

Modeling and Stability Analysis of Closed Loop Current-Mode Controlled Ćuk Converter using Takagi-Sugeno Fuzzy Approach

Kamyar Mehran, *Member IEEE*, Damian Giaouris, *Member IEEE*, and Bashar Zahawi, *Senior Member IEEE*
School of Electrical, Electronic and Computer Engineering,
Newcastle University, Newcastle upon Tyne, UK
E-mail: {kamyar.mehran@ncl.ac.uk}

Abstract—The current-mode controlled Ćuk converter, being a fourth-order nonlinear nonsmooth system, does not lend itself to any simple analysis and the complexity lies mainly in the modeling of the circuit to capture all the essential nonlinearities that occur in fast time scale. This paper, for the first time, extends the Takagi-Sugeno fuzzy modeling approach to capture fast time-scale nonlinearities in the converter like border-collision bifurcation and chaos. Then, non-smooth Lyapunov theory is employed to study the stability and actually pinpoint the operating point at which the converter loses its stable period-1 operation to bifurcation.

I. INTRODUCTION

DC-DC SWITCHING converters are a traditional benchmark for testing different nonlinear modeling and stability analysis approaches, due to their inherent nonsmooth characteristics. Closed loop current-mode controlled Ćuk converter is a fourth-order nonsmooth dynamical system that shows a variety of nonlinear phenomena. Up to this date, the number of modeling methods proposed to capture all the nonlinearities in power electronic converters like period-doubling bifurcation, subharmonics and chaos, although powerful, is limited. Accurate mathematical modeling turns out to be even more essential for power electronic circuits like Ćuk converter. It is mainly because analysis and simulations for the converter, as a fourth-order system, will become rather complicated if we intend to identify all nonlinear phenomena occurring on fast-time scale and not to use the traditional averaged modeling technique, merely capable of characterizing low-frequency behavior [1]. In exploring new methods for nonlinear modeling of the converter, a novel Takagi-Sugeno fuzzy modeling approach is proposed to study the instabilities in the Ćuk converter, which occurs at clock frequency. Despite other complicated approaches based on discrete nonlinear modeling [1], [2], the proposed TS fuzzy modeling approach seems a promising method to simulate high-frequency behavior of the converter to acceptable accuracy and with low computational time [3].

At the same time, the traditional approach for the stability analysis of the converter is to obtain the period- n equilibrium orbit or limit cycle by setting $x(0) = x(nT)$ acquired from iterated map. Then, the stability of the limit cycle can be determined by checking if the magnitude of all characteristic multipliers at the limit cycle are less than unity [1], [4].

However, using the classical iterative map approach will be ended up with the fourth-order iterative map which makes the stability analysis for higher period-doubling bifurcation fairly knotty. In another attempt, Filippov's method is successfully applied to study the stability of the free-running current controlled Ćuk converter [2]. Even though the method is essentially used for slow-scale stability of the converter, the complexity is inevitable. As an alternative approach, for the first time in this paper, the proposed Takagi-Sugeno model-based system is employed to accurately investigate the stability of the nonsmooth model of the converter and predict the operating point when the converter loses its stability to border collision bifurcation. The stability analysis will be based on nonsmooth Lyapunov approach which seems realistic [5] for the nonsmooth nature of the model and be formulated numerically as a Linear Matrix inequality (LMI) system. Less computational complexity and more direct analysis by checking the feasibility of the LMI system, take an edge comparing to the former analysis using nonlinear iterative map. Further more, based on LMI stability conditions which will be presented in this paper, a new control strategy can be elaborated to extend the period-1 behavior to the wide range of operating-point excursion. The method has been previously applied to non-autonomous system like boost and buck converter and showed promising results [3], [6].

II. THE ĆUK CONVERTER AND ITS MATHEMATICAL MODEL

The closed loop current-mode controlled Ćuk converter (1) is a non-autonomous nonsmooth dynamical system that conventionally is controlled by comparing sum of the inductor current, $i_{L1} + i_{L2}$, with a reference current I_{ref} to generate the OFF driving signal for the switch. The switch S is turned on at the beginning of the cycle $t = nT$ and stay there until $i_{L1} + i_{L2}$ reaches the value of I_{ref} , then, the switch S is turned OFF, and stay off until the next cycle begins. The control scheme can be formulated as follows [1]:

$$I_{ref} - (i_{L1} + i_{L2})_n = \left[\frac{E}{L_1} + \frac{v_{C2,n} - v_{C1,n}}{L_2} \right] d_n T \quad (1)$$

where subscript n denotes values at $t = nT$. Therefore

$$d_n = \frac{I_{ref} - (i_{L1} + i_{L2})_n}{\left(\frac{E}{L_1} + \frac{v_{C2,n} - v_{C1,n}}{L_2} \right) T} \quad (2)$$

