CONTROL OF PERIOD DOUBLING BIFURCATIONS IN DC-DC CONVERTERS

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Abstract: The dc to dc buck converter operating with a voltage controller is a nonlinear, nonsmooth system that presents different circuit topologies within the same switching cycle. In this paper, we propose two new bifurcation or chaos control strategies based on the use of the saltation matrix and the location of the floquet multipliers to stabilize the period-1 orbit of the circuit regardless of the input voltage of the converter. The new methods are analytically and numerically validated and their sensitivity to parameter variations is assessed.

Keywords: Control, Chaos, Electric power system, Nonlinearities, Limit cycles

1. INTRODUCTION

Dc to dc converters, which are some of the most widely used circuits in power electronics, are inherently nonlinear, nonsmooth time varying systems which exhibit subharmonic oscillations and chaos following various bifurcation pathways. The analysis and control of these phenomena are of great importance since small changes in the system's parameters (for example the supply voltage) can have catastrophic results on the system dynamics. There is extensive literature on these issues and many researchers have proposed various methods of analysis, control and/or exploitation of these behaviors (Banerjee and Verghese, 2001; Fossas and Olivar, 1996; Tse and Bernardo, 2002). The output voltage and current of these systems are usually periodic waveforms around a constant DC value. The nominal operating mode of such systems is a limit cycle whose period equals the period of the PWM clock used in the controller. This pattern is called period-1. By altering various system parameters the converter may undergo period doubling bifurcations. A succession of such bifurcations may lead to subharmonic oscillations, crises, and even chaotic behavior (Chakrabarty *et al.*, 1996). It is generally felt desirable to avoid such unusual modes in power converters, which makes it necessary to develop methodologies to control period doubling bifurcation.

The study of these limit cycles is conventionally based on the Poincaré map which captures the essential properties of the system. In some cases this map can be obtained analytically (for example in the current mode controlled boost converter), but in most other converter configurations (like the voltage controlled buck converter), where the model includes transcendental equations, the discrete map can only be calculated numerically. The authors have previously proposed a different method to study the nonlinear behavior of a buck converter whose Poincaré map can only be calculated numerically (Giaouris *et al.*, n.d.) and (Giaouris *et al.*, 2005). We have shown that this method is more powerful mainly because it can follow unstable limit cycles, which play a crucial role in the appearance of crises. To achieve this, we used the floquet multipliers of the system which are the eigenvalues of the monodromy matrix. The monodromy matrix of piecewise smooth systems can be calculated by using the fundamental solution matrices before, after and during the switching (Leine *et al.*, 2000; Leine and Nijmeijer, 2004). We have shown that the fundamental solution matrix during the switching is very important and it can totally alter the behavior of the overall system (Giaouris *et al.*, n.d.). We refer to that matrix as the saltation matrix, a name first proposed by Leine (Leine and Nijmeijer, 2004).

There is a large body of literature on the control of chaos in dynamical systems, such as the Ott-Grebogi-Yorke method and the Pyragas method (Banerjee and Verghese, 2001). These methods are aimed at stabilizing one of the unstable periodic orbits that exist in any chaotic system. On the other hand there are methods which are based on the empirical observation that an application of a periodic perturbation (Zhou *et al.*, 2003) that can stabilize a system in a periodic orbit. Previous work has shown how the chaos control method presented in (Zhou *et al.*, 2003) works and how it can be improved by optimally placing the eigenvalues at predefined locations (Giaouris *et al.*, n.d.).

The first part of this paper briefly analyzes the behavior of the buck converter based on the monodromy matrix and proves that when the orbit is period-1, it is impossible to have attractive sliding modes (as was also observed by Di Bernardo (Bernardo *et al.*, 1998)). The second part presents two novel bifurcation control methods based on the saltation matrix and describes its influence on the overall system. These two methods stabilize the period-1 orbit regardless of the input voltage. Hence even though we call these methods *bifurcation control* they can easily be applied to suppress chaos by stabilizing one of the infinite unstable periodic orbits that are in a strange attractor. Robustness tests were carried out to test the controller sensitivity to parameter variation like the load resistance.

