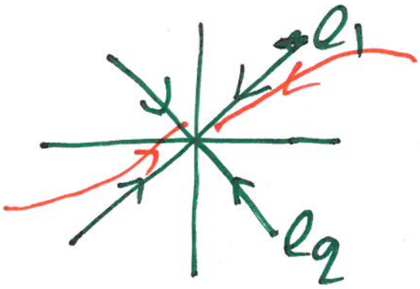
Revision

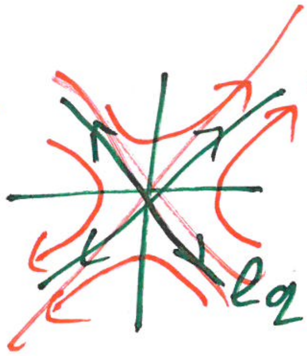
$\dot{X} = AX \quad , \quad X \in \mathbb{R}^{2\times 1}$

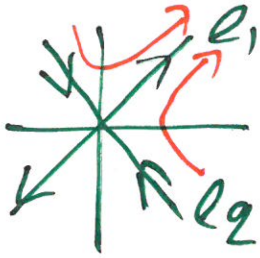
- $r_1 \neq r_2 \in \mathbb{R}$

$\downarrow \qquad \downarrow$

$e_1 \qquad e_2$







$r_1, r_2 < 0$ — stable Node

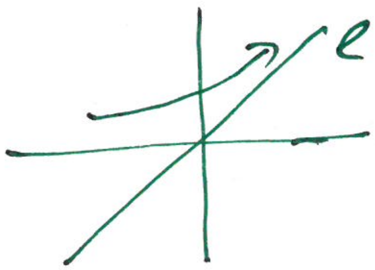
$r_1, r_2 > 0$ — unstable Node

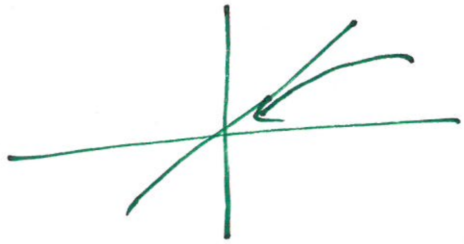
$r_1 > 0, r_2 < 0$ — Saddle

- $r_1 = r_2$

$\downarrow$

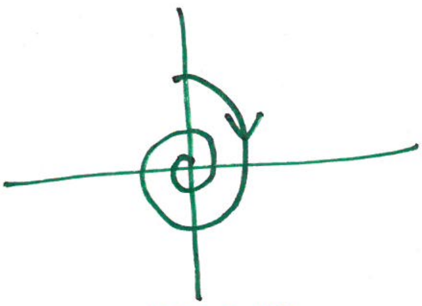
e_1



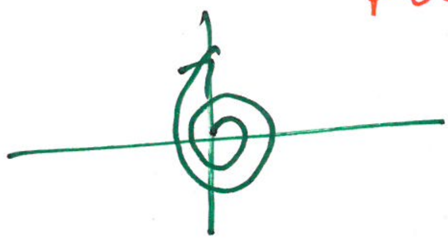


- $r = a \pm bi$

Focus



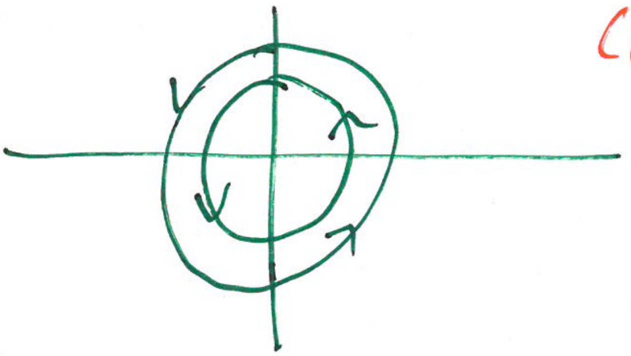
$a < 0$



$a > 0$

- $r = \pm bi$

Centre



F.P. = E.P. = S.P.

$\dot{X} = 0 \xrightarrow{u=0} X_{EP} = 0$

$\hookrightarrow X_{EP} = -A^{-1} \cdot B \cdot u$

$\dot{x} = f(x,a)$

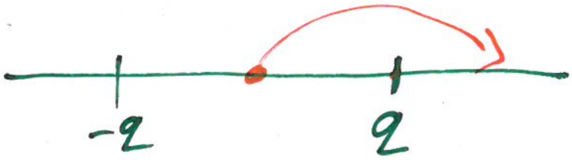
1) if $x_1 = \text{sol}_n \not\Rightarrow x_2 = k \cdot x_1$ is a sol_n

2) F.P.s.