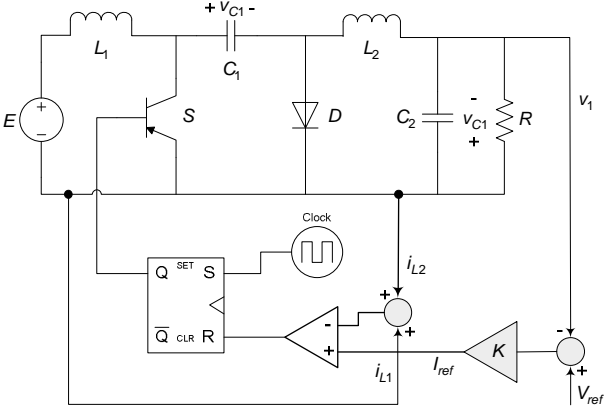


Fig. 1. Ćuk converter under original current-mode control scheme

The dynamic of the system can be described by four sets of differential equations:

$$\frac{dv_{C1}(t)}{dt} = \begin{cases} \frac{-1}{RC2}v_{C1} + \frac{1}{C2}i_{L1}, & S \text{ is OFF} \\ \frac{-1}{RC2}v_{C1} + \frac{1}{C2}i_{L1}, & S \text{ is ON} \end{cases} \quad (3)$$

$$\frac{dv_{C2}(t)}{dt} = \begin{cases} \frac{-1}{C1}i_{L1}, & S \text{ is OFF} \\ \frac{-1}{C2}i_{L2}, & S \text{ is ON} \end{cases} \quad (4)$$

$$\frac{di_{L1}(t)}{dt} = \begin{cases} \frac{-1}{L2}v_{C1} + \frac{1}{L2}v_{C2}, & S \text{ is OFF} \\ \frac{-1}{L2}v_{C1}, & S \text{ is ON} \end{cases} \quad (5)$$

$$\frac{di_{L2}(t)}{dt} = \begin{cases} \frac{1}{L1}v_{in}, & S \text{ is OFF} \\ \frac{-1}{L1}v_{C2} + \frac{1}{L1}v_{in}, & S \text{ is ON} \end{cases} \quad (6)$$

If we define the state vector as $x = [v_{C1} \ v_{C2} \ i_{L1} \ i_{L2}]$ and u as an input voltage V_{in} , the equations (3), (4), (5) and (6) can be written as:

$$\dot{x} = \begin{cases} A_1x + Bu, & (I_{ref} - A(i_{L1} + i_{L2})) < i_{ramp}(t), \\ A_2x + Bu, & (I_{ref} - A(i_{L1} + i_{L2})) > i_{ramp}(t). \end{cases}$$

where:

$$A_1 = \begin{bmatrix} -1/RC2 & 0 & 1/C2 & 0 \\ 0 & 0 & -1/C1 & 0 \\ -1/L2 & 1/L2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L1 \end{bmatrix} \quad (7)$$

$$A_2 = \begin{bmatrix} -1/RC2 & 0 & 1/C2 & 0 \\ 0 & 0 & 0 & 1/C1 \\ -1/L2 & 0 & 0 & 0 \\ 0 & -1/L1 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L1 \end{bmatrix} \quad (8)$$

In this paper, the proportional feedback controller is used where $i_{ref} = K(V_{ref} - v_1)$, K is the control parameter and V_{ref} is the reference voltage. The outer closed loop determine the reference current I_{ref} based on the value of v_1 . Under this control scheme the normal output of the converter will be a periodic ripple with the peak value of I_{ref} and the period that