2. THE BUCK CONVERTER

The buck converter (Fig. 1) consists of a switch S which is controlled by a pulse-width modulated signal to achieve the required output. The controller creates a compensating signal $V_{con} = A(v_o - V_{ref})$ which is compared with a suitable periodic sawtooth waveform V_{ramp} and a switching occurs when these two signals become equal. A is the feedback amplifier gain and the ramp signal can be written as $V_{\text{ramp}}(t) = V_L + (V_U - V_L)\left(\frac{t}{T} \mod 1\right)$. To experimentally test and analyze the proposed control schemes we used a buck converter with the following parameters $L = 20$mH, $C = 47\mu$F, $R = 22\Omega$, $A = 8.4$, $V_{\text{ref}} = 11.3$V, $V_L = 3.8$V, $V_U = 8.2$V, and $T = 400\mu$s.

The system is governed by two sets of linear differential equations related to the *ON* and *OFF* states of the controlled switch. The inductor current $i_L(= x_2)$ and

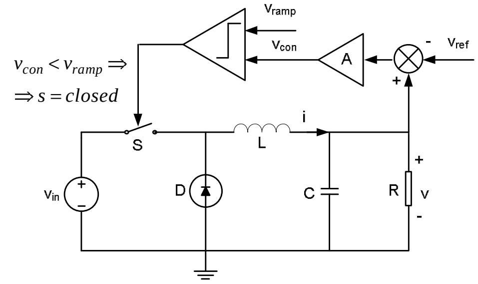


Fig. 1. The voltage mode controlled buck dc-dc converter

the capacitor voltage $v_o(= x_1)$ are taken as state variables. The individual state equations can be written as

$$\dot{x}_2 = \begin{cases} \dfrac{V_{\text{in}} - x_1(t)}{L}, & A(x_1(t) - V_{\text{ref}}) < V_{\text{ramp}}(t), \\ -\dfrac{x_1(t)}{L}, & A(x_1(t) - V_{\text{ref}}) > V_{\text{ramp}}(t). \end{cases} \tag{1}$$

$$\dot{x}_1 = \frac{x_2(t) - x_1(t)/R}{C} \tag{2}$$

which in a matrix form are:

$$\dot{\mathbf{X}} = \mathbf{A}_s\mathbf{X} + \mathbf{B}u \tag{3}$$

$$\dot{\mathbf{X}} = \mathbf{A}_s\mathbf{X} \tag{4}$$

and they have a solution of the form:

$$\mathbf{X}(t) = \Phi(t, d'T)\mathbf{X}(d't) + \int_{d'T}^{t} \Phi(t-\tau, d'T)\mathbf{B}\, u\, d\tau \tag{5}$$

$$\mathbf{X}(t) = \Phi(t, 0)\mathbf{X}(0) \tag{6}$$

where

$$\mathbf{X} = \begin{bmatrix} v_0 & i_L \end{bmatrix}^T, \quad \mathbf{A}_s = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & \dfrac{-1}{L} \end{bmatrix}^T,$$

u is the input voltage V_{in}, Φ is the state transition matrix and $d' = 1 - d$, d is the duty cycle.

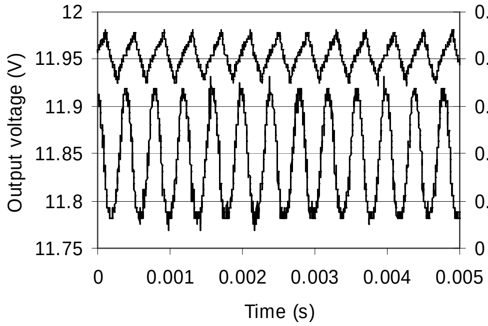
In general, the circuit gives an average dc output voltage close to the desirable value with a periodic ripple equal to the period of the driving clock (the ramp) as shown in Fig. 2a. As the input voltage of the system is increased, the circuit exhibits a period doubling bifurcation, Fig. 2b, and then by successive bifurcations it enters into a chaotic regime (Chakrabarty *et al.*, 1996).

