New Model for Three-Phase Converter Operating Under Supply Unbalanced Conditions

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Abstract- A rigorous analysis of the converter bridge circuit, operating under practical operating conditions, when connected to an unbalanced three-phase supply is presented in this paper. The time domain model takes full account of system losses and supply impedance on circuit current and voltage waveforms, allowing for DC current ripple and the changing states of conduction of the converter devices, and is valid for all types of loads and converter conduction modes, including mode 3 overlap when the angle of overlap extends beyond 60° and the number of conducting devices alternate between 3 and 4. Results are verified and display very good agreement with simulation results.

Keywords- AC/DC converters, Modeling; Balanced & Unbalanced Supply.

1. Introduction

In most power electronics applications, the AC input power supply, in the form of 50- or 60-Hz sine-wave AC voltage provided by electric utility, is converted to a DC voltage using six-pulse AC-DC converters. Three-phase AC-DC converters employed various applications, such as switching DC power supplies, AC motor derives, static frequency converters, DC servo drives, and so on, [1]-[4]. Different models developed for these circuits in the case of balanced, [5]-[13], or unbalanced, [14]-[17], supply conditions. Unbalance describes a situation in which either the voltages of a three-phase source are not identical in magnitude, or the phase differences between them are not 120 electrical degrees, or both, [4]. Many models published for six-pulse rectifier in the case of balance input voltages and a few others are in the case of unbalanced input voltages. However, in reference [5], an investigation onto the performance characteristics of mercury-arc rectifier was done in the case of balance voltages. A commutation angle was included in these investigations. In reference [6], six-pulse rectifier was modeled taking into account the influence of common AC reactance on the operation of the rectifier. In reference [7], must of the important parameters in the circuit are included in the model. The model is developed to use thyristors in Reference [8], where a new parameter, which is firing angle, is included.

Reference [9], proposed an analytical method based on frequency-domain to calculate the harmonic currents for the mode of continuous current on the DC side. In reference [10], the same authors has proposed an analytical method to calculate the harmonic currents for the continuous and the discontinuous modes of conduction, taking into account the AC and DC side impedances. The problem of the last two models is the deriving of the ripple part of the DC-link current is not easy task. Also, the switching function defined in these models is not taking the commutation into account. So that, in reference [11], an improvement is made to include the commutation in the switching function.

Reference [12], provided a method to determine the harmonic distortion level produced by six-pulse thyristor bridge taking into account the DC ripples. The load of the circuit is a constant DC source and the commutation angle is included. Reference [13], shows analytical models for harmonic analysis of line current for six-pulse diode bridge rectifier when powered by non-sinusoidal voltage for continuous and discontinuous mode of conduction.

A proposed model to analyze six-pulse diode bridge rectifier in the case of balance and unbalance input line voltages was made in reference [14]. The proposed model was based on the frequency-domain and switching function method. DC current is assumed to be ripple-free and commutation angle was neglected. In reference [15], the proposed model was taking into account the commutation angle and the ripple component of the DC current. Derive of analytical relationships; using symmetrical components was done in reference [16], which can be used to approximate the fundamental frequency characteristics of line-commutated converters during unbalanced operating conditions. The proposed model was ignored the effect of commutation angle. Reference [17], shows a proposal models for six-pulse rectifier to estimate the emission of non-characteristic line harmonics in the presence of high DC current ripple. The commutation angle is assumed to be neglected.

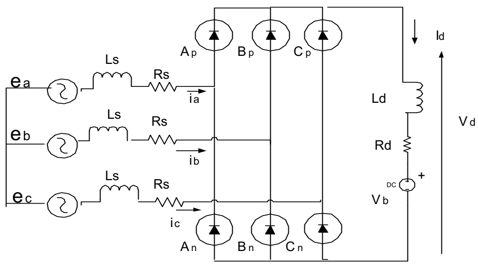

Fig. 1. Three-phase bridge rectifier.

From the literature survey it is seen that different developed models are derived based on different assumptions. From the other side very little models has been published related to the unbalanced supply conditions. This paper proposes a new model for balanced and unbalanced supply conditions, which is based on the time-domain methods. It takes into account all circuit parameters (switches, *V, R, L* & *C*) with no assumptions. These parameters are:

1. *The commutation angles.*
2. *It takes into account the drop on the valves (diodes, thyristors, IGBT,..etc).*
3. *It takes into account source resistance & reactance.*
4. *It is valid for balance & unbalance supply.*
5. *Easy develop to concern any new parameter inter the circuit.*

More generally though, the system modeled not assuming a constant DC current (infinite inductance) or a constant DC voltage (infinite capacitance), but rather solving system differential equations for different diode conduction states allowing fully for the impedance of the load (inductive or capacitive), the impedance of the supply (including supply resistance), diode voltage drops, and supply voltage unbalance.

