Stability Analysis of DC-DC Buck Converter using Takagi-Sugeno Fuzzy Model-based Approach

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Abstract— **This paper develops a Takagi-Sugeno fuzzy approach for modeling a DC-DC voltage-mode controlled buck converter as a nonlinear, nonsmooth system to capture all the essential fast-scale nonlinearities that occur at controller clock frequency. A tractable mathematical stability analysis, employing nonsmooth Lyapunov functions, is proposed for identifying converter fast-scale instabilities and the onset of bifurcation phenomena. The search for the Lyapunov functions is formulated as a Linear Matrix Inequalities (LMI) feasibility problem which may be solved using interior point methods.**

I. Introduction

The most conventional approach for studying the stability of limit cycles in nonsmooth DC-DC converters is nonlinear discrete modeling [1] which generally captures the essential properties of periodic orbits. In many cases, including the voltage-mode controlled buck converter, the analytical derivation of the discrete map is impossible and one has to resort to numerical methods. A powerful numerical technique based on the application of Fillipov's method in combination with Floquet theory, has been proposed [2] to determine the stability of periodic limit cycles in a DC-DC buck converter operating in continuous conduction mode, allowing the stability of the circuit to be directly inferred from analyzing the behavior of the system in response to small perturbations.

In this paper, a model-based Takagi-Sugeno (TS) fuzzy system design [3], [4] is developed as an alternative, efficient approach for the accurate mathematical modeling and rigorous stability analysis of the converter to automatize the whole procedure. To date, there have been a number of successful applications of TS fuzzy methods in power electronic converters [5]–[7]. In terms of the model-based fuzzy approach that blends fuzzy logic and the theory of modern control, the earlier papers approximated the dynamical model of the converter by a TS fuzzy model obtained using the averaging technique. While this is suitable for deriving some information about the stability and dynamic behavior in slow-time scale, it cannot capture the events that occurs at clock frequency. Thus, all instabilities that occur in fast-time scale cannot be taken into account and be studied.

The TS fuzzy model method is extended in this paper to capture all the possible nonlinear phenomena that take place at fast time scale including subharmonic oscillations, crises and chaotic behavior [1], [2]. Stability conditions are discussed based on nonsmooth Lyapunov functions and the analysis used to locate the deviation from period-1 stable operation via the resulting LMI feasibility problem.

II. The buck converter and its mathematical model

The voltage mode controlled buck converter circuit shown in Fig. 1 is a nonsmooth dynamical system described by two sets of differential equations:

$$\frac{dv(t)}{dt} = \begin{cases} \frac{v_{in}-v(t)}{L}, & \text{S is blocking} \\ -\frac{v(t)}{L}, & \text{S is conducting} \end{cases} \tag{1}$$

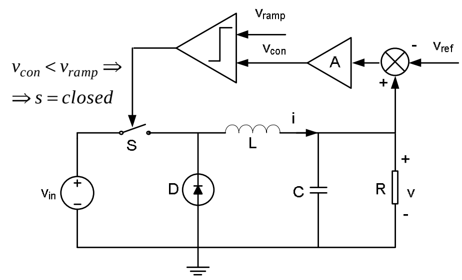

Fig. 1. Voltage mode controlled buck dc-dc converter.

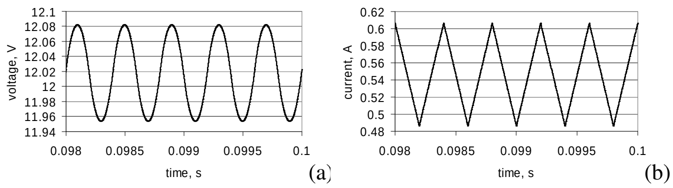

Fig. 2. Nominal period-1 operation of the buck converter: (a) output voltage, (b) output current; fixed parameter values $V_{\text{in}} = 24V$, $V_{\text{ref}} = 11.3V$, $L = 20mH$, $R = 22\Omega$, $C = 47\mu F$, $A = 8.4$, $T = 1/2500s$ and the ramp signal varies from 3.8V to 8.2V.

$$\frac{dv(t)}{dt} = \frac{i(t) - \frac{v(t)}{R}}{C} \tag{2}$$

The operation of this PWM controlled circuit is expounded in [1], [2]. Normally, the output of the converter is a dc voltage with a mean value close to the desired voltage and a period that is equal to the period of the PWM ramp signal (referred to as a period-1 waveform), as shown in Fig. 2. In Fig. 3, the same stable period-1 orbit is illustrated in $v - i$ space. The time-varying switching surface returns to the so-called fixed point of the cycle with a Poincaré map $X(0)$ [1], repeating the periodic cycle. It has been shown that if a system parameter (say, the input voltage) is varied, the circuit may lose stability through successive period-doubling bifurcations leading to chaos [1], [2] as apparent in Fig. 4.