3. FILIPPOV THEORY

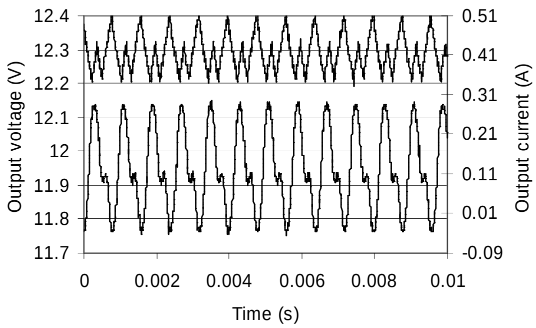
The theory of Filippov gives a generalized definition of the solution of state equations involving switching. Consider a piecewise smooth system with one switching manifold:

$$\dot{\mathbf{X}} = \begin{cases} \mathbf{f}_-(\mathbf{X}, t) & \text{for } \mathbf{X} \in V_- \\ \mathbf{f}_+(\mathbf{X}, t) & \text{for } \mathbf{X} \in V_+ \end{cases} \tag{7}$$

where $\mathbf{f}_-$ and $\mathbf{f}_+$ are the two n dimensional smooth vector fields before and after the $n-1$ dimensional hypersurface Σ which is defined by a scalar function



(a)



(b)

Fig. 2. Experimental results, (a) period 1 pattern and (b) period 2 pattern

$h(\mathbf{X},t)$. The state space is thus separated into three subsets: $\mathbb{R}^n = V_- \cup \Sigma \cup V_+$ where

$$\begin{cases} V_- &= \mathbf{X} \in \{\mathbb{R}^n \mid h(\mathbf{X},t) < 0\} \\ \Sigma &= \mathbf{X} \in \{\mathbb{R}^n \mid h(\mathbf{X},t) = 0\} \\ V_+ &= \mathbf{X} \in \{\mathbb{R}^n \mid h(\mathbf{X},t) > 0\} \end{cases}$$

To define the behavior of the system while it is on Σ we have to extend the previous piecewise system to a differential inclusion (Filippov, 1988):

$$\dot{\mathbf{X}} \in \mathbf{F}(\mathbf{X},t) = \begin{cases} \mathbf{f}_-(\mathbf{X},t) & \text{for } \mathbf{X} \in V_- \\ \overline{co}\{\mathbf{f}_-(\mathbf{X},t), \mathbf{f}_+(\mathbf{X},t)\} & \text{for } \mathbf{X} \in \Sigma \\ \mathbf{f}_+(\mathbf{X},t) & \text{for } \mathbf{X} \in V_+ \end{cases} \tag{8}$$

where $\overline{co}\{\mathbf{f}_-, \mathbf{f}_+\} = \{(1-q)\mathbf{f}_- + q\mathbf{f}_+\}, \forall q \in [0,1]$

The extension of a discontinuous system (7) into a convex differential inclusion (8) is known as Filippov's convex method. The existence of the solution can be guaranteed if $\mathbf{F}(\mathbf{X},t)$ is upper semi-continuous. The uniqueness is guaranteed if the solution crosses the hypersurface transversally. A necessary condition for a transversal intersection at Σ is:

$$\mathbf{n}^T\mathbf{f}_-(\mathbf{X},t) \times \mathbf{n}^T\mathbf{f}_+(\mathbf{X},t) > 0 : \mathbf{X} \in \Sigma$$

where $\mathbf{n}$ is the normal vector onto Σ i.e. the gradient of h: $\mathbf{n} = \nabla h(\mathbf{X},t)$ and $\mathbf{n}^T\mathbf{f}_-, \mathbf{n}^T\mathbf{f}_+$ are the projections of the smooth vector fields onto Σ.

4. APPLYING FILIPPOV THEORY TO THE BUCK CONVERTER

Theorem: The system described by (1) and (2) admits a unique period-1 orbit.