- Mult. F.P.s.

$\dot{x} = x^2 - 4$

$\dot{x} = 0 \Rightarrow x = \pm 2$

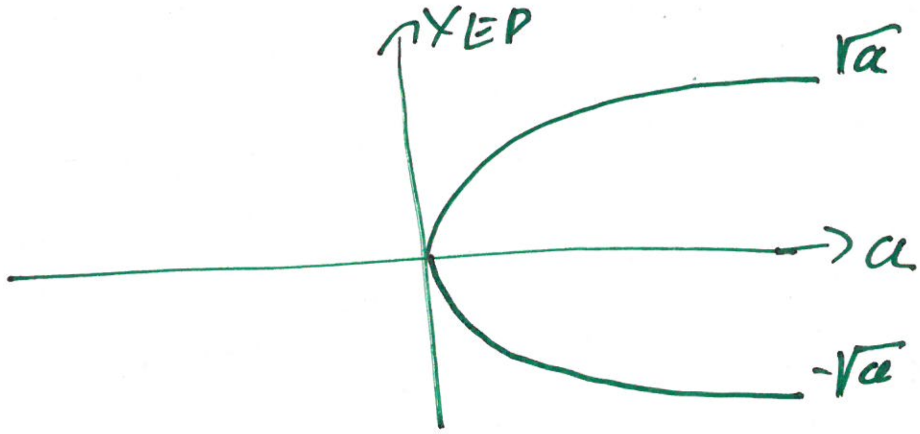


- No F.P.s.

$\dot{x} = x^2 + 4 \quad \nexists\, x : \dot{x} = 0$

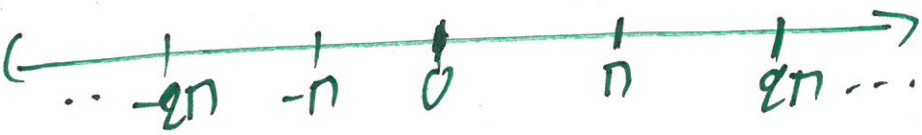
- $\dot{x} = x^2 - a$ $\xrightarrow{a>0}$ $x_{EP} = \pm\sqrt{a}$

$\xrightarrow{a<0}$ $\cancel{x_{EP}}$



$\dot{x} = \sin(x)$

$\dot{x} = 0 \Rightarrow \sin(x) = 0 \Rightarrow x_{EP} = \pi \pm k\pi \quad k = 0, 1, \ldots$



$\dot{x}_1 = x_1 - x_2$

$\dot{x}_2 = x_1^2 + x_2^2 - 2$

$\dot{x}_1 = 0 \Leftrightarrow x_1 = x_2$

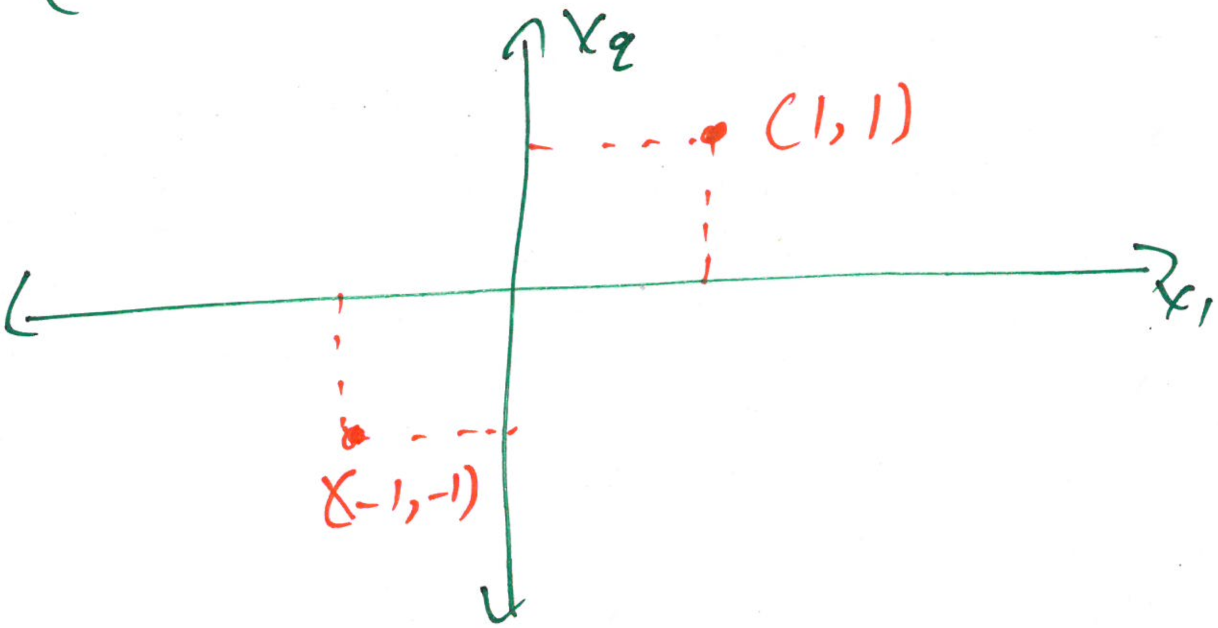
$\dot{x}_2 = 0 \Leftrightarrow x_1^2 + x_2^2 - 2 = 0$