If we define $x_1(t) = v(t)$ and $x_2(t) = i(t)$, (1) and (2) can be written as:

$$\dot{x} = \begin{cases} A_s x + Bu, & (Ax_1(t) - V_{ref}) < v_{ramp}(t), \\ A_s x, & (Ax_1(t) - V_{ref}) > v_{ramp}(t). \end{cases} \tag{3}$$

where we can define the state matrices:

$$A_s = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{\text{in}} \tag{4}$$

The switching hypersurface(h) can be written as:

$$h(x(t), t) = x_1(t) - V_{\text{ref}} - \frac{v_{\text{ramp}}}{A} = 0, \; A \neq 0. \tag{5}$$

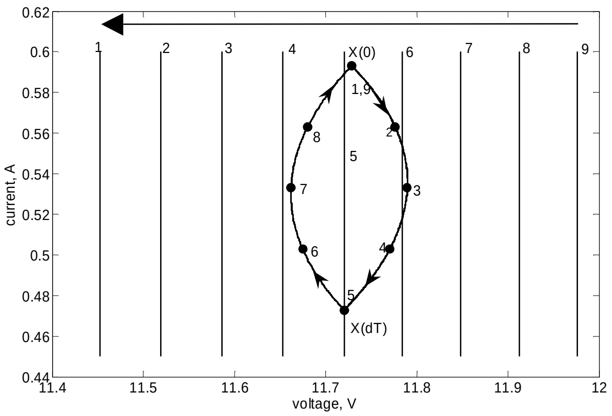

Fig. 3. Period-1 limit cycle indicating the location of the time varying switching surface.

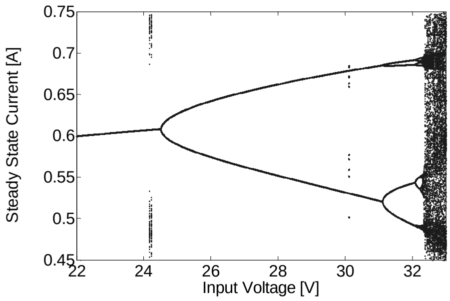

Fig. 4. Bifurcation diagram of the buck converter with the input voltage as the bifurcation parameter.

where the periodic sawtooth waveform is defined as:

$$v_{\mathrm{ramp}}(t) = V_L + (V_U - V_L)(\frac{t}{T}\ mod\ 1) \tag{6}$$

III. TS FUZZY MODEL OF THE BUCK CONVERTER FOR FAST-SCALE ANALYSIS

The behavior of conventional TS fuzzy models is described by a set of rules of the form

$$\begin{aligned} \text{Rule } j \quad &: \quad \text{IF } x_1 \text{ is } F_1^j \text{ AND...AND } x_q \text{ is } F_q^j \\ & \quad\ \ \text{THEN } \dot{x} = A^j x + B^j u + a^j, \ \ j = 1, \ldots, l \end{aligned}$$

and the dynamics of this system can be described by

$$\dot{x} = \textstyle\sum_{j=1}^{l} w^j(x)(A^j x + B^j u + a^j) \tag{7}$$

where $w^j(x)$ are normalized membership functions of the rule antecedents satisfying $0 \leq w^j(x) \leq 1$, $\sum_{j=1}^{l} w^j(\theta) = 1$ and l is the number of rules. The stability of these systems is based on the existence of a common quadratic Lyapunov function for all linear subsystems and sufficient stability conditions based on that Lyapunov function [4]. The finesse of this analysis as a numerical approach comes from the fact that the search for Lyapunov functions can be formulated as linear matrix inequalities (LMI's). The optimization problem can then be solved efficiently with less computational effort using a widely available software tool like MATLAB.