Proof: For the voltage controlled closed loop buck converter, the switching hypersurface (h) is given by

$$h(\mathbf{X}(t),t) = x_1(t) - V_{\text{ref}} - V_{\text{ramp}}(t)/A = 0. \tag{9}$$

By restricting the analysis to period-1 limit cycles we can assume that $t \in (0,T)$ hence:

$$V_{\text{ramp}}(t) = V_L + (V_U - V_L)t/T \tag{10}$$

The normal vector $\mathbf{n}$ is given as

$$\mathbf{n} = \nabla h(\mathbf{X},t) = \begin{bmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \end{bmatrix}^T \tag{11}$$

The set valued function is defined as:

$$\dot{\mathbf{X}} \in \mathbf{F}(\mathbf{X},t) = \begin{cases} \mathbf{f}_-(\mathbf{X},t) & \text{for } \mathbf{X} \in V_- \\ \overline{co}\{\mathbf{f}_-(\mathbf{X},t), \mathbf{f}_+(\mathbf{X},t)\} & \text{for } \mathbf{X} \in \Sigma \\ \mathbf{f}_+(\mathbf{X},t) & \text{for } \mathbf{X} \in V_+ \end{cases}$$

where

$$\mathbf{f}_-(\mathbf{X}(t)) = \begin{bmatrix} x_2(t)/C - x_1(t)/RC \\ (V_{\text{in}} - x_1(t))/L \end{bmatrix},$$

$$\mathbf{f}_+(\mathbf{X}(t)) = \begin{bmatrix} x_2(t)/C - x_1(t)/RC \\ -x_1(t)/L \end{bmatrix}.$$

The convex hull is defined as:

$$\overline{co}\{\mathbf{f}_-(\mathbf{x}(t)), \mathbf{f}_+(\mathbf{x}(t))\} = \begin{bmatrix} x_2(t)/C - x_1(t)/RC \\ \overline{co}\left\{\frac{V_{\text{in}} - x_1(t)}{L}, -\frac{x_1(t)}{L}\right\} \end{bmatrix}$$

The projections of the two vector fields onto $\mathbf{n}$ are: $\mathbf{n}^T\mathbf{f}_- = \dfrac{x_2 - x_1/R}{C}$ and $\mathbf{n}^T\mathbf{f}_+ = \dfrac{x_2 - x_1/R}{C}$. Hence $\mathbf{n}^T\mathbf{f}_- \times \mathbf{n}^T\mathbf{f}_+ > 0$ and therefore we have a transversal intersection which guarantees a unique Filippov solution, as shown in Fig. 3.

Since there is a transversal intersection when $t \in (0,T)$ it is impossible to have a period-1 orbit involving sliding solution. This was also observed by Di Bernardo (Bernardo *et al.*, 1998) who proved that a sliding mode (or infinite stretching as it is also called) can only occur when $V_{ramp}(0) = V_{cont}(0)$ and $V'_{ramp}(0) = V'_{cont}(0)$. Since the previous theorem assumed that $t \in (0,T)$ it does not contradict with the work in (Bernardo *et al.*, 1998). □

The monodromy matrix of the system can be calculated as:

$$\mathbf{M}(0,\mathbf{X}(0),T) = \Phi(T,d'T)\mathbf{S}\Phi(d'T,0) \tag{12}$$

where $\mathbf{S}$ is the saltation matrix and is defined (for this transition) as:

$$\mathbf{S} = \mathbf{I} + \frac{(\mathbf{f}_+(\mathbf{X}(d'T)) - \mathbf{f}_-(\mathbf{X}(d'T)))\mathbf{n}^T}{\mathbf{n}^T\mathbf{f}_-(\mathbf{X}(d'T)) + \frac{\partial h}{\partial t}\big|_{t=d'T}} \tag{13}$$

By using eqns. (5), (6) and (9) at $t = d'T$ it is possible to create a nonlinear smooth function of d' which was solved numerically using a Newton-Raphson method

