Revision

$\dot{x} = f(x, t)$

$\dot{x} = 0 \Rightarrow f(x_{EP}, t) = 0$

Stability (Local)

$$\Delta \dot{x} = \underbrace{\left.\frac{\partial f}{\partial x}\right|_{x = x_{EP}}}_{A} \cdot \underbrace{\Delta x}_{x - x_{EP}}$$

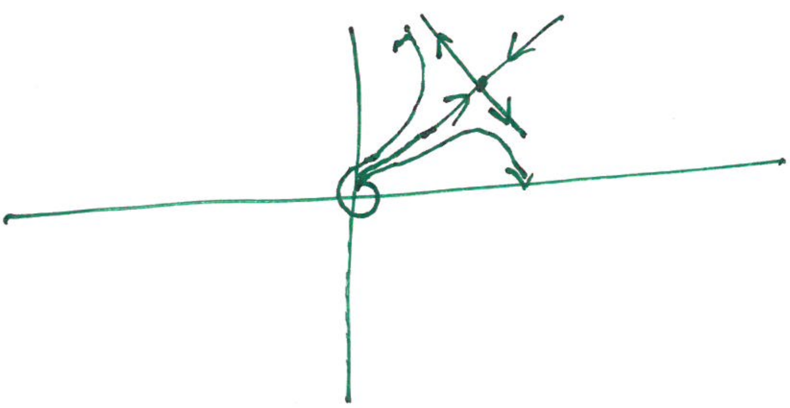
Assume $f(x_1, x_2) = \begin{bmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{bmatrix}$.

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

e.g.

$f = \begin{bmatrix} x - y \\ x + y - 2 \cdot x \cdot y \end{bmatrix}$ F.P. $(1, 1)$ $\lambda = \pm 1.4$, $e = \ldots$

$(0, 0)$ $\lambda = 1 \pm 2i$



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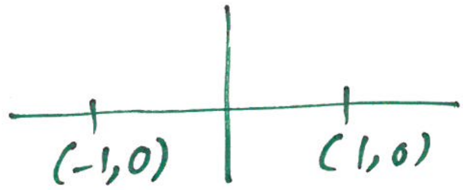
$\dot{x} = y \cdot e^{y} \qquad \dot{y} = 1 - x^2$

$y \cdot \exp(y)$

$y \cdot e^{y} = 0 \Rightarrow y = 0$

$1 - x^2 = 0 \Rightarrow x = \pm 1$



$$f(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} = \begin{bmatrix} y \cdot e^{y} \\ 1 - x^2 \end{bmatrix}$$

$$\frac{\partial f}{\partial (x,y)} = \begin{bmatrix} 0 & e^{y} + y e^{y} \\ -2 \cdot x & 0 \end{bmatrix}$$

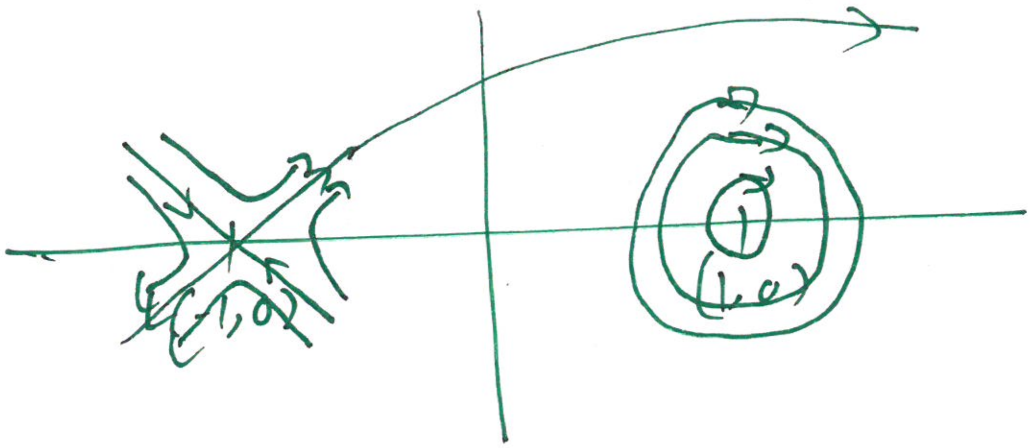
$$A(1,0) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \rightarrow \lambda = \pm 1.41 i$$