The approach would be to write down the system differential equations for each diode conduction state (i.e. which diodes are conducting) using Kirchhoff's laws and then solving the equations numerically to obtain system currents, voltages, power flows, etc. On detecting a change in diode conduction states, a new set of differential equations needs to be solved, and so on.

2. Possible Input States

In the diode bridge, current cannot transfer immediately from one supply phase to the next because of supply inductance; the angle of overlap μ is determined by the time taken to complete commutation. As in many other diode applications where the source has high reactance, rectifier overlap can prolonged and extend into the more complex modes. There are three modes of operation illustrated in Table 1.

Table 1. Three Mode of Operation.

Mode 1	$\mu < 60^o$
Mode 2	$\mu = 60^o$
Mode 3	$\mu > 60^o$

In normal operation only two or three diodes conduct at any instant of time (mode 1 & 2), diode-commutation overlap rarely is extending beyond 60 (mode 3), where three or four diodes conducts. Hence there are twelve possible supply states which may be defined by diode-conduction patterns, and one non-conducting state (state 13) in which no diode conducts, as can happen when the DC-link current become discontinuous. The twelve conducting states fall into two types each repeated cyclically six times. The odd-numbered states, 1 to 11, occur when only two diodes conduct, the even numbered, 2 to 12, when three diodes conduct, i.e. with commutation still in progress. The conditions for transition between one conduction state and another must be clearly defined before calculations. Initially, no current flows and the conducting state are determined by the relative magnitudes of supply voltages. Therefore, transitions from one input state to the next depend on the type of conducting state applying at the time.

Table II. Possible Diodes Conducting States for Mode-1, (Two or Three Diodes Conduct).

State	Upper Diode	Lower Diode
1	C_p	B_n
2	A_p C_p	B_n
3	A_p	B_n
4	A_p	C_n B_n
5	A_p	C_n
6	B_p A_p	C_n
7	B_p	C_n
8	B_p	A_n C_n
9	B_p	A_n
10	C_p B_p	A_n
11	C_p	A_n
12	C_p	B_n A_n

Table III. Possible Diodes Conducting States for Mode-2, (Three Diodes Conduct).

State	Upper Diode	Lower Diode
1	C_p	B_n A_n
2	A_p C_p	B_n
3	A_p	C_n B_n
4	B_p A_p	C_n
5	B_p	A_n C_n
6	C_p B_p	A_n

Table IV. Possible Diodes Conducting States for Mode-3, Three or Four Diodes Conduct).

State	Upper Diode	Lower Diode
1	C_p B_p	B_n A_n
2	C_p	B_n A_n
3	A_p C_p	B_n A_n
4	A_p C_p	B_n
5	A_p C_p	C_n B_n
6	A_p	C_n B_n
7	B_p A_p	C_n B_n
8	B_p A_p	C_n
9	B_p A_p	A_n C_n
10	B_p	A_n C_n
11	C_p B_p	A_n C_n
12	C_p B_p	A_n

Transition from an odd-numbered to an even-numbered state occurs when a non-conducting diode becomes forward-biased, thus signaling the start of a new commutation process. The opposite transition occurs when the current in a diode falls to zero, making the end of commutation.

A. Conduction States – Mode1

In the first conducting state at mode-1, only two diodes will conduct the current (C_p & B_n). Diodes modeled as a constant voltage V_f and resistance R_f in series. So that, we have three equations could simulate the operation of the circuit.

$$i_a = 0 \quad (1)$$

$$i_b + i_c = 0 \quad (2)$$

$$(v_c - v_b) - [R_d + 2R_f + R_c + R_b + (L_d + L_c + L_b)p]i_c - (E_b + 2V_f) = 0 \quad (3)$$

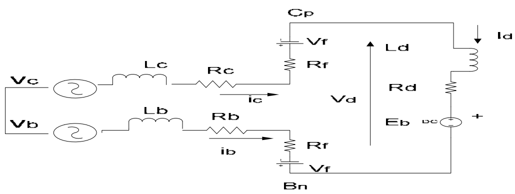

Fig. 2. Rectifier circuit at conduction state 1 (mode 1).