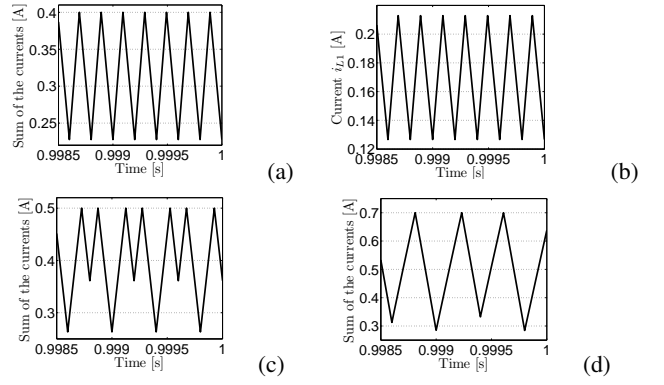


Fig. 2. The nominal period-1 operation of the Ćuk converter when $I_{ref} = 0.4A$ for (a) $i_{L1} + i_{L2}$ (b) i_{L1} (c) the output current becomes unstable and behaves in period-2 operation when $I_{ref} = 0.5A$ (d) the output current behaves chaotically when $I_{ref} = 0.7A$. The fixed parameter values are $V_{in} = 15V$, $L1 = L2 = 1mH$, $R = 75\Omega$, $C = 47\mu F$, $T = 1/2500s$.

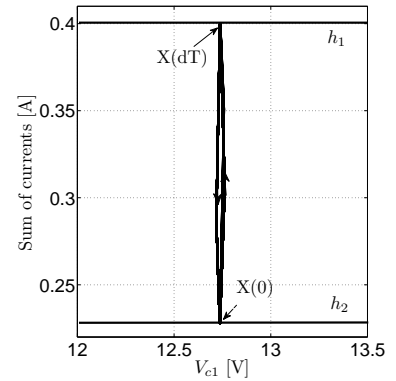


Fig. 3. Period 1 limit cycle:Phase space and the transversal intersection

is equal to the period of the PWM ramp signal. Figure 2a and 2b, shows this so-called stable period-1 operation of the converter for sum of the currents $i_{L1} + i_{L2}$ and i_{L1} respectively. Figure 2c shows the unstable period-2 operation when border collision bifurcation occurs and Figure 2d illustrates the chaotic behavior under reference-current variation. The example Ćuk converter has two switching manifold on ON and OFF switching instances. Figure 3 illustrates the stable period-1 orbit in $v - i$ space, where the orbit periodically switches between $X(dT)$ and the fixed point of the cycle with Poincaré map $X(0)$. If a system parameter like reference current I_{ref} is varied, the circuit becomes unstable through period-doubling bifurcation and then chaos, which is apparent from Figure 4 [1].

III. TS FUZZY MODEL OF THE BUCK CONVERTER FOR FAST-SCALE ANALYSIS

The fuzzy inference system of Takagi-Sugeno Fuzzy models [7], [8] in order to approximate *smooth* dynamical functions is generally described by a set of rules in the form

$$\text{Rule } j : \text{ IF } x_1 \text{ is } F_1^j \text{ AND...AND } x_q \text{ is } F_q^j \\ \text{ THEN } \dot{x} = A^j x + B^j u, \quad j = 1, \dots, l$$

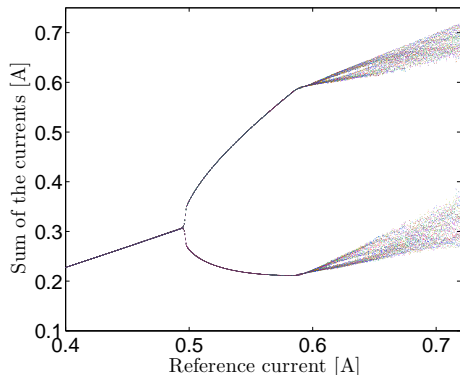


Fig. 4. Bifurcation diagram of the Ćuk converter under closed loop current-mode control (Figure 1), varying reference current as a parameter.

and the dynamics of this system can be described by:

$$\dot{x} = \sum_{j=1}^l w^j(x)(A^j x + B^j u) \quad (9)$$

where $w^j(x)$ are normalized membership functions of the rule antecedents satisfying $0 \leq w^j(x) \leq 1$, $\sum_{j=1}^l w^j(\theta) = 1$ and l is the number of rules. It is well-known that the model structure above, being famous as affine TS models, is the universal approximator of smooth nonlinear functions to arbitrary accuracy [8], [9]. As discussed in [3], [6], approximating the nonsmooth dynamical function as the example converter with TS fuzzy model structure (9) cannot be realized. The TS model (9) designed to approximate smooth nonlinear functions is fundamentally incapable to represent the inherent discontinuity already observed in the set of equations (3),(4),(5) and (6). In the mathematical point of view, existence (and uniqueness) of the TS fuzzy model of the form (9) is not being hold should the Ćuk converter's non-smooth dynamics modeled with this approach [3], [6]. Nevertheless, deriving the averaged model of the system in Figure 1, makes possible to model the converter with TS modeling approach (9). At the same time, taking advantage of averaging technique to derive a smooth model of the converter, the nonlinearities occurs on fast-time scale will be thoroughly ignored. Therefore, the border-collision bifurcation and chaos occur at the clock frequency [1] is not observable by any means.