Center.

$$A(-1,0) = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$

$\rightarrow \lambda_1 = -1.4, \quad e_1 = [-0.5 \;\; 0.8]^T$

$\rightarrow \lambda_2 = +1.4, \quad e_2 = [0.5 \;\; 0.8]^T$



$\dot{x} = (1+x-2\cdot y)\cdot x = x + x^2 - 2\cdot x\cdot y$

$\dot{y} = (x-1)\cdot y = x\cdot y - y$

or $x-1=0 \Rightarrow x=1 \Rightarrow (1+1-2\cdot y)\cdot 1 = 0$

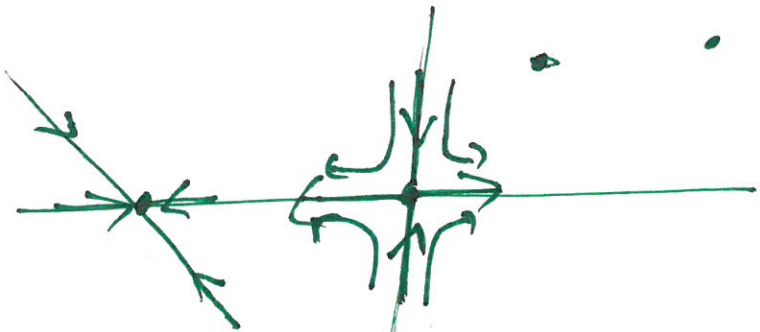
$2-2y=0 \Rightarrow y=1.$

$\boxed{(1,1)}$

or $y=0$

$(1+x-2\cdot 0)\cdot x = 0$

- $x=0 \Rightarrow \boxed{(0,0)}$
- $1+x=0 \Rightarrow \boxed{x=-1} \Rightarrow \boxed{(-1,0)}$



$$A = \begin{bmatrix} 1+2x-2y & -2x \\ y & x-1 \end{bmatrix}$$

saddle

$$A(0,0) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{cases} \lambda_1 = 1 \rightarrow e_1 = [1 \ \ 0]^T \\ \lambda_2 = -1 \rightarrow e_2 = [0 \ \ 1]^T \end{cases}$$

Stable Node

$$A(-1,0) = \begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix} \quad \begin{cases} \lambda_1 = -1 \rightarrow e_1 = [1 \ \ 0] \\ \lambda_2 = -2 \rightarrow e_2 = [-0.8, 0.4] \end{cases}$$

$$A(1,1) = \begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix} \rightarrow \lambda = 0.5 \pm 1.32i$$