In the second conducting state at mode-1, only three diodes will conduct the current, commutation, (A_p, C_p & B_n). So that, we have three equations could simulate the operation of the circuit.

$$i_a + i_b + i_c = 0 \quad (4)$$

$$(v_a - v_c) - (R_f + R_c + L_c p)i_c - (R_f + R_a + L_a P)i_a = 0 \quad (5)$$

$$(v_c - v_b) - [R_d + R_f + R_b + (L_d + L_b)p]i_b + (R_f + R_c + L_c p)i_c - (E_b + 2V_f) = 0 \quad (6)$$

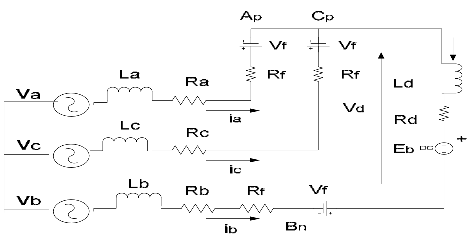

Fig. 3. Rectifier circuit at conduction state 2 (mode 1).

The remaining states could be written in same way. Where p is the operator $p = d/dt$.

B. Conduction States – Mode2

In the first conducting state at mode-2, only three diodes will conduct the current, commutation, (C_p & B_n, A_n). So that, we have three equations could simulate the operation of the circuit.

$$i_a + i_b + i_c = 0 \quad (7)$$

$$(v_b - v_a) - (R_f + R_b + L_b p)i_b - (R_f + R_a + L_a P)i_a = 0 \quad (8)$$

$$(v_c - v_b) - [R_d + R_f + R_c + (L_d + L_c)p]i_c + (R_f + R_b + L_b p)i_b - (E_b + 2V_f) = 0 \quad (9)$$

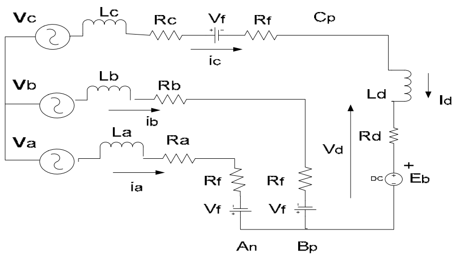

Fig. 4. Rectifier circuit at conduction state 1 (mode 2).

In this mode three diodes operate at any instant of time so that, the remaining states could be written in the same way.

C. Conduction States – Mode3

In the first conducting state at mode-3, only four diodes will conduct the current, commutation, (C_p, B_p & B_n, A_n). So that, we have three equations could simulate the operation of the circuit.

$$(v_c - v_b) - [R_c + 2R_f + R_b + (L_c + L_b)p]i_c + (R_b + L_b p)i_b - R_f I_d = 0 \quad (10)$$

$$(v_b - v_a) + (R_a + L_a p)i_a - (R_b + L_b p)i_b + R_f I_d = 0 \quad (11)$$

$$(E_b + 2V_f) - [(R_d + 2R_f) + L_d p]I_d + R_f i_a + R_f i_c = 0 \quad (12)$$

In the second conducting state at mode-3, only three diodes will conduct the current, commutation, (C_p & B_n, A_n). So that, we have three equations could simulate the operation of the circuit.

$$i_a + i_b + i_c = 0 \quad (13)$$

$$(v_b - v_a) - (R_f + R_b + L_b p)i_b - (R_f + R_a + L_a P)i_a = 0 \quad (14)$$

$$(v_c - v_b) - [R_d + R_f + R_c + (L_d + L_c)p]i_c + (R_f + R_b + L_b p)i_b - (E_b + 2V_f) = 0 \quad (15)$$

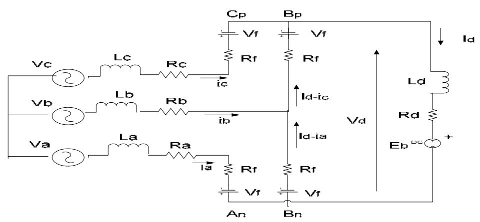

Fig. 5. Rectifier circuit at conduction state 1 (mode 3).

The remaining states could be written in same way.
Voltages and currents of the system can be obtained in the time domain. This normally involves solving differential equations. By transforming the differential equations into algebraic equations using phasors or complex frequency representation, the analysis can be simplified.

