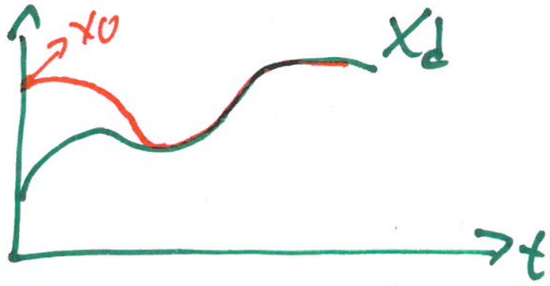
$\ddot{x} + A\dot{x} + Bx = u.$, $A, B \to$ known

$\Downarrow$

$r^2 + Ar + B = 0 \begin{cases} \to r_1 < 0 \\ \to r_2 < 0 \end{cases}$

$x_d = ?$



$\boxed{x_d = e^{-t} \cdot \cos t}$

$u = ? \quad : \quad x \to x_d$

$u = ? \quad : \quad \tilde{x} \to 0 \quad , \quad \tilde{x} = x - x_d$

$u = \ddot{x}_d + A\dot{x}_d + B \cdot x_d$

$\ddot{x} + A\dot{x} + Bx = \ddot{x}_d + A\dot{x}_d + Bx_d$

$(\ddot{x} - \ddot{x}_d) + A(\dot{x} - \dot{x}_d) + B(x - x_d) = 0$

$\ddot{\tilde{x}} + A \cdot \dot{\tilde{x}} + B \cdot \tilde{x} = 0$

$\Downarrow$

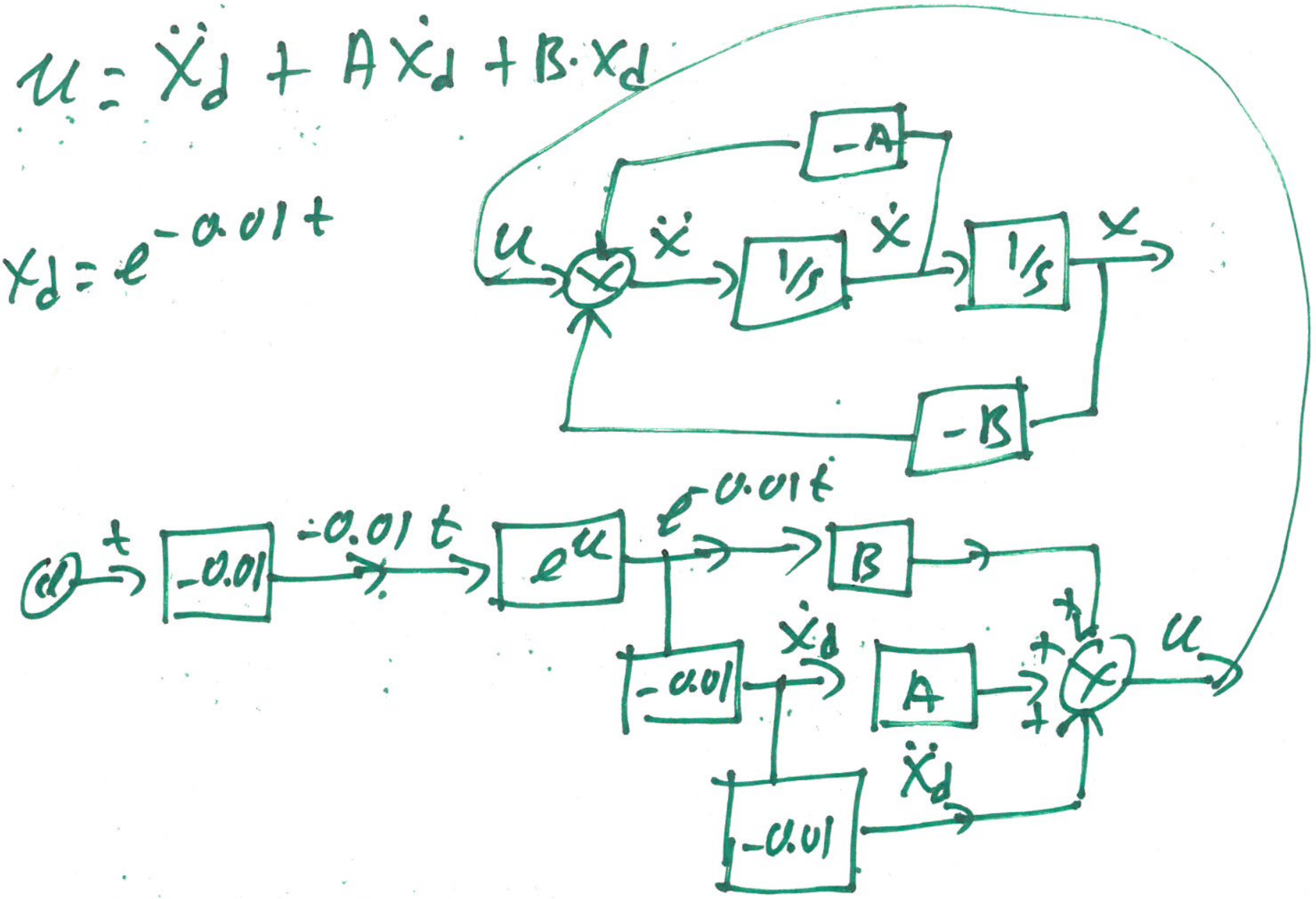
$r^2 + Ar + B = 0 \begin{cases} \to r_1 < 0 \\ \to r_2 < 0 \end{cases}$

