e^{rt} e^{(1)} + e^{rt} \cdot \boxed{(r e^{(2)} + e^{(1)})}$$

$$= e^{rt} \cdot t \cdot (A e^{(1)}) + e^{rt} \cdot \boxed{(A e^{(2)})}$$

- $r e^{(1)} = A e^{(1)} \Rightarrow (A - rI) e^{(1)} = 0$
- $r e^{(2)} + e^{(1)} = A \cdot e^{(2)}$

$$r \cdot e^{(2)} + e^{(1)} = A \cdot e^{(2)}$$

$$e^{(1)} = (A - rI)\, e^{(2)} \Rightarrow$$

$$(A - rI)^2 \cdot e^{(2)} = 0$$

Gen. eigenvector

$$x' = A \cdot x \qquad x^{(1)} = e^{rt} \cdot e^{(1)}$$

$r_1, r_2, r_3 \ldots\ldots$

- If $r_1 \neq r_2 \ldots$ $\quad X(t) = \sum_{i=1}^{n} x^{(i)} \cdot e^{(i)} e^{r_i t}$

- If $r = r_1 = r_2$

in ODEs $\rightarrow$ $x = t \cdot e^{rt}$

in S.S. $\rightarrow$ $X = t\, e^{rt} \cdot e^{(1)} + e^{(2)} e^{rt}$

$$(A - rI)^2 \cdot e^{(2)} = 0$$

$$(A - rI)\, e^{(1)} = 0$$

$\dot{X} = A \cdot X + \cancel{Bu}$

$X = e^{At} \cdot X_0$

where $e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!}$

$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$ $\lambda_1, \lambda_2 \in \mathbb{R}$.

$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

Is it pos. $\left.\begin{array}{l} \dot{X} = AX + Bu \\ Y = C \cdot X + Du \end{array}\right\} \Longrightarrow$ Easier

$X = T \cdot Z$ or $\dot{X} = T \cdot \dot{Z}$

or or

$Z = T^{-1} \cdot X$ $\dot{Z} = T^{-1} \dot{X}$

$\dot{X} = AX + B \cdot u \iff T \cdot \dot{Z} = A \cdot T \cdot Z + Bu$

$\dot{Z} = \underbrace{T^{-1} \cdot A \cdot T}_{\substack{n \times n \\ \hat{A}}} \cdot Z + \underbrace{T^{-1} \cdot B}_{\hat{B}} u$

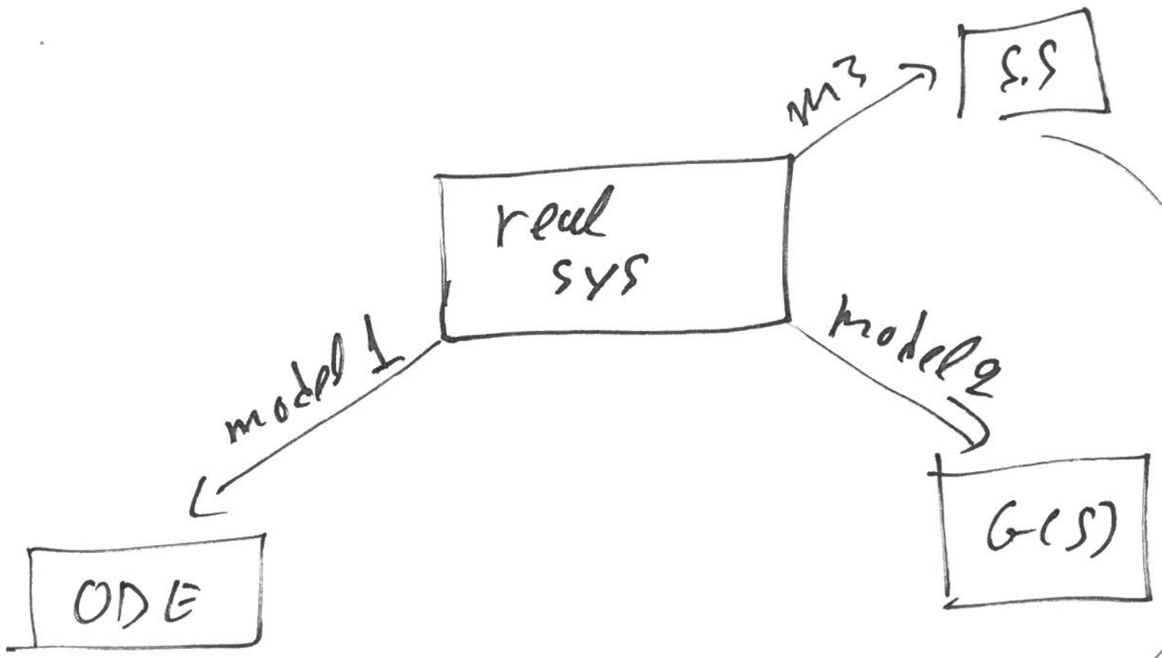
$u \in \mathbb{R}^{p \times 1}$

~~$B \in n \times p$~~

$B \in \mathbb{R}^{n \times p}$

$\underset{n \times n}{T^{-1}} \; \underset{n \times p}{B} = \underset{n \times p}{\hat{B}}$