Universal approximation capability of the fuzzy models of the form (7) is discussed in [4]. It has been shown that the affine structure originally proposed in [3] and later in many other applications can approximate any smooth nonlinear function to arbitrary accuracy. However, the function approximation capability of the fuzzy models of the form (7) is fundamentally inadequate to represent the discontinuous dynamics and the ensuing nonlinear events in the example buck converter. To empower the TS fuzzy modeling approach to mathematically represent any switching events, we need to introduce discrete states to interpolate with their associated continuous states. Moreover, the conventional TS fuzzy model can only approximate the functions satisfying local Lipschitz conditions for any interval. However, the mathematical model of the buck converter (1), (2) does not fulfil this property at the point of discontinuity according to the definition of the Lipschitz condition [8]. For this reason we need an extra element (discrete events) to hold the existence and uniqueness of the fuzzy approximation representing nonsmooth functions [8].

To overcome the shortcomings stated above, a novel TS fuzzy modeling approach is synthesized and presented here to enable modeling of the nonsmooth dynamical equations of the buck converter. The behavior of these models can be described by:

$$\begin{aligned} \text{Rule } j \quad &: \quad \text{IF } x_1 \text{ is } F^j \\ \text{THEN } \dot{x} \quad &= \quad \begin{cases} A^j(m_i)x + B^j(m_i)u \\ m^+ = \varphi(x, m), \ \ j = 1, 2, \ \ i = 1, 2 \end{cases} \end{aligned} \tag{8}$$

and by the appropriately restricting the inference parameters, the dynamics of the discontinuous fuzzy system can be described by:

$$\begin{cases} \dot{x} = \sum_{j=1}^{l} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u) \\ m^+ = \varphi(x, m) \end{cases} \tag{9}$$

where $x \in R^n$ is the continuous state, $m \in M = \{m_1, m_2\}$ is the discrete state, $A^j(m_i) \in \Re^{n \times n}$, $B^j(m_i) \in \Re^n$, $w^j : \Re^n \times M \rightarrow [0\ \ 1]$, $j \in I_l$, are continuous weighting functions which satisfy $\sum_{j=1}^{l} w^j(x, m) = 1$, l is the number of fuzzy rules and F^j are fuzzy sets. The state space is the Cartesian product $\Re^n \times M$. The function $\varphi : \Re^n \times M \rightarrow M$ describes the dynamics of the discrete state. The notation m^+ means the next state of m. Any value of discrete state $m_i \in M$ is associated with an affine subsystem like:

$$\text{if } \forall x \in A(m_i)x + B(m_i) + a(m_i) \text{ then } m_i \in M, \ i \in \{1, 2\}$$

Remark 1: In general a value of m_i could be associated with a subset of subsystem as:

if $\forall x \in \{\sum_{j \in \{1,2,\ldots\}} w^j(x, m_i)(A^j(m_i)x + B^j(m_i)u + a^j(m_i))\}$ then $m_i \in \{m_1, m_2, \ldots, m_N\}$ when N is possibly infinite $\square$

The transition from one discrete state to another means the abrupt change from one set of fuzzy subsystems representing a continuous vector field to another set, formally described by the function φ. For convenience, this transition can be defined by a set of *switch sets* which in fact represent the hypersurface (5). So a switch set can in general be described as:

$$S_{i,k} = \{x \in R^n | m_k = \phi(x, m_i)\}, \ m_i \neq m_k, \ i, k \in I_N \tag{10}$$

and, referring to the hypersurface equation (5), the switch set can be defined as:

$$\begin{aligned} S_{1,2} = \{x \in R^n | x_1(dT) - V_{\mathrm{ref}} < \tfrac{v_{\mathrm{ramp}}}{A}\}, \\ S_{2,1} = \{x \in R^n | x_1(dT) - V_{\mathrm{ref}} > \tfrac{v_{\mathrm{ramp}}}{A}\} \end{aligned} \tag{11}$$

where d is the duty ratio at each instant. Now we define two membership functions to exactly represent each vector field of the buck converter as follows:

$$F^1(x_1(t)) = \frac{1}{2} + \frac{x_1(t) - X_1(0)}{22.6}, \ F^2(x_1(t)) = 1 - F^1(x_1(t))$$

where the state $X(0) = [12.0747, 0.6220]^T$ is the stable fixed point of the system, i.e. an intersection point of limit cycle with the poincaré map (Fig. 3) (see [1], [2] for the detailed derivation using the Newton-Raphson method). The main reason for selecting the fixed point for constructing the membership functions is to minimize the error of the fuzzy approximation at the switching instants.