$\Rightarrow \quad x \to x_d \quad$ or $\quad \tilde{x} \to 0$

$$\ddot{x} + A\dot{x} + B \cdot x = u.$$

$$u = \ddot{x}_d + A\dot{x}_d + B \cdot x_d$$

$$x_d = e^{-0.01t}$$



$\ddot{x} + A\dot{x} + Bx = u$, $x_d \to \ldots$

$A, B \to r_1 > 0$

$u = ?$: $x \to x_d$

$u = ?$: $\tilde{x} \to 0$

$$\ddot{\tilde{x}} + A\dot{\tilde{x}} + B\cdot\tilde{x} = 0 \xrightarrow{u=?} \ddot{\tilde{x}} + C\dot{\tilde{x}} + D\tilde{x} = 0$$

$$\Downarrow$$

$$r^2 + C\cdot r + D = 0$$

$$\Downarrow$$

$r_1 < 0$

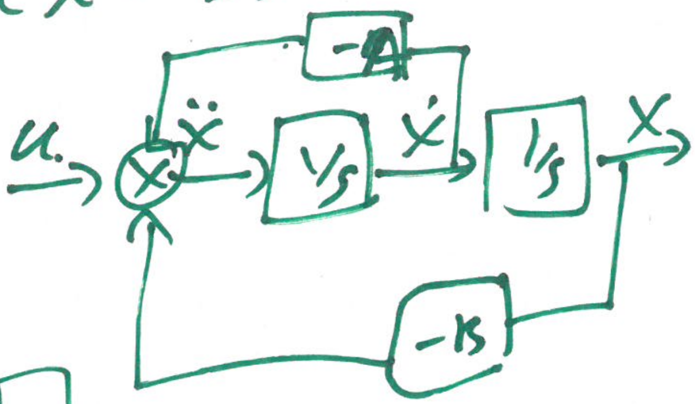
$r_2 < 0$

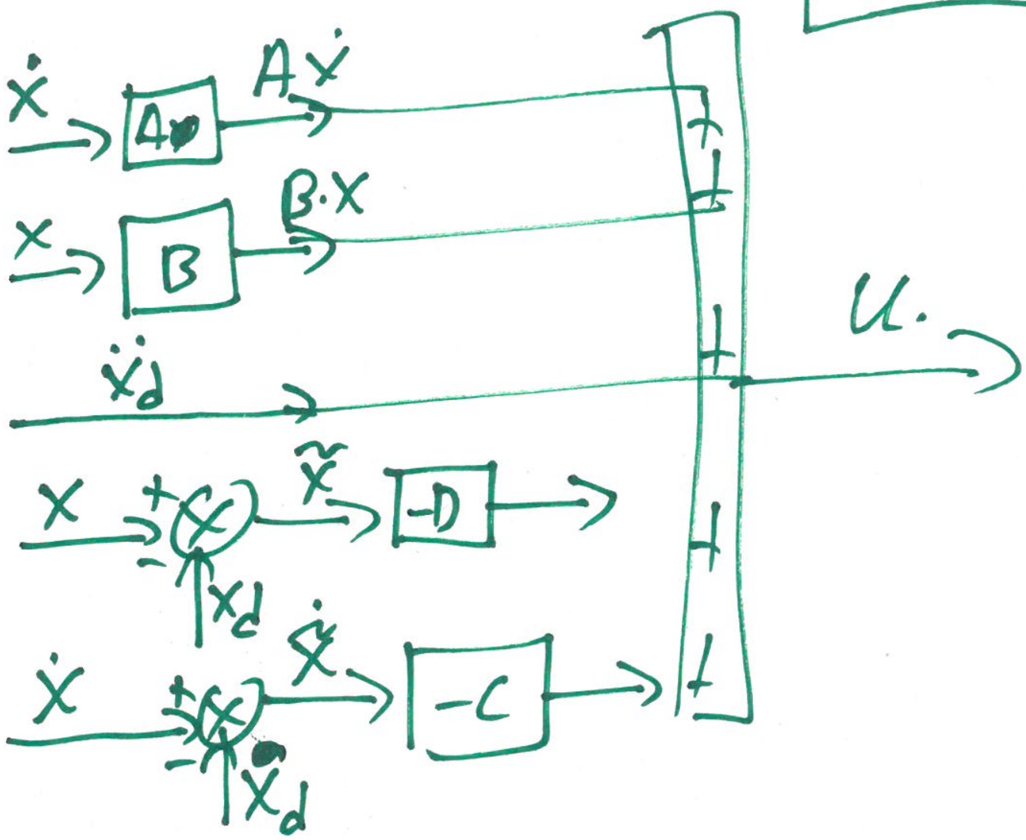
$$\ddot{x} + \cancel{A\dot{x}} + \cancel{B\cdot x} = \underbrace{\cancel{A\dot{x}} + \cancel{Bx} + \ddot{x}_d - C\dot{\tilde{x}} - D\tilde{x}}_{u} = 0$$

$$\Rightarrow \ddot{\tilde{x}} + C\dot{\tilde{x}} + D\tilde{x} = 0$$

$$\ddot{x} + A\dot{x} + Bx = u$$

$$u = A\dot{x} + Bx + \ddot{x}_d - C\dot{\tilde{x}} - D\tilde{x}$$





$\ddot{x} + A\dot{x} + Bx = u$

$A, B \to$ stable

$u = \ddot{x}_d + A\dot{x}_d + Bx_d$

$x^{(n)} + p_{n-1} x^{(n-1)} + \dots\dots = u$

$u = +x_d^{(n)} + x_d^{(n-1)} p_{n-1} \dots\dots$

$\ddot{x} + A\dot{x} + Bx = u$

$A, B \to$ unstable

$u = A\dot{x} + Bx + \ddot{x}_d - C\dot{\tilde{x}} - D\tilde{x}$

$$\dddot{x} + A\ddot{x} + B\dot{x} + Cx = A\ddot{x} + B\dot{x} + Cx - E\dot{\tilde{x}} - F\tilde{x} + \dddot{x}_d$$