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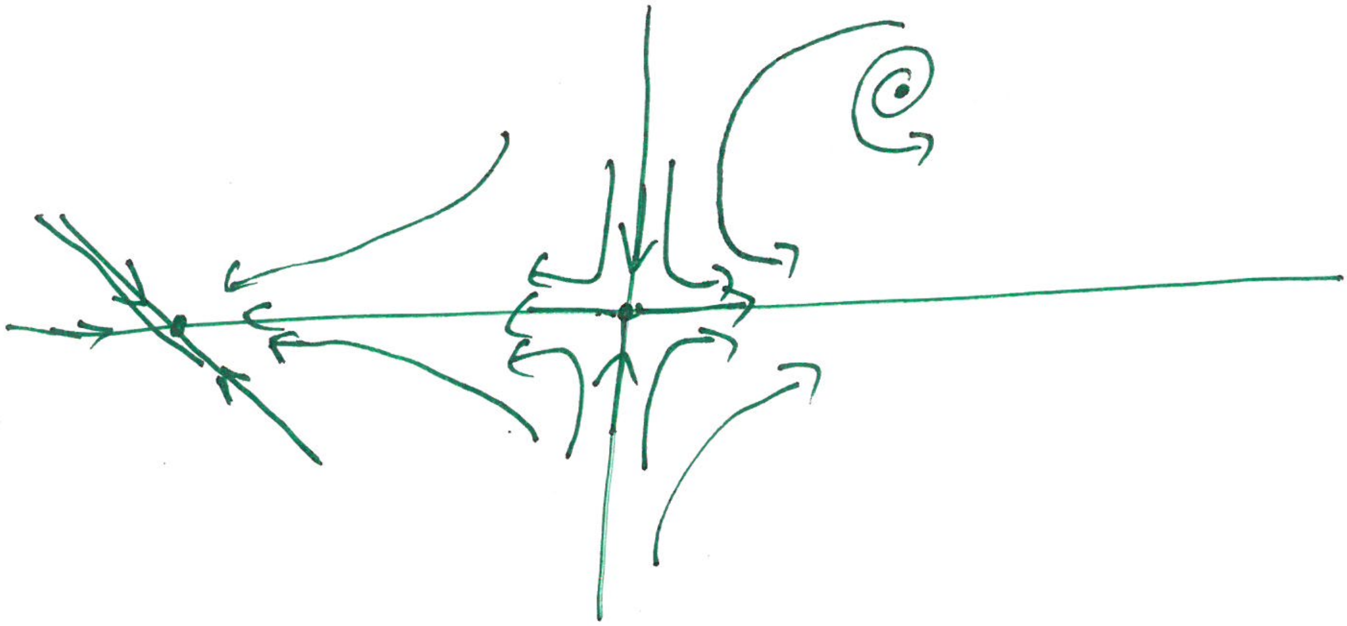
unstable Focus.



$\dot{x} = a(y-x)$

$\dot{y} = bx - y - xz$

$\dot{z} = xy - cz.$

Find F.Ps.
Show that if $b \leq 1$
then the origin $(0,0,0)$
is the only F.P.
while if $b>1 \Rightarrow 3$ F.P.

$a, b, c > 0$

$y = x$

$x \cdot y - c \cdot z = 0$

$x^2 - c \cdot z = 0 \Rightarrow z = \dfrac{x^2}{c}$

$b \cdot x - y - x \cdot z = 0$

$b \cdot x - x - x\dfrac{x^2}{c} = 0$

$x(b-1) - \dfrac{x^3}{c} = 0$

• $x = 0 \Rightarrow (x,y,z) = (0,0,0)$

• $x \neq 0 \Rightarrow (b-1) \cdot c - x^2 = 0$

$x = \pm\sqrt{(b-1) \cdot c}$

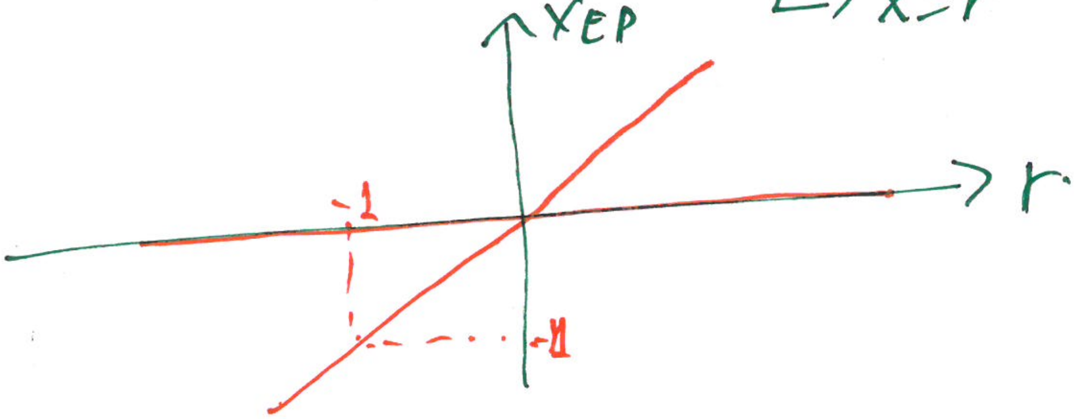
$(x,y,z) = (\sqrt{(b-1) \cdot c}, \sqrt{(b-1) \cdot c}, b-1)$