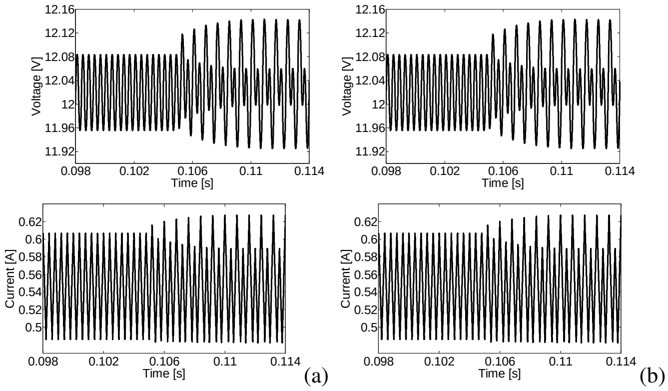

Fig. 5. System output voltage and current time responses showing a loss of period-1 stability at $time = 0.105s$ caused by an increase of input voltage from $24V$ to $25V$ (a) original model (b) TS fuzzy model.

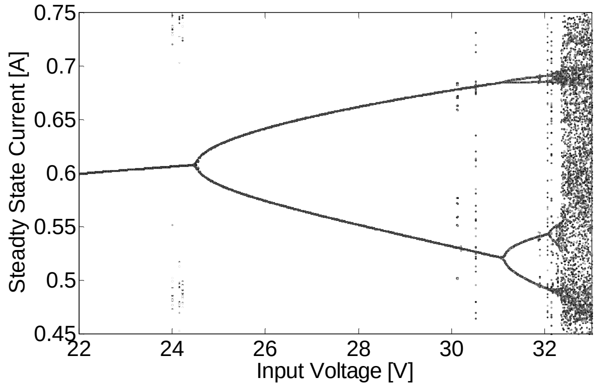

Fig. 6. Bifurcation diagram of the TS fuzzy model of the buck converter with the input voltage as the bifurcation parameter.

The fuzzy model matrices are constructed directly using (4) as $A^1(m_1) = A^2(m_1) = A^1(m_2) = A^2(m_2) = A_s$, $B(m_1) = B$ and $B(m_2) = [0 \ \ 0]^T$. The discrete state m_1 is associated with the switch-off vector field and m_2 is associated with the switch-on vector field of the converter.

To verify the accuracy of how the new modeling approach is able to represent the fast-scale nonlinearities of the system, the time response of the TS fuzzy model of the converter and the original system (Fig. 1) is compared under voltage mode control. Figure 5, shows how the TS fuzzy model exactly predicts the behavior of the original system both in stable period-1 and when the system loses its stability to a period doubling bifurcation by changing the input voltage from 24V to 25V. The bifurcation diagram produced using the new model (Fig. 6) also shows the same successive subharmonics and chaotic region when compared to the original system (Fig. 4).

IV. Exponential Stability analysis

The exponential stability of a linear TS fuzzy system approximating a smooth function is thoroughly treated in [4]. Considering the fact that the proposed TS fuzzy model of the converter represents a nonsmooth dynamical system, any LMI formulation for finding a *global* and *smooth* quadratic Lyapunov function candidate $V(x) = x^{\mathrm{T}}Px$ in the entire fuzzy state space is very conservative. Even in the case of TS fuzzy modeling of smooth dynamical systems, a global smooth quadratic Lyapunov function fails to exist while the system is actually stable [9]. Hence, very few efficient methods are available (assuming these are applicable) to formulate LMI stability conditions based on smooth Lyapunov functions. To overcome this conservative formulation for the stability analysis of the nonsmooth TS fuzzy model of the buck converter, two natural extension are applied. First, the Lyapunov function candidates are selected as *discontinuous* or *piecewise smooth* functions. Second, the fuzzy state space can be partitioned into *different flexible regions* for the system of the form (9).

The method described in this section can be applied to the proposed TS fuzzy model of the buck converter, and is based on formulating the stability condition as LMIs.

For the sake of relaxing the conservative formulation of stability conditions, we assume that the fuzzy state space is partitioned into Δ detached regions $\Omega_q, q \in I_\Delta$ where $I_\Delta = \{1, \ldots, \Delta\}$. The candidate Lyapunov function will be piecewise quadratic, meaning that each local Lyapunov function has the structure:

$$V(x) = V_q(x) = \tilde{x}^T \tilde{P}_q \tilde{x} \ \text{ when } \ (x, m) \in \Omega_q \tag{12}$$

where $\tilde{x} = \begin{bmatrix} x \\ 1 \end{bmatrix}$, $\tilde{P}_q = \begin{bmatrix} P_q & p_q \\ p_q^T & \pi_q \end{bmatrix}$, $\pi_q \in \Re$, $p_q \in \Re^n$, $P_q = P_q^T \in \Re^n \times \Re^n$ and $q \in I_\Delta$.