$$x^{(n)} = f(x, \dot{x}, \dots, x^{(n-1)}, t) + g(x, \dot{x}, \dots, x^{(n-1)}, t) \cdot u$$

$$x^{(n)} = f \cdot + g \cdot u$$

$$u = \frac{1}{g(\;)}\left(\_\_\_\_\_\_\right)$$

$$x_d^{(n)} - f(\;) - \cancel{h(x^{(n-1)}, x^{(n-2)})} - h(\tilde{x}, \tilde{\dot{x}}, \dots, \tilde{x}^{(n-1)}, t)$$

$$\tilde{x}^{(n)}$$

$$x^{(n)} = f(\;) + \cancel{g(\;)} \cdot \frac{\left(x_d^{(n)} - f(\;) - h(\;)\right)}{\cancel{g(\;)}}$$

$$x^{(n)} = \cancel{f} + \left(-\cancel{f} - h + x_d^{(n)}\right)$$

$$x^{(n)} = -h(\;) + x_d^{(n)}$$

$$\tilde{x}^{(n)} + h(\;) = 0$$

e.g.

$$\tilde{x}^{(n)} + p_{n-1}\tilde{x}^{(n-1)} + p_{n-2}\tilde{x}^{(n-2)} + \dots + p_1 \tilde{x} = 0$$

$\ddot{x} = f(x, \dot{x}, t) + g(x, \dot{x}, t) \cdot u$

$x_d = \dots \qquad \tilde{x} = x - x_d.$

mapped

ODE of error

$u = \dots$ stable

$\tilde{x} \to 0$

$\dot{\tilde{x}} \to 0$

$s = \dot{\tilde{x}} + \lambda \cdot \tilde{x} \quad , \lambda \in \mathbb{R}$

$u = ?$ $\quad s \to 0$ or Ode of s stable

$u = ?$ $\quad : \exists\, V(s) > 0$
and $\dot{V}(s) < 0$

Find u such as I can have a $V(s)$ with $V(s) > 0$

$\frac{dV}{dt} < 0$

Then ODE of s is stable

Then $s \to 0$, then both $\dot{\tilde{x}}, \tilde{x} \to 0$

Assume $V(s) = \frac{1}{2} s^2$

$\dot{V} = s \cdot \dot{s}$

$u = ?$: $\dot{V}(s) < 0 \Rightarrow$ ODE of s stable

$\Rightarrow s \to 0, \tilde{x} \to, \dot{\tilde{x}} \to 0$

I want $\dot{V} < 0$

$\dot{V} = s \cdot \dot{s}$

$u = ?$: $\dot{V}(s) = -s^2$

if $s = -\dot{s} \Rightarrow \dot{V}(s) = -s^2$

$u = ?$: $s = -\dot{s}$

$s = \dot{\tilde{x}} + \lambda \tilde{x} \Rightarrow -s = -\dot{\tilde{x}} - \lambda \tilde{x}$

$\dot{s} = \ddot{\tilde{x}} + \lambda \dot{\tilde{x}}$

$\ddot{x} - \ddot{x}_d + \lambda \dot{\tilde{x}}$

$f + g \cdot u - \ddot{x}_d + \lambda \dot{\tilde{x}}$

$\dot{s} = -s$

$f + g\cdot u - \ddot{x}_d + \lambda\dot{\tilde{x}} = -\dot{\tilde{x}} - \lambda\tilde{x}$

$g\cdot u = -f + \ddot{x}_d - \lambda\dot{\tilde{x}} - \dot{\tilde{x}} - \lambda\tilde{x}$

$u = \frac{1}{g}\left(-f + \ddot{x}_d + \dot{\tilde{x}}(-\lambda-1) - \lambda\tilde{x}\right)$

if $u = \ldots$ find the ODE of $\tilde{x}$

prove that it is stable.