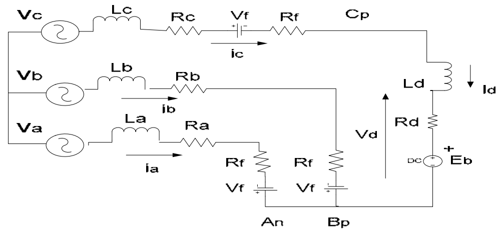

Fig. 6. Rectifier circuit at conduction state 2 (mode 3).

3. Calculated Results

To test the calculation accuracy of the proposed model, the results obtained by the proposed model are compared with those obtained by time-domain simulation method-Simulink. This calculation is tested for mode-1, mode-2 and mode-3. The data used in this study are as follows:

$$v_a(\omega t) = v_{s1}.\sin(\omega t - \varphi_a)$$

$$v_b(\omega t) = v_{s2}.\sin(\omega t - 120 - \varphi_b)$$

$$v_c(\omega t) = v_{s3}.\sin(\omega t - 240 - \varphi_c)$$

Balance Input voltages:

$v_{s1} = v_{s2} = v_{s3} = 50v$, $R_a = R_b = R_c = 0.836\Omega$

$E_b = 0$, $\varphi_a = \varphi_b = \varphi_c = 0^o$, $L_d = 10\mu H$,

$R_d = 2.4\Omega$, $V_f = 0.5\Omega$, $R_f = 0.5664\Omega$ $f = 50Hz$

Unbalance Input voltages: $v_{s1} = 50v$, $v_{s2} = 53v$,

$v_{s3} = 48v$, $f = 50Hz$, $R_a = R_b = R_c = 0.836\Omega$,

$L_d = 10\mu H$, $\varphi_a = 0^o$, $\varphi_b = 5^o$, $\varphi_c = -5^o$,

$E_b = 0$, $R_d = 2.4\Omega$, $V_f = 0.5\Omega$, $R_f = 0.5664\Omega$.

Mode-1: $L_a = L_b = L_c = 1.6mH$ ($\mu < 60^o$)

Mode-2: $L_a = L_b = L_c = 7.75mH$ ($\mu = 60^o$)

Mode-3: $L_a = L_b = L_c = 15mH$ ($\mu > 60^o$)

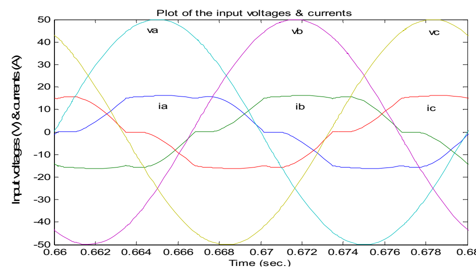

Fig. 7 Input voltages & currents in the case of balanced input voltages using simulation method. (Mode-1)

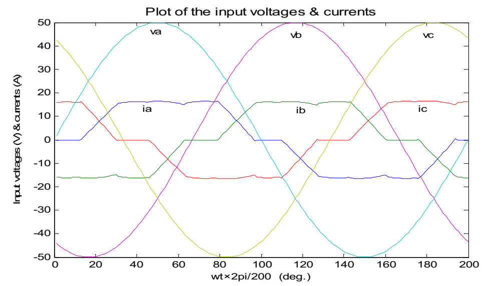

Fig. 8 Input voltages & currents in the case of balanced input voltages using proposed model. (Mode-1)

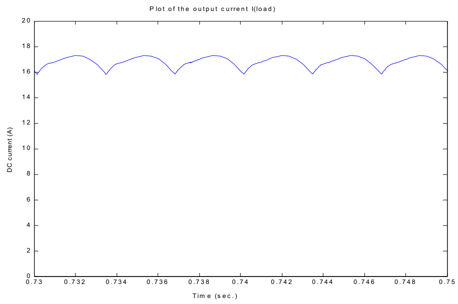

Fig. 9 DC output current in the case of balanced input voltages using simulation method. (Mode-1)

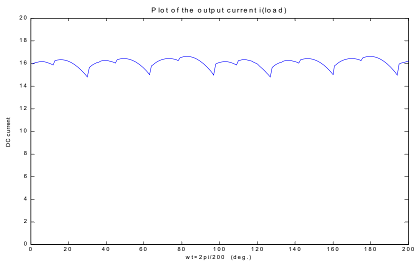

Fig. 10 DC output current in the case of balanced input voltages using proposed model. (Mode-1)