$(x,y,z) = (-\sqrt{(b-1) \cdot c}, -\sqrt{(b-1) c}, b-1)$

$$\frac{dx}{dt} = r \cdot x - x^2$$

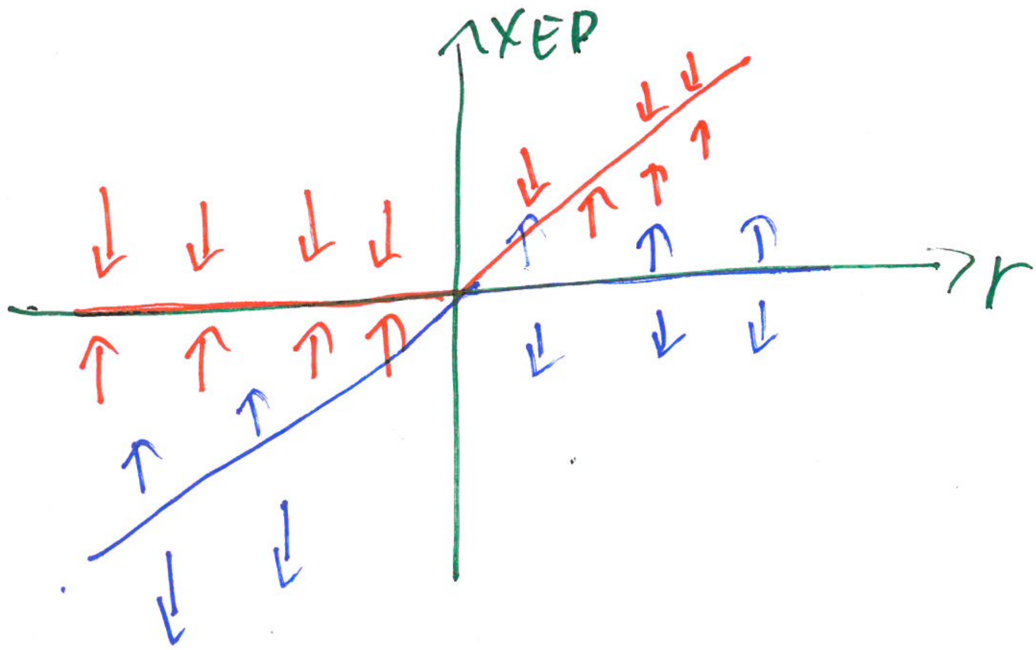
$$\frac{dx}{dt} = 0 \Rightarrow r \cdot x_{EP} - x_{EP}^2 = 0$$

$$\Rightarrow x(r-x) = 0 \quad \begin{cases} \to x = 0 \\ \to x = r \end{cases}$$



$$f(x,r) = rx - x^2$$

$$\frac{\partial f}{\partial x} = r - 2x \quad \begin{cases} \xrightarrow{x=0} A = r \\ \xrightarrow{x=r} A = -r \end{cases}$$



Lyapunov's S. T.

$\dot{x} = f(x, u)$, $x \in \mathbb{R}^{n\times 1}$

if $V(x)$ → $V(0) = 0$
→ $V(x) > 0$
→ $\frac{dV}{dt} < 0$

⇒ $\dot{x} = f(\ )$ is stable

$\dot{x} = -x + y - x \cdot y^2$

$\dot{y} = -2x - y - x^2 \cdot y$

$V(x, y) = x^2 + y^2 > 0$

$\frac{dV}{dt} = \dots \quad < 0$

$\frac{dV}{dt} = 2 \cdot x \cdot \dot{x} + 2 \cdot y \cdot \dot{y}$

$= 2 \cdot x(-x + y - x \cdot y^2) + 2 \cdot y \cdot (-2x - y - x^2 \cdot y)$

$= -2x^2 + 2 \cdot x \cdot y - 2x^2 \cdot y^2 - 4 \cdot yx - 2y^2 - 2 \cdot x^2 \cdot y^2$

$= \dots = -2(x + y)^2 - 4 \cdot x^2 \cdot y^2 < 0$