The new TS fuzzy modeling approach (11) initially proposed in [3], [6] is capable of representing the essential dynamics of the Ćuk converter (discontinuous), which has the form:

$$\begin{aligned} \text{Rule } j &: \text{ IF } (x_3 + x_4) \text{ is } F^j \\ \text{THEN } \dot{x} &= \begin{cases} A^j(m_i)x + B^j(m_i)u \\ m^+ = \phi(x, m), \quad j = 1, 2, \quad i = 1, 2 \end{cases} \end{aligned} \quad (10)$$

and by the appropriately restricting the inference parameters, the dynamics of the discontinuous fuzzy system can be described by:

$$\begin{cases} \dot{x} = \sum_{j=1}^l w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u) \\ m^+ = \phi(x, m) \end{cases} \quad (11)$$

where $x \in R^n$ is the continuous state, $m \in M = \{m_1, m_2\}$ is the discrete state, $A^j(m_i) \in R^{n \times n}$, $B^j(m_i) \in R^n$, $w^j: R^n \times M \rightarrow [0, 1]$, $j \in I_l$, are continuous weighting

functions which satisfy $\sum_{j=1}^l w^j(x, m) = 1$, l is the number of fuzzy rules and F^j are fuzzy sets. The state space is the Cartesian product $R^n \times M$. The function $\phi: R^n \times M \rightarrow M$ describes the dynamics of the discrete state. The notation m^+ means the next state of m . Any value of discrete state $m_i \in M$ is associated with an affine subsystem like:

$$\text{if } \forall x \in A(m_i)x + B(m_i)u \text{ then } m_i \in M, \quad i \in \{1, 2\}$$

Remark 1: In general a value of m_i could be associated with a subset of subsystem as:

if $\forall x \in \{\sum_{j \in \{1, 2, \dots\}} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u)\}$ then $m_i \in \{m_1, m_2, \dots, m_N\}$ when N is possibly infinite \square

As it is inferred from (10) and (11), function ϕ formally describes the transition of the discrete states. Each transition involves with jumping from the subset of fuzzy subsystems, which from now on is called fuzzy sub-vector field, to another fuzzy sub-vector field. Alternatively, the transition can be defined by a set of *switch sets* used to mathematically define the switching manifold. The general formulation of the *switch sets* is defined as:

$$S_{i,k} = \{x \in R^n | m_k = \phi(x, m_i)\}, \quad m_i \neq m_k, \quad i, k \in I_N \quad (12)$$

which in the case of the Ćuk converter, the switching hypersurface at the switching OFF

$$h(X, (dT)) = x_3(dT) + x_4(dT) - K(V_{\text{ref}} - x_1) \quad (13)$$

in the form of switch sets can be describe as:

$$\begin{aligned} S_{1,2} &= \{x \in R^n | x_3(dT) + x_4(dT) + Kx_1 < KV_{\text{ref}}\}, \\ S_{2,1} &= \{x \in R^n | x_3(dT) + x_4(dT) + Kx_1 > KV_{\text{ref}}\} \end{aligned} \quad (14)$$

where d is the duty ratio at each instant and T is the switching period.

To construct the TS model (10) of the converter, the membership functions are defined to exactly represent each fuzzy sub-vector as follows:

$$\begin{aligned} F^1(x_3(t) + x_4(t)) &= \frac{1}{2} \left(1 + \frac{X_1(0) - x_3(t) - x_4(t)}{2l} \right), \\ F^2(x_3(t) + x_4(t)) &= \frac{1}{2} \left(1 + \frac{X_1(0) - x_3(t) - x_4(t)}{2l} \right) \end{aligned}$$