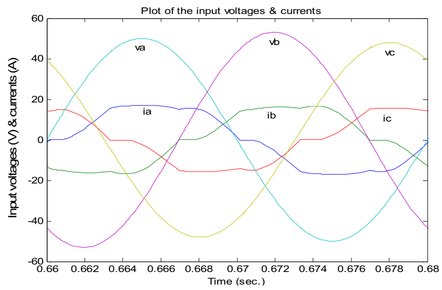

Fig. 11 Input voltages & currents in the case of unbalanced input voltages using simulation method. (Mode-1)

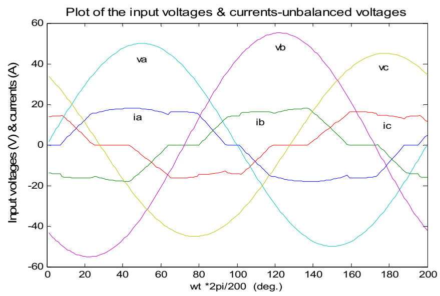

Fig. 12 Input voltages & currents in the case of unbalanced input voltages using proposed model. (Mode-1)

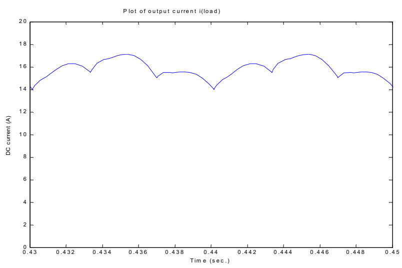

Fig. 13 DC output current in the case of unbalanced input voltages using simulation method. (Mode-1)

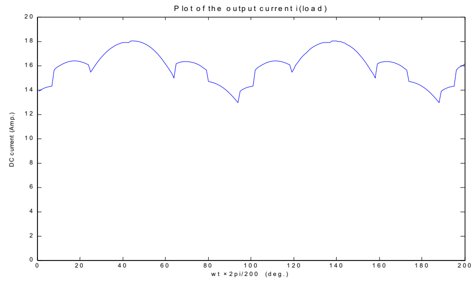

Fig. 14 DC output current in the case of unbalanced input voltages using proposed model. (Mode-1)

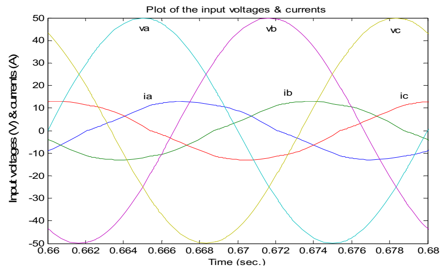

Fig. 15 Input voltages & currents in the case of balanced input voltages using simulation method. (Mode-2)

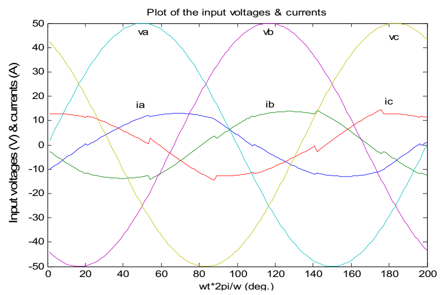

Fig. 16 Input voltages & currents in the case of balanced input voltages using proposed model. (Mode-2)

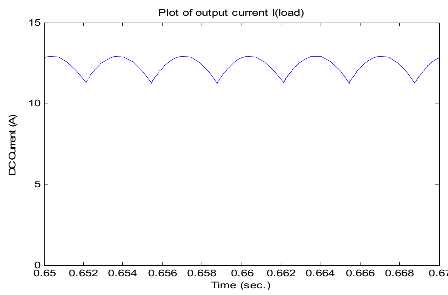

Fig. 17 DC output current in the case of balanced input voltages using simulation method. (Mode-2)

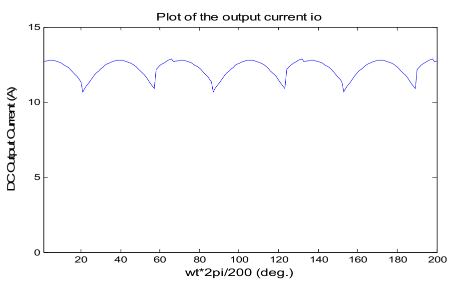

Fig. 18 DC output current in the case of balanced input voltages using proposed model. (Mode-2)