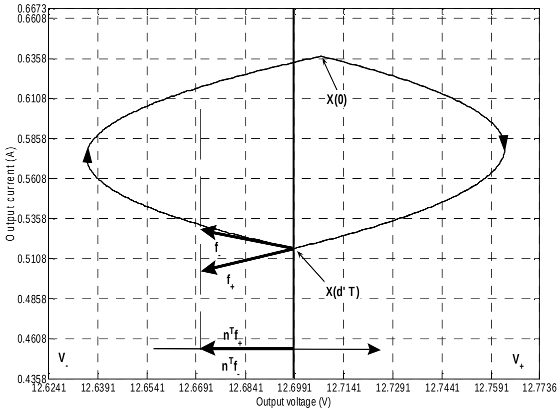


Fig. 3. Phase space and the transversal intersection

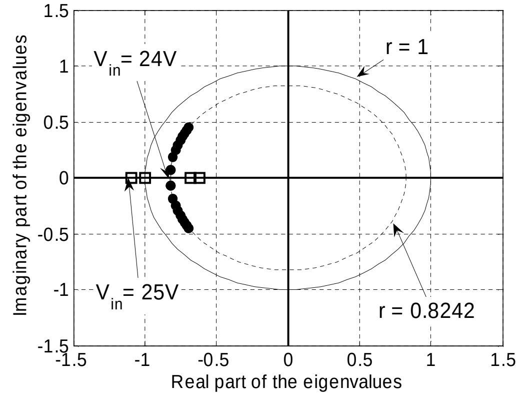


Fig. 4. Eigenvalue location for various values of V_{in}

for various values of the input voltages. The value of d' was found to be 0.4993 for $V_{in} = 24V$ and from that we can calculate the values of the state vector while the solution is on the hypersurface, $x_1(d'T) = 12.0139V$ and $x_2(d'T) = 0.4681A$. Using these values we calculated the saltation matrix as

$$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ -0.4639 & 1 \end{bmatrix}.$$

Since this is a piecewise linear model of the converter, $\Phi(d'T,T) = e^{\mathbf{A}_s dT}$ and $\Phi(0,d'T) = e^{\mathbf{A}_s d'T}$, where $\mathbf{A}_s$ is the state matrix. By using these in (12) we calculated the monodromy matrix and we found its eigenvalues: $0.8211 \pm 0.0708j$ which implies that the system is stable as expected and shown in Fig. 2a. Fig. 4 shows the evolution of the eigenvalues for various values of the supply voltage showing very good agreement with the experimental results shown in Fig. 2b.

Furthermore, when the supply voltage is increased from 24 to 25V, the change in the duty cycle is very small, and the change in the state vector at $t = 0$ and at $t = d'T$ is even smaller (Giaouris *et al.*, n.d.). This implies that the changes of $\Phi(T,d'T)$ and $\Phi(d'T,0)$ are minor and hence we should expect that the monodromy matrix would also remain almost unchanged with the same stability properties. This obviously contradicts with the experimental and numerical results shown before. The answer to that peculiar circuit behavior lies on the (2,1) element of the saltation matrix which changed from -0.4639 to -0.4744. This change forced the floquet multipliers to move outside the unit circle which caused the instability. This sensitivity of the system on the saltation matrix will be used at the following sections as a guide to controlling the system.

5. BIFURCATIONS AND CHAOS CONTROL ANALYSIS

Using Filippov's method of analysis of the stability of the period-1 limit cycle, we now analyze the method proposed in (Zhou *et al.*, 2003). Based on that analysis we will propose and rigorously analyze another novel control method based on the saltation matrix. A second simpler method which is easier to implement will also be presented based on the suitable choice of the feedback gain A which will ensure that the eigenvalues of the monodromy matrix remain inside the unit circle.