Here, l is a constant denoting the range of sum of the currents, i.e. $i_{L1}(t) + i_{L2}(t) \in \{0.4 - l, 0.4 + l\}$. We let $l = 0.2$. The state vector $X(0) = [12.7187 \ 27.5608 \ 0.1012 \ 0.1268]$ is the transversal intersection of the periodic orbit with the switching manifold when the switch is OFF (see Figure 3). Normally we would pick the equilibrium point of the system if destined for modeling a smooth or the average model of the converter [10]. However, the nonsmooth model is the target here so the transversal intersection point should take into consideration in the membership function to accurately represents switching between two sub-vector fields. Since the linear vector fields are dealt with in this case, the matrices in (7) and (8) can immediately be used in building the exact TS fuzzy models of the converter as $A^1(m_1) = A^2(m_1) = A_1$, $A^1(m_2) = A^2(m_2) = A_2$ and $B(m_1) = B(m_2) = B$ where the discrete states m_1 and m_2 is actually representing ON

and OFF state of the converter respectively. Figure 5a and 5b comparing with the original behavior of the converter in different operation modes (Figure 2) shows that the proposed TS fuzzy model under current-mode control can well represents the nonsmooth switching and capture any phenomena occurring at the clock cycle like border collision bifurcation and chaos. The bifurcation diagram in Figure 6 makes it more evident the strength of the new TS fuzzy modeling method in capturing all nonlinearities by showing exactly the same quantitative behavior compared with Figure 4.

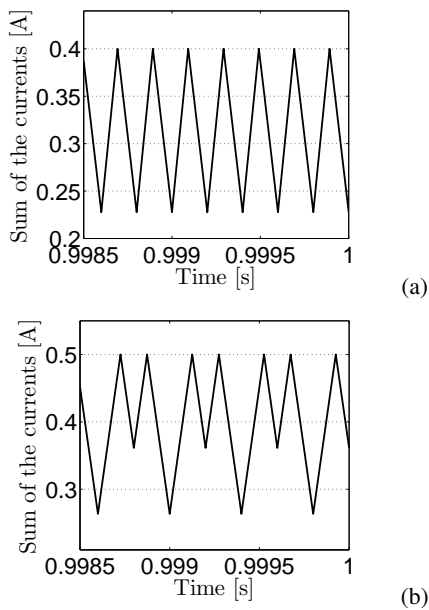


Fig. 5. (a) The nominal period-1 operation of the TS fuzzy model of the Ćuk converter under original current-mode control when $I_{\text{ref}} = 0.4A$ (b) The nominal period-2 operation of the TS fuzzy model of the converter when $I_{\text{ref}} = 0.5A$.

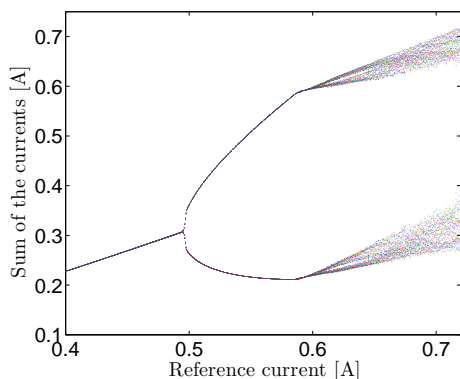


Fig. 6. Bifurcation diagram of the Ćuk converter modeled with TS fuzzy approach, varying reference current as a parameter.

IV. EXPONENTIAL STABILITY ANALYSIS

The stability of the limit cycle in power electronics circuits will be predominately determined using Poincaré maps [11]. At a selected switching surface, a perturbation of the switching point is mapped through the switching cycle back to the

same switching surface. This is a discrete mapping, where the eigenvalues of the mapping matrix determines the stability of the limit cycle. Employing this approach with an engagement of Filippov's method, the stability analysis of an autonomous Ćuk converter is already studied [2].

A. Nonsmooth Lyapunov method to show the stability of limit cycle

Considering the fact that the proposed TS fuzzy model of the converter represents a nonsmooth dynamical system, a Lyapunov method is employed for the first time in this section for the bifurcation analysis of the example converter. The traditional approach of finding a smooth global Lyapunov function to show the stability of TS fuzzy models approximating a smooth nonlinear system have been around for long time in the model-based fuzzy control literature [8]. The shortcomings of finding global quadratic Lyapunov candidates in the entire fuzzy state space and the subsequent conservative LMI formulation for TS fuzzy models of the smooth nonlinear systems and naturally for the proposed TS fuzzy model (11) is also already hashed over [3], [6], [12]. Briefly, to relax the Linear Matrix Inequalities (LMI) formulation for the stability analysis of the nonsmooth model of the converter, the Lyapunov function candidate should be given as discontinuous or nonsmooth functions. We also let the fuzzy state space to be partitioned to different *flexible* regions. A switching function partitions the state space into two regions, separated by the switching surface $h(X, (dT)) = 0$. Followingly, we let the fuzzy state space \mathcal{F} is partitioned into two detached region $\Omega_q, q \in I_2$ where $I_2 = \{1, 2\}$. Therefore

$$\begin{aligned} \Omega_1 &= \{(x, m) \in \mathcal{F} \mid x \in \mathbb{R}^n, m = m_1\} \\ \Omega_2 &= \{(x, m) \in \mathcal{F} \mid x \in \mathbb{R}^n, m = m_2\} \end{aligned} \quad (15)$$