Let Ω_q^x denote the continuous state of x in Ω_q. $V_q : \mathbf{cl}\Omega_q^x \to \Re, q \in I_\Delta$, is a scalar function which is assumed to be continuously differentiable on closure of region Ω_q (cl. denotes the closure of a set, which is the smallest closed set containing the set). In fact, the scalar function $V_q(x,t)$ is used to measure the fuzzy system's energy in a local region Ω_q. A trajectory initiated in any region at time $t_k, k = 1, 2, ...$ can pass through another region if $t_k < t_{k+1}$. We define Λ_{qr} as a neighboring region which means:

$$\Lambda_{qr} = \{x \in \Re^n | \exists t < t_0, \text{ such that } x(t^-) \in \Omega_q, x(t) \in \Omega_r\} \tag{13}$$

Λ_{qr} is given by the hypersurface of the form (5). Therefore, if $\Lambda_{qr} \neq \emptyset$, Ω_q and Ω_r must be neighboring sets. As a sufficient condition let

$$\mathrm{I}_\Lambda = \{(q, r) | \Lambda_{qr} \neq \emptyset\} \tag{14}$$

which is a set of tuples indicating that there is at least one point for which the trajectory passes from Ω_q to Ω_r. Considering the above fuzzy region portioning, (12) is a discontinuous Lyapunov function at the neighboring regions Λ_{qr}, $(q, r) \in I_\Lambda$. Assuming $t_k < t_{k+1}$ for every trajectory with initial point in any region, $V(x)$ is *piecewise continuous* function with respect to time.

A. Stability conditions as LMI for bifurcation analysis

Stability conditions presented in this section are confined to a part of the fuzzy continuous state space. This is practicable by expressing a region as positive (quadratic) functions and employing a so-called S-procedure technique [10], to substitute the confined conditions with unconfined conditions. The procedure is essential to formulate all of the stability conditions to LMIs [11]. To formulate the stability theorem in the form of confined conditions in one LMI feasibility problem, all conditions in the stability theorem should be described by $Q_0(x) \geq 0$, where $Q_0(x) = x^T \begin{bmatrix} Z_0 & c_0 \\ c_0^T & d_0 \end{bmatrix} x$ is a quadratic function. The first condition is defined by two inequalities $Q_0(x) = \tilde{x}^T(\tilde{P}_q - \alpha\tilde{I})\tilde{x} \geq 0$ and $Q_0(x) = \tilde{x}^T(\beta\tilde{I} - \tilde{P}_q)\tilde{x} \geq 0$ where α and β are constants which represent class $\mathcal{K}$ function $\alpha : \Re^+ \to \Re^+$ and $\beta : \Re^+ \to \Re^+$ of $\|x\|$ [8] and $\tilde{I} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$. The second condition is $Q_0(x) = -\tilde{x}^T(\tilde{A}(m)^T\tilde{P}_q + \tilde{P}_q\tilde{A}(m) + \gamma\tilde{I})\tilde{x} \geq 0$ where $\gamma > 0$ is a scalar constant, $\tilde{A}(m_i) = \begin{bmatrix} A(m_i) & B(m_i) \\ 0 & 0 \end{bmatrix}$ and $\tilde{I}$ as defined in the first condition. The third inequality condition is $Q_0(x) = \tilde{x}^T(\tilde{P}_r - \tilde{P}_q)\tilde{x} \geq 0$. The first and second conditions of the stability theorem should be satisfied in regions Ω_q^x and Ω_q^{x,m_i} respectively. These conditions can be substituted by the unconfined condition of $Q_0(x)$. The third condition is satisfied on the hypersurface Λ^x_{qr} given by $Q_k(x) = 0,\ k \in I_s$ [11].