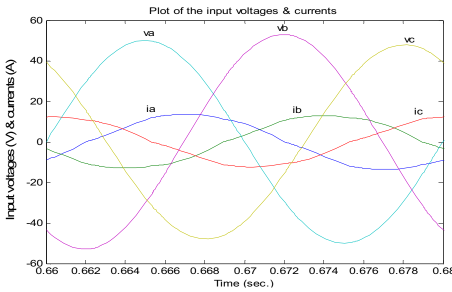

Fig. 19 Input voltages & currents in the case of unbalanced input voltages using simulation method. (Mode-2)

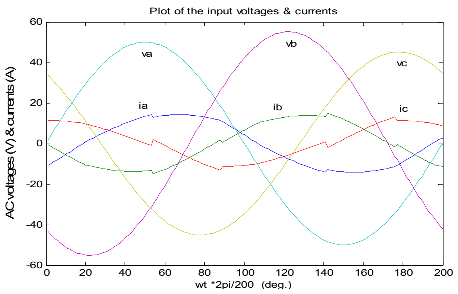

Fig. 20 Input voltages & currents in the case of unbalanced input voltages using proposed model. (Mode-2)

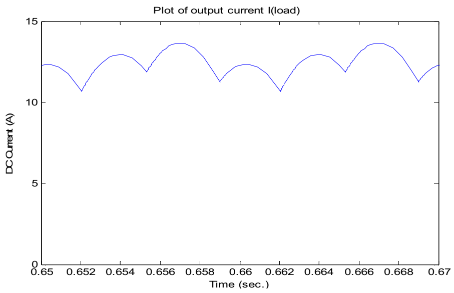

Fig. 21 DC output current in the case of unbalanced input voltages using simulation method. (Mode-2)

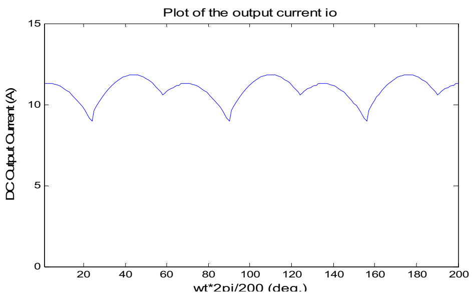

Fig. 22 DC output current in the case of unbalanced input voltages using proposed model. (Mode-2)

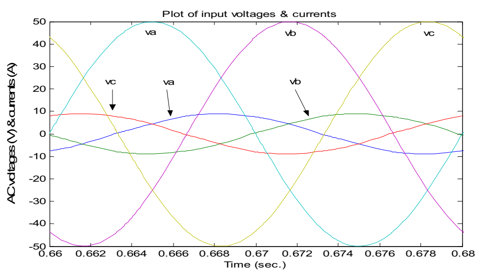

Fig. 23 Input voltages & currents in the case of balanced input voltages using simulation method. (Mode-3)

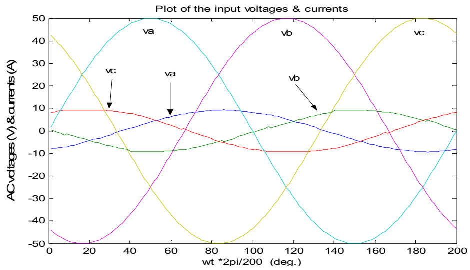

Fig. 24 Input voltages & currents in the case of balanced input voltages using proposed model. (Mode-3)

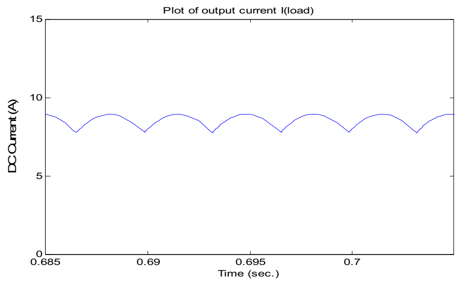

Fig. 25 DC output current in the case of balanced input voltages using simulation method. (Mode-3)