A trajectory initiated in any region at time $t_k, k = 1, 2, \dots$ can pass through another region if $t_k < t_{k+1}$. We define Λ_{qr} as a neighboring region which means:

$$\Lambda_{qr} = \{x \in \mathbb{R}^n \mid \exists t < t_0, \text{ such that } x(t^-) \in \Omega_q, x(t) \in \Omega_r\} \quad (16)$$

Λ_{qr} is given by the hypersurface of the form (13). In fact, conditions for the switching from mode m_1 to mode m_2 at time τ are

$$\begin{cases} h(X, \tau) = 0 \\ \sum_{j \in \{1, 2, \dots\}} w^j(x, m_i) (A^j(m_i)x + B^j(m_i)u) \cdot \nabla h(X, \tau) < 0 \end{cases} \quad (17)$$

Hence if $\Lambda_{qr} \neq \emptyset$, Ω_q and Ω_r must be neighboring sets. As a sufficient condition let:

$$I_\Lambda = \{(q, r) \mid \Lambda_{qr} \neq \emptyset\} \quad (18)$$

which is a set of tuples indicating that there is at least one point for which the trajectory passes from Ω_q to Ω_r .

Now, we define a local quadratic Lyapunov function for each region, which has the structure:

$$V(x) = V_q(x) = \tilde{x}^T \tilde{P}_q \tilde{x} \quad \text{when } (x, m) \in \Omega_q \quad (19)$$

where $\tilde{x} = \begin{bmatrix} x \\ 1 \end{bmatrix}$, $\tilde{P}_q = \begin{bmatrix} P_q & p_q \\ p_q^T & \pi_q \end{bmatrix}$, $\pi_q \in \mathbb{R}$, $p_q \in \mathbb{R}^n$, $P_q = P_q^T \in \mathbb{R}^n \times \mathbb{R}^n$ and $q \in I_2$.

Let Ω_q^x denote the continuous state of x in Ω_q . $V_q : \text{cl}\Omega_q^x \rightarrow \mathfrak{R}, q \in I_\Delta$, is a (scalar) function which is assumed to be continuously differentiable on closure of region Ω_q (cl. denotes the closure of a set, which is the smallest closed set containing the set). In fact, the scalar function $V_q(x, t)$ is used to measure the fuzzy system's energy in a local region Ω_q . As it is apparent, the overall Lyapunov function V (19) is a discontinuous Lyapunov function at the hypersurface (13) or at the neighboring regions $\Lambda_{qr}, (q, r) \in I_\Delta$. Assuming $t_k < t_{k+1}$ for every trajectory with initial point in any region, $V(x)$ is *piecewise continuous* function with respect to time.

B. LMI formulation for stability and bifurcation analysis

The first stage in formulating the Lyapunov stability conditions into LMI conditions is to define fuzzy state-space partitions by (positive) quadratic functions. This is realizable using the so-called S-procedure technique to substitute the confined conditions with unconfined conditions [13]. To explain the procedure in general terms and more clearly, let Q_0, \dots, Q_s , be quadratic functions of the variable $x \in \mathfrak{R}^n$ on the form:

$$Q_k(x) = x^T Z_k x + 2c_k^T x + d_k, \quad k = 0, \dots, s, \quad (20)$$

where $Z_k = Z_k^T$. We consider the following condition on Q_0 :

$$Q_0(x) \geq 0 \text{ in the region } \{x \in \mathfrak{R}^n | F_k(x) \geq 0, k \in I_s\} \quad (21)$$

The confined condition (21) can be substituted by an unconfined condition in the following way:

Lemma [13]: if there exist $\delta_k \geq 0, k \in I_s$, such that

$$\forall x \in \mathfrak{R}^n, Q_0(x) \geq \sum_{k=1}^s \delta_k Q_k(x) \quad (22)$$

then (21) holds. Hence, by introducing additional variables $\delta_k \geq 0, k \in I_s$, condition (21) can be turned into an LMI which can be written as:

$$x^T \begin{bmatrix} Z_0 & c_0 \\ c_0^T & d_0 \end{bmatrix} x \geq \sum_{k=1}^s \delta_k x^T \begin{bmatrix} Z_k & c_k \\ c_k^T & d_k \end{bmatrix} x \quad (23)$$

The replacement of (21) by **Lemma** may be conservative. However it can be shown that the converse is true in case of single quadratic form, $s=1$ [13] contingent on the existence of some x such that $Q_1(x) > 0$. In case of hypersurface which can be defined by $Q_k(x) = 0, k \in I_s$, Lemma is true without the restriction $\delta_k \geq 0$.

Here, all the conditions in the stability theorem are immediately described by $Q_0(x) \geq 0$, where $Q_0(x)$ is a quadratic function defined by (20).

$$\begin{cases} Q_0(x) = \tilde{x}^T (\tilde{P}_q - \alpha \tilde{I}) \tilde{x} \geq 0, \forall x \in \Omega_q^x, q \in I_2 \\ Q_0(x) = \tilde{x}^T (\beta \tilde{I} - \tilde{P}_q) \tilde{x} \geq 0, \forall x \in \Omega_q^x, q \in I_2 \\ Q_0(x) = -\tilde{x}^T (\tilde{A}(m)^T \tilde{P}_q + \tilde{P}_q \tilde{A}(m) + \gamma \tilde{I}) \tilde{x} \geq 0, \forall (x, m) \in \Omega_q \\ Q_0(x) = \tilde{x}^T (\tilde{P}_r - \tilde{P}_q) \tilde{x} \geq 0, \forall x \in \Lambda_{qr}^x, (q, r) \in I_\Delta \end{cases} \quad (24)$$

In the conditions above, α and β are constants which originally represent class \mathcal{K} function $\alpha(\|x\|), \alpha : \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$ and $\beta(\|x\|), \beta : \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$. (for definition of class \mathcal{K} function see [14]) and $\gamma > 0$ is a scalar constant. The third condition is satisfied on the hypersurface Λ_{qr}^x which can be given by $Q_k(x) =$

$0, k \in I_s$, where each $Q_k(x) = 0$ has the form (20) with no limitation on δ_k as mentioned before. All the conditions can be substituted by the unconfined condition (23); however, if the switching manifold cannot exactly be represented by $Q_k(x) = 0, k \in I_s$ in the last condition, it is possible to represent such a region with quadratic functions satisfying $Q_k(x) \geq 0$, in which the additional variables δ_k should be limited to $\delta_k \geq 0$.

Now, all the stability conditions for bifurcation analysis can be recasted to LMI conditions:

LMI problem: If there exist $\tilde{P}_q, q \in I_\Delta$, constants $\alpha > 0, \mu_k^q \geq 0, \nu_k^{qij} \geq 0, \eta_k^{qr}$ and a solution to $\min \beta$ subject to the three conditions:

$$\begin{aligned} & \bullet \quad \alpha \tilde{I} + \sum_{k=1}^{s_q} \mu_k^q \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix} \leq \tilde{P}_q \\ & \tilde{P}_q \leq \beta \tilde{I} + \sum_{k=1}^{s_q} \mu_k^q \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix}, \quad q \in I_\Delta \\ & \bullet \quad (q, i, j) \in I_\Omega, (\tilde{A}^j)^T \tilde{P}_q + \tilde{P}_q \tilde{A}^j \\ & \quad + \sum_{k=1}^{s_{qij}} \nu_k^{qij} \begin{bmatrix} Z_k^q & c_k^q \\ (c_k^q)^T & d_k^q \end{bmatrix} \leq -\tilde{I}, \quad q \in I_\Delta \\ & \bullet \quad \tilde{P}_r \leq \tilde{P}_q - \sum_{k=1}^{s_{qr}} \eta_k^{qr} \begin{bmatrix} Z_k^{qr} & c_k^{qr} \\ (c_k^{qr})^T & d_k^{qr} \end{bmatrix}, \quad (q, r) \in I_\Delta \end{aligned}$$

Then the fixed point 0 is exponentially stable in the sense of Lyapunov ¹.