$2 \cdot x_1^2 - 2 = 0$

$x_1 = \pm 1 \Rightarrow x_2 = \pm 1$

$(x_1, x_2) = (1, 1)$

$(x_1, x_2) = (-1, -1)$



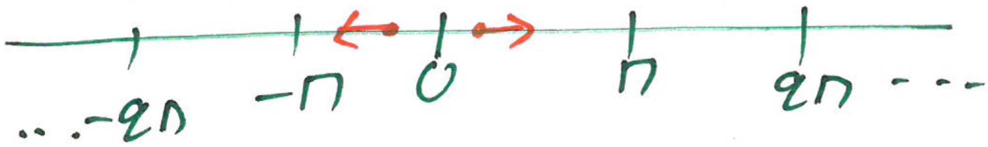
L.Sys stable or Unstable sys

NL.Sys stable or Unstable F.Ps.

$\dot{x} = \sin(x)$

$\sin(x) = 0$

$x_{EP} = \pi \pm k\pi$



Taylor Series

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\dot{x} \simeq x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \qquad x \in [-2, 2]$$

$$\dot{x} \simeq x \Rightarrow x = e^t \qquad x \in [-0.5, 0.5]$$

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0}(x - x_0) + \cancel{\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=0}(x - x_0)^2} + \dots$$

$$f(x) \simeq f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0}(x - x_0)$$

$$\dot{x} = f(x) \simeq f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0}(x - x_0)$$

$$\dot{x} \simeq \underbrace{f(x_{EP})}_{0} + \underbrace{\left.\frac{\partial f}{\partial x}\right|_{x=x_{EP}}}_{A}\underbrace{(x - x_{EP})}_{\Delta x}$$

$$\dot{x} \simeq A \cdot \Delta x$$

$$\dot{x} - \dot{x}_{EP} \simeq A \cdot \Delta x$$

$$(x - x_{EP})' \simeq A \cdot \Delta x$$

$\Rightarrow \Delta\dot{x} \simeq A \cdot \Delta x$

$$\frac{df}{dx}\Big|_{x=x_{EP}} \rightarrow \text{Jacobian of } f.$$

$\dot{x} = -x^2+1 = f(x)$

F.P. $\dot{x} = 0 \Rightarrow x = \pm 1$

$$A = \frac{df}{dx}\Big|_{x=x_{EP}}$$

$$= -2 \cdot x \Big|_{x_{EP}}$$

$x_{EP}=1$: $-2 \cdot x \rightarrow \Delta\dot{x} = -2\Delta x \rightarrow \Delta x = e^{-2 \cdot t} \cdot C$ stable

$x_{EP}=-1$: $2x \rightarrow \Delta\dot{x} = +2\Delta x \rightarrow \Delta x = e^{2t} \cdot D$ Unstable