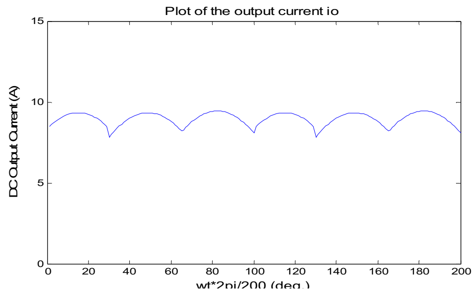

Fig. 26 DC output current in the case of balanced input voltages using proposed model. (Mode-3)

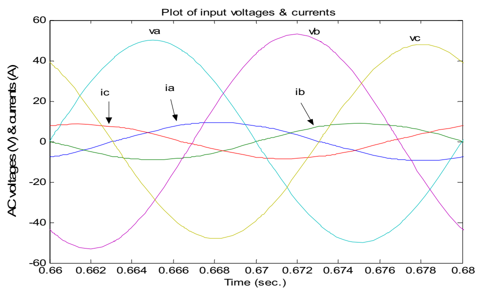

Fig. 27 Input voltages & currents in the case of unbalanced input voltages using simulation method. (Mode-3)

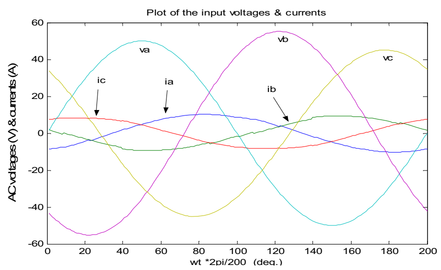

Fig. 28 Input voltages & currents in the case of unbalanced input voltages using proposed model. (Mode-3)

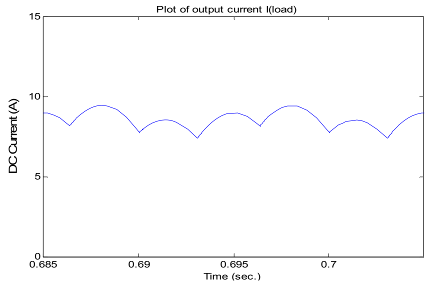

Fig. 29 DC output current in the case of unbalanced input voltages using simulation method. (Mode-3)

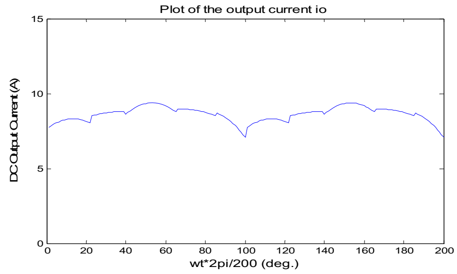

Fig. 30 DC output current in the case of unbalanced input voltages using proposed. (Mode-3)

The comparison of the proposed model and the simulation method are shown in Figs. 7-30. Figs. 7, 8, and 9, 10 illustrate AC input currents and DC output current in the case of balanced input voltages respectively (mode-1).It is seen that the collected current waveforms from the proposed model agree well with simulation model results. Also, the current waveforms not have a flat top due to load resistance with peak-to-peak value -16.3 to 16.3A. While DC output current have six segments with maximum value 16.3A. Also, Figs. 19, 20, & 21, 22 illustrate AC input currents and DC output current in the case of unbalanced input voltages respectively (mode-1). Also, a comparison for mode-2 & mode-3 was illustrated in Figs 11 to 18 and 23 to 30, respectively.

It is seen that the current waveforms collected from the proposed model agree well with simulation model results taking into consideration that more smooth curves could attain by reduce step of time change (*wt*). Moreover, the proposed model are faster then simulation program.

It is clear seen that the maximum current values for the proposed and simulation models are approximately the same in each mode. Also, it is seen that input current reduced with increasing in supply inductance. Also, the proposed model could be used for different load type (i.e. *R, R-L, R-L-*E_b*,*...etc). Moreover it could be use for different type of semiconductor switches (diode, thyristor, IGBT,..etc).

4. Conclusion

This paper proposed a new model for six-pulse bridge rectifier. The proposed model is used to study the circuit in the case of balanced and unbalanced input voltages. The model is based on Kirchhoff's Law with no assumptions. The analytical equations for the model in different modes and different states are derived. Also, the proposed model is a useful model in the case of harmonic analysis. Different data output could be received from using this model (Current, Voltage, Power factor, Apparent power, Active power, Reactive power, THD, Harmonic spectrum,etc) at any point at the circuit. In order to validate the proposed model, comparisons are made to the simulation model. There is a very good agreement between the results obtained by the two methods for the various values of the system parameters.

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