Remark 2: Without loss of generality, it is assumed that the origin is a fixed point of the fuzzy system (11). For the buck converter, the fixed point mentioned above is the fixed point of limit cycle with a stroboscopic map [15] \square

Let the fixed parameter values as mentioned in Figure 2, for a specified desired current $I_{\text{ref}} = 0.42$, which is the stable period-1 operation of the converter as apparent from the bifurcation diagram in Figure 6, the system converges exponentially to the stable limit cycle, which will be verified by solving LMI problem.

$$\tilde{\mathbf{P}}_1 = \begin{bmatrix} 43.8488 & 41.3433 & -93.4600 & 0 & 0 \\ 41.3433 & 42.5052 & -90.0070 & 0 & 0 \\ -93.46 & -90.00 & 810.9474 & 0 & 0 \\ 0 & 0 & 0 & 1.33 & -833.36 \\ 0 & 0 & 0 & -833.36 & 2.08 \end{bmatrix} \quad (25)$$

$$\tilde{\mathbf{P}}_2 = \begin{bmatrix} 1.0210 & 0 & -1.8009 & 0 & 0 \\ 0 & 1.6300 & 0 & -0.0050 & -24.44 \\ -1.8009 & 0 & 269.87 & 0 & 0 \\ 0 & -0.0050 & 0 & 554.86 & -0.14 \\ 0 & -24.44 & 0 & -0.147 & 949.35 \end{bmatrix} \quad (26)$$

with the optimal value of $\alpha = 0.36$ and $\beta = 0.43$. If the value of reference current is intentionally changed to $I_{\text{ref}} = 0.5$, the LMI problem finds infeasible and no optimal value of α and

¹The proof of this theorem is out of the scope of this paper and it will present in later publications.

β can be found. Actually, the infeasibility of LMI problem, can detect the edge of unstable period doubling bifurcation as for all the values of $I_{ref} < 0.5$, the LMI problem can found a feasible solution with the optimum value of β . This readily determine the range of the stable period-1 operation of the converter without using nonlinear discrete mapping approach.

Letting the fuzzy state space to be partitioned into flexible detached region is of prime importance in finding the actual stability of the nonsmooth system like the example Ćuk converter. An attempt to let the overall Lyapunov function V measure the system's energy for the entire fuzzy state space means a common Lyapunov function for all discrete states has to be sought. Then, the neighboring region Λ_{gr} are all empty and the system will be stable regardless of the switchings. However, as can be noticed from table below, searching for the feasible solution to the LMI problem is not possible even for the stable period-1 excursion of the converter. Partitioning the fuzzy state space into more regions may end up in different optimal value of β , although it can accurately show the stable region of the converter and the operating point when bifurcation occurs by the LMI feasibility problem. Deciding a suitable region partitioning is optional. Nevertheless, it should be avoided to make numerous partitions or a partitioning so complex as in practice, verifying the stability conditions in the LMI problem turns out to be impossible for heavy computational burden.

Number of Partitions in \mathcal{F}	LMI feasibility	Optimum value of β	Numerical complexity
1	not feasible	N/A	low
2	feasible	0.4330	low
4	feasible	0.4540	medium
8	feasible	0.4540	medium
16	feasible	0.6450	high
32	not feasible	N/A	very high

V. CONCLUSION

The Takagi-Sugeno fuzzy modeling approach proposed in this paper is specifically synthesized to incorporate the typical switching events of power electronic converters. Application on the example Ćuk converter, showed the prowess of new TS fuzzy modeling approach to capture all nonlinear phenomena occurs at the clock cycle like fast-scale bifurcation and chaos.

A rigorous mathematical stability analysis based on the piecewise Lyapunov function candidates introduced to delve into the stability of limit cycles. All stability conditions formulated in Linear Matrix Inequalities (LMI) conditions to evidently predict the onset of unstable period-doubling bifurcation. Presented nonsmooth Lyapunov theory seems essential to tackle the discontinuous dynamics of the new TS fuzzy model of the converter. Fuzzy state-space partitioning to relaxed, detached regions reduce the possibility of conservative LMI formulation to bare minimum although a suitable region partitioning is ad-hoc to each case. The proposed stability analysis is a new way to get an insight of all the fast-scale instabilities of the converter over and above the traditional discrete nonlinear mapping normally employed for specially high-ordered systems.

This paper demonstrates another successful attempt of the proposed modeling and stability analysis approach on power electronic circuits. Not only the whole approach can be extended to tackle the nonsmooth mechanical systems with state jumps, it is also a solid bedrock to coin new switching fuzzy model-based controllers to preserve the stable period-1 behavior of the power electronic circuits for large range of parameter variation.

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