5.1 Control by Using a Sinusoidal Signal

The method in (Zhou *et al.*, 2003) forces the period one limit cycle to remain stable for a wide range of the bifurcation parameter by slightly changing the hypersurface. This is archived by superimposing a small sinusoidal signal onto h:

$$h(\mathbf{x}(t),t) = x_1(t) - V_{\text{ref}} - aV_{\text{ref}} \sin \omega t - \frac{V_{\text{ramp}}(t)}{A} = 0.$$

Since the value of a is very small ($\simeq -0.0004$ for $V_{\text{in}} = 25V$) it has almost no effect on $h(\mathbf{X},T)$ and therefore almost no change on the value of the duty cycle. But the saltation matrix uses the partial derivative of h with respect to time and hence the saltation matrix will have the term $\partial h/\partial t = -V_{\text{ref}}\, a\, \omega \cos \omega d'T - (V_U - V_L)/AT$ instead of $\partial h/\partial t = -(V_U - V_L)/AT$ and hence the eigenvalues of the monodromy matrix will be a function of a. Fig. 5a shows the location of the maximum absolute value of the monodromy matrix eigenvalues when $V_{\text{in}} = 25V$. It is clear that we can place the eigenvalues of the system so that the magnitude is less than unity by appropriately choosing the value of a. By using the saltation matrix we were able to place the eigenvalues at a predefined location. While (Zhou *et al.*, 2003) proposes no theoretical criterion for the choice of a, the above approach allowed us to propose the method of choosing a optimally (Giaouris *et al.*, n.d.).

5.2 Control by Using a Signal Proportional to the Output Voltage

Our aim in this section is to propose, apply and rigourously justify a novel chaos or bifurcation control method based on the saltation matrix. To better understand the derivation of this control method we restrict our studies to the area where the first period doubling occurs approximately at 24.5V. At 24V the system is stable, i.e., there is a stable period 1 limit cycle,

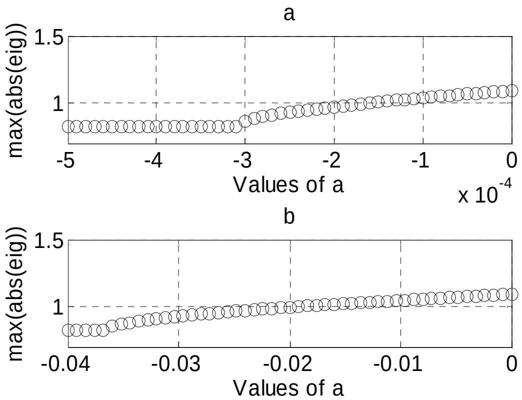


Fig. 5. Maximum absolute eigenvalue for various values of a for the two control schemes

while at 25V the system is unstable, i.e., there is one unstable period 1 and one stable period 2 limit cycle. As it has been proved above the instability occurred from the sensitivity of the system on the saltation matrix, From eqn. 13 it can be seen that this matrix depends on $\mathbf{f}_{-}$, $\mathbf{f}_{+}$, $\mathbf{n}$ and $\partial h/\partial t$. The two smooth vector fields depend on the system structure and hence they cannot be altered. Zhou et al. (Zhou *et al.*, 2003) effectively changed the term $\partial h/\partial t$ by adding a small time varying component in h. In this paper we propose that we can alter the location of the eigenvalues by changing the normal vector $\mathbf{n}$. To do that we can use another control law (easily implemented with a DSP) where the controlling signal is not $A(x_1 - V_{\text{ref}})$ but is $A(x_1(1+a) - V_{\text{ref}})$. This will effectively alter the switching manifold to

$$h(\mathbf{X}(t),t) = x_1(t) - V_{\text{ref}} + a x_1(t) - \frac{V_{\text{ramp}}(t)}{A} = 0. \quad (14)$$

Hence the normal vector will be:

$$\mathbf{n} = \nabla h(\mathbf{X},t) = \begin{bmatrix} 1+a \\ 0 \end{bmatrix} \quad (15)$$

This implies that the new saltation matrix will be

$$\mathbf{S} = \begin{bmatrix} 1 & 0 \\ \dfrac{(1+a)V_{\text{in}}/L}{(1+a)\dfrac{x_2(t_\Sigma) - x_1(t_\Sigma)/R}{C} - \dfrac{V_U - V_L}{AT}} & 1 \end{bmatrix} \quad (16)$$

It is again obvious that the eigenvalues will be a function of a, and thus it is possible to stabilize the period 1 limit cycle even if the system nominally exhibits period 2 patterns, Fig. 5b. Fig. 6 shows the response of the system when the input voltage changes suddenly from 24 to 25V. It is clear that the system after an initial transient will settle down to the stable period 1 limit cycle.