$\dot{x} = \sin(x)$ $\qquad x_{EP} = \pi \pm k\pi$

$f(x) = \sin(x)$

$A = \frac{\partial f}{\partial x} = \cos x.$

$$f(x) \simeq \sin(x_{EP}) + \cos x_{EP} \cdot (x - x_{EP})$$

• $x_{EP} = 0$

$$f(x) \simeq 0 + 1 \cdot (x - 0)$$

$$\Delta\dot{x} \simeq \Delta x \Rightarrow \Delta x = e^{t} \cdot C.$$

unstable

• $x_{EP} = \pi.$

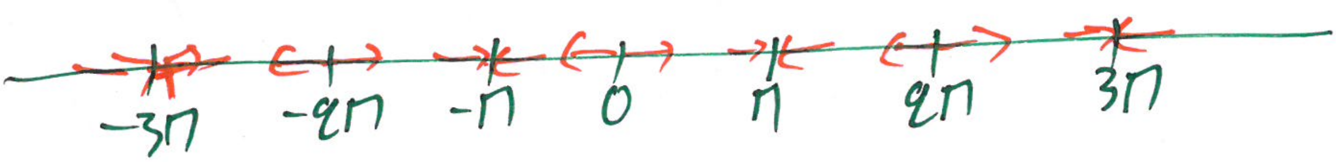
$$f(x) \simeq \cancel{\sin(\pi)}^{\to 0} + \cancel{\cos\pi}^{\to -1} \overbrace{(x - \pi)}^{\to \Delta x}$$

$$\Delta\dot{x} = -\Delta x \Rightarrow \Delta x = e^{-t} \cdot D$$

• $x_{EP} = -\pi$

$$f(x) \simeq \cancel{\sin(-\pi)}^{\to 0} + \cancel{\cos(-\pi)}^{\to -1} \cdot \overbrace{(x + \pi)}^{\to \Delta x}$$

$$\Delta\dot{x} = -\Delta x \Rightarrow \Delta x = e^{-t} \cdot P$$



$$\dot{x} = x - y$$

$$\dot{y} = x + y - 2 \cdot x \cdot y$$

$$f(x,y) = \begin{bmatrix} x - y \\ x + y - 2 \cdot x \cdot y \end{bmatrix}$$

$$f(x,y) = 0 \Rightarrow x = y$$

$$x + y - 2 \cdot xy = 0$$

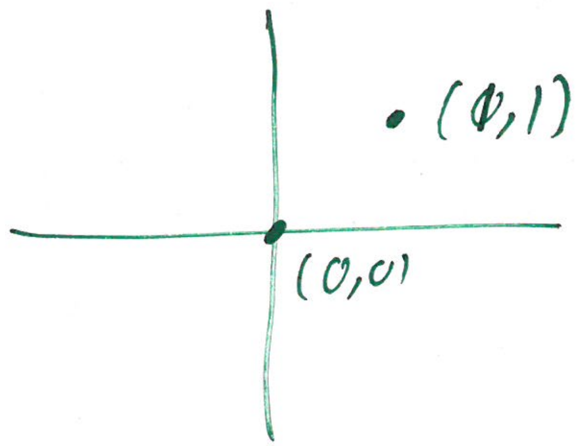
$$2 \cdot x - 2 \cdot x^2 = 0$$

$$2 \cdot x (1 - x) = 0$$

- $x = 0$
- $1 - x = 0 \Rightarrow x = 1$

$(x,y) = (0,0)$,

$(x,y) = (1,1)$



$$f(x,y) = \begin{bmatrix} x-y \\ x+y-2\cdot x\cdot y \end{bmatrix} = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix}$$

$$A = \frac{\partial f}{\partial(x,y)} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x} = 1 \qquad \frac{\partial f_1}{\partial y} = -1$$

$$\frac{\partial f_2}{\partial x} = 1+0-2\cdot y \qquad \frac{\partial f_2}{\partial y} = 1-2\cdot x,$$

$$A = \begin{bmatrix} 1 & -1 \\ 1-2y & 1-2x \end{bmatrix}$$

$$A(0,0) = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\Delta\dot{x} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \Delta x$$

The part. near $(0,0)$

(45)

$$A(1,1) = \begin{bmatrix} 1 & -1 \\ 1-2 & 1-2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$\Delta\dot{x} = \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix} \cdot \Delta x$$ pert. near (1,1)

$A(0,0) \rightarrow \lambda_{1,2} = 1 \pm 2i$ unstable Focus

$A(1,1) \rightarrow \lambda_1 = 1.41 \rightarrow e_1 = \begin{bmatrix} -2.41 \\ 1 \end{bmatrix}$

$\rightarrow \lambda_2 = -1.41 \rightarrow e_2 = \begin{bmatrix} -1 \\ -2.41 \end{bmatrix}$ Saddle