$\dot{Z} = \hat{A} \cdot Z + \hat{B} \cdot u$



ss: $\dot{x} = Ax + Bu$

$y = C\cdot x + D\cdot u$

$\dot{z} = \hat{A}\cdot z + \hat{B}\cdot u$

$y = \hat{C}\cdot z + \hat{D}\cdot u$

T: inv. $x = T\cdot z \Rightarrow$

$\dot{z} = \hat{A}\cdot z + \hat{B}\cdot u$

$y = \underbrace{C\,T}_{\hat{C}}\cdot z + \underbrace{D}_{\hat{D}}\cdot u$

$\hat{A} = T^{-1} A\cdot T$

$\hat{B} = T^{-1} B$

$G_A(s) = C\cdot(sI - A)^{-1} B$

$G_{\hat{A}}(s) = \hat{C}\cdot(sI - \hat{A})^{-1}\hat{B}$

$$G_A = C\cdot(sI-A)^{-1}\cdot B$$

$$= C\cdot I\cdot(sI-A)^{-1}\cdot I\cdot B$$

$$= C\cdot(T\cdot T^{-1})\cdot(sI-A)^{-1}\cdot(T\cdot T^{-1})\cdot B$$

$$= \underbrace{C\cdot T}_{\bar{C}}\cdot T^{-1}\cdot(sI-A)^{-1}\, T\cdot \underbrace{T^{-1}\, B}_{\bar{B}}$$

$$\bar{C}\cdot T^{-1}\cdot(sI-A)^{-1}\cdot T\cdot \bar{B}$$

I want to prove that

$$T^{-1}\,(sI-A)^{-1}\,T = (sI-\bar{A})^{-1}$$

$$C^{-1} \quad B^{-1} \quad A^{-1}$$

$$(ABC)^{-1} = C^{-1}\cdot B^{-1}\cdot A^{-1}$$

$$(A\cdot B\cdot C)^{-1}$$

$$(T^{-1}\,(sI-A)\cdot T)^{-1}$$

$$(\underbrace{T^{-1}\cdot sI\cdot T}_{} - T^{-1}A\cdot T)^{-1}$$

$$(sI-\bar{A})^{-1}$$

$\dot{x} = A \cdot x \qquad \dot{z} = \hat{A} \cdot z$

if $A = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \quad e^{At} = I + At + \dots$

$\hat{A} = \begin{bmatrix} \square & 0 \\ 0 & \square \end{bmatrix} \quad e^{\hat{A}t} = \begin{bmatrix} e^{\square t} & 0 \\ 0 & e^{\square t} \end{bmatrix}$

zeros

prev. T must be inv.

$+ \dots$

$+ \dots$

$x = T \cdot z \longrightarrow \hat{A} = \begin{bmatrix} \square & 0 \\ 0 & \square \end{bmatrix}$

$A \cdot e_1 = \lambda_1 e_1$

$A e_q = \lambda_q \cdot e_q$

$\cancel{A \cdot \underbrace{[e_1 \quad e_q]}_{T} = \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_q \end{bmatrix}}_{\hat{A}} \cdot \underbrace{[e_1 \quad e_q]}_{T}}$

$A \cdot [e_1 \quad e_q] = \underbrace{[e_1 \quad e_q]}_{T} \cdot \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_q \end{bmatrix}}_{\hat{A}}$.

$T^{-1} A \cdot T = \hat{A}$

$\hat{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

$(\hat{A} - \lambda_1 I) e_1 = 0$

$$\left(\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} - \lambda_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) e_1 = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \lambda_2 - \lambda_1 \end{bmatrix} \cdot e_1 = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & \lambda_2 - \lambda_1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = 0$$

$0 \cdot X + (\lambda_2 - \lambda_1) \cdot Y = 0$

$X = \text{any}$ $Y = 1.$ $\therefore \lambda_1 \rightarrow e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\therefore \lambda_2 \rightarrow e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$e^{At} = I + At + \frac{(At)^2}{2!} + \frac{(At)^3}{3!} + \dots$$ (20)

$$e^{\hat{A}t} = I + \hat{A}t + \frac{(\hat{A}t)^2}{2} + \dots$$

$$\hat{A} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \qquad \hat{A}^2 = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix}$$

$$\hat{A}^3 = \begin{bmatrix} \lambda_1^3 & 0 \\ 0 & \lambda_2^3 \end{bmatrix} \dots$$

$$e^{\hat{A}t} = \underbrace{I}_{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} t + \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} \cdot \frac{t^2}{2} + \begin{bmatrix} \lambda_1^3 & 0 \\ 0 & \lambda_2^3 \end{bmatrix} \cdot \frac{t^3}{3!}$$

$$= \begin{bmatrix} 1 + \lambda_1 t + \frac{\lambda_1^2 t^2}{2} + \frac{\lambda_1^3 \cdot t^3}{3!} + \dots & 0 \\ 0 & 1 + \lambda_2 t + \frac{\lambda_2^2 t^2}{2} + \dots \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

$$e^{\lambda t} = 1 + \lambda t + \frac{\lambda^2 t^2}{2} + \frac{\lambda^3 \cdot t^3}{3!} + \dots$$

nth $\lambda_1 \neq \lambda_2 \neq \lambda_3 \ldots\ldots$

$$\hat{A} = \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ 0 & \cdots & - & - & - \\ - & \cdots & - & \cdots & \lambda_n \end{bmatrix}$$

$$e^{\hat{A}t} = \begin{bmatrix} e^{\lambda_1 t} & 0 & 0 & \cdots \\ \cdot & - & - & - \\ & \cdots & \cdots & e^{\lambda_n t} \\ - & - & \cdot & \cdot \end{bmatrix}$$