A similar method has been proposed by (Marius-F. Danca, 2004) on simple nonsmooth systems but without any rigorous justification of why it works.

5.3 *Control by Appropriately Changing the Feedback Gain*

The second controller which we present in this section is again based upon the fact that small changes of V_{in}

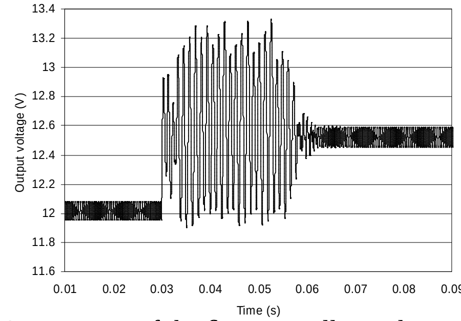


Fig. 6. Response of the first controller under a sudden input voltage change

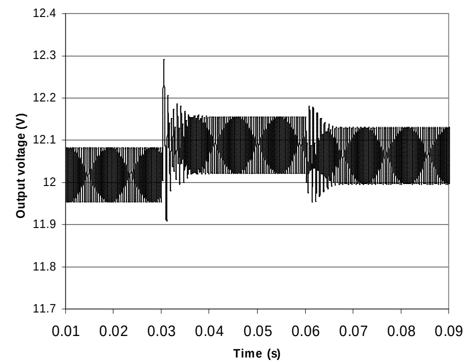


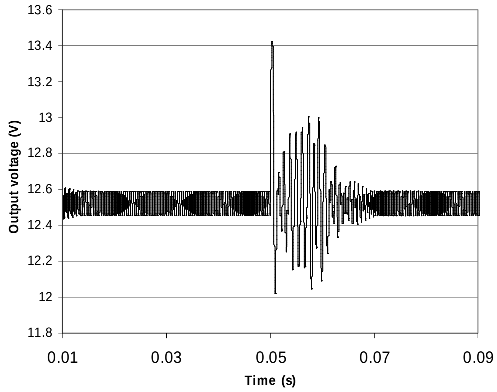
Fig. 7. Response of the second controller under a sudden input voltage change.

will cause small changes in the duty cycle and that the instability occurs due to the changes of the saltation matrix. By appropriately changing the saltation matrix we can force the eigenvalues of the monodromy matrix to remain close to a circle of radius 0.82, i.e. close to the eigenvalues of the system when the supply voltage is 24V. If we use the feedback gain as the control parameter, we can set the gain at:

$$A = \frac{V_U - V_L}{T\left(\dfrac{x_2(t_\Sigma) - x_1(t_\Sigma)/R}{C} - \dfrac{V_{in}}{\mathbf{S}(2,1)L}\right)} \quad (17)$$

where $\mathbf{S}(2,1)$ is the element (2,1) of the saltation matrix when the system is stable. Since at 24V the value of $\mathbf{S}(2,1)$ was -0.4638, we use this value. The results of this method (Fig. 7 where voltage perturbation was applied at 0.03s) show that the system remained stable. We have also used the values of V_L, V_U, T as control parameter — with similar outcomes.

A small drawback with this method is that when we calculate A we have to use the values of x_1 and x_2 at $t = d'T$. In the results of Fig. 7 we have kept these values constant at the nominal values of the system, i.e. at the values when the supply voltage is 24V. The direct result of that is the big decrease of the proportional gain which caused a big steady state error. To improve the behavior of the system we created a look-up table with the values of x_1 and x_2 for different values of V_{in} and we have used that to update the controller. For example when V_{in} is 25V the values of x_1 and x_2 are those of the unstable period 1 limit cycle and not of the previous stable one as we did



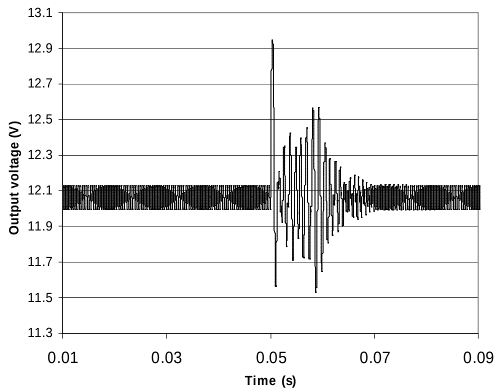


Fig. 8. Sensitivity test of the two controllers under a sudden resistance change when $V_{\text{in}} = 25V$.

before. This improvement led to a controller that can suppress chaos but it has smaller steady state error, as shown in Fig. 7 (after 0.06s).

5.4 Sensitivity to system's parameter changes

To further validate the chaos controllers that were presented in the previous section we subjected the system to a sudden change of the load resistance, i.e., while the controller was forcing the system to be stable at 25V we increased the load resistance by 50%. This is something that can happen to applications were the converter is feeding a variable dc load like a dc machine. It can be seen from Fig. 8 that the two controllers will satisfactory force the system back to a stable period 1 pattern.

6. CONCLUSIONS

We have proposed two novel control methods based on the location of the floquet multipliers which forces the system to be stable regardless of the value of the bifurcation parameter. The bifurcation phenomena have been proven with simulated and experimental results. These control methods use the concept of the saltation matrix to change stability properties of the period 1 limit cycle. Advantages and limitations of these methods over other existing ones have been discussed and analyzed.

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REFERENCES

Banerjee, S. and Verghese, G. C., Eds. (2001). *Nonlinear Phenomena in Power Electronics: Attractors, Bifurcations, Chaos, and Nonlinear Control*. IEEE Press. New York, USA.

Bernardo, M. Di, F. Garofalo, L. Glielmo and F. Vasca (1998). Switchings, bifurcations and chaos in DC-DC converters. *IEEE Transactions on Circuits and Systems-I* **45**(2), 133–141.

Chakrabarty, K., G. Poddar and S. Banerjee (1996). Bifurcation behaviour of the buck converter. *IEEE Transactions on Power Electronics* **11**(3), 439–447.

Filippov, A. F. (1988). *Differential equations with discontinuous right-hand sides*. Kluwer Academic Publishers. Dortrecht.

Fossas, E. and G. Olivar (1996). Study of chaos in the buck converter. *IEEE Transactions on Circuits and Systems-I* **43**(1), 13–25.

Giaouris, D., B. Zahawi and V. Pickert (2005). On the derivation of the monodromy matrix of the buck converter. In: *5th International Conference on Technology and Automation 2005*. Thessaloniki - Greece. pp. 222–226.

Giaouris, D., S. Banerjee, B. Zahawi and V. Pickert (n.d.). Stability analysis of power electronic dc-dc converters using complete-cycle solution matrix. *Under submision at: IEEE Transactions on Circuits and Systems - I*.

Leine, R. I. and H. Nijmeijer (2004). *Dynamics and Bifurcations of Non-Smooth Mechanical Systems*. Springer. Berlin.

Leine, R. I., D. H. Van Campen and B. L. Van de Vrande (2000). Bifurcations in nonlinear discontinuous systems. *Nonlinear Dynamics*.

Marius-F. Danca (2004). Controlling chaos in discontinuous dynamical systems. *Chaos,Solutions and Fractals* **22**(3), 605–612.

Tse, C. K. and M. Di Bernardo (2002). Complex behavior in switching power converters. *IEEE Proceedings* **90**(5), 768–781.

Zhou, Y., C. K. Tse, S. S. Qiu and F. C. M. Lau (2003). Applying resonant parametric perturbarion to control chaos in the buck dc/dc converter with phase shift and frequency mismatch considerations. *International Journal of Bifurcation and Chaos* **13**(11), 3459–3471.