LMI problem: If there exist $\tilde{P}_q,\ q \in I_\Delta$, constants $\alpha > 0$, $\mu^q_k \geq 0$, $\nu^{qij}_k \geq 0$, η^{qr}_k and a solution to $min\ \beta$ subject to the three conditions:

- $\alpha\tilde{I} + \sum_{k=1}^{s_q} \mu^q_k \begin{bmatrix} Z^q_k & c^q_k \\ (c^q_k)^T & d^q_k \end{bmatrix} \leq \tilde{P}_q$

  $\tilde{P}_q \leq \beta\tilde{I} + \sum_{k=1}^{s_q} \mu^q_k \begin{bmatrix} Z^q_k & c^q_k \\ (c^q_k)^T & d^q_k \end{bmatrix},\ q \in I_\Delta$
- $(q,i,j) \in I_\Omega, (\tilde{A}^j)^T\tilde{P}_q + \tilde{P}_q\tilde{A}^j$

  $+\sum_{k=1}^{s_{qij}} \nu^{qij}_k \begin{bmatrix} Z^q_k & c^q_k \\ (c^q_k)^T & d^q_k \end{bmatrix} \leq -\tilde{I},\ q \in I_\Delta$
- $\tilde{P}_r \leq \tilde{P}_q - \sum_{k=1}^{s_{qr}} \eta^{qr}_k \begin{bmatrix} Z^{qr}_k & c^{qr}_k \\ (c^{qr}_k)^T & d^{qr}_k \end{bmatrix},\ (q,r) \in I_\Lambda$

Then the fixed point 0 is exponentially stable in the sense of Lyapunov [1].

Remark 2: Without loss of generality, it is assumed that the origin is a fixed point of the fuzzy system (9). For the buck converter, the fixed point mentioned above is the fixed point of the limit cycle with a stroboscopic map [2] □

In order to verify the analysis presented above, the fuzzy state-space $\mathcal{F}$ is first partitioned into $\Delta = 2$ detached regions ($I_\Delta = \{1, 2\}$):

$$\begin{array}{l} \Omega_1 = \{(x,m) \in \mathcal{F}|\ x \in \Re^n, m = m_1\} \\ \Omega_2 = \{(x,m) \in \mathcal{F}|\ x \in \Re^n, m = m_2\} \end{array} \tag{15}$$

Solving the LMI problem for the value of supply voltage V_{in} = 24V results in a solution:

$$\tilde{\mathbf{P}}_\mathbf{1} = \begin{bmatrix} 2.2526 & -12.8865 & -39.1678 \\ -12.8865 & 0.0026 & -103.3283 \\ -39.1678 & -103.3283 & 0.0004 \end{bmatrix} \tag{16}$$

$$\tilde{\mathbf{P}}_\mathbf{2} = \begin{bmatrix} 2.2526 & 12.8865 & -39.1678 \\ 12.8865 & 387.3544 & 103.3283 \\ -39.1678 & 103.3283 & 2235.9155 \end{bmatrix} \tag{17}$$

with the optimal value of $\beta = 2.4962$. Finding the feasible solution to the LMI problem clearly means that the system is exponentially stable as readily perceived from Fig. 6 showing the stable period-1 response corresponding to this operating point. By changing the supply voltage to V_{in} = 25V, no feasible solution can be found for the LMI problem, which obviously implies instability of the new operating point. The stability analysis via Fillipov method reconfirms the unstable period-1 orbit coexisting with a period-2 orbit for V_{in} = 25V [2] and affirms the prowess of the new method for fast-scale stability analysis of the converter.

It is worth noting that by single partitioning, no feasible solution can be found for the LMI problem while the converter response is actually in a stable period-1 orbit. This indicates the essential role of flexible region partitioning of the fuzzy state-space in the case of nonsmooth systems like the DC-DC buck converter.

V. Conclusion

A novel Takagi-Sugeno fuzzy modeling approach has been synthesized to represent the discrete switching events of nonsmooth dynamical systems such as the DC-DC buck converter. The new modeling approach has been shown to accurately predict all nonlinear phenomena induced by these switching events including period-doubling bifurcation, crises and chaos.

Based on the resulting TS fuzzy model, a rigorous mathematical stability analysis is presented to give an insight into all aforementioned fast-scale instabilities affiliated with the buck converter. Stability conditions are formulated as an LMI problem using piecewise Lyapunov functions to deal with the discontinuous dynamics of the new TS fuzzy representation and to relax the conservative formulation leading to an infeasible solution. The TS stability analysis successfully locates the onset of period-doubling bifurcation, which till now have only been possible using complicated discrete nonlinear modeling methods.

The proposed TS fuzzy modeling and stability analysis approach can also be applied to other types of power electronic converters. Most importantly, the whole modeling and stability analysis approach can be employed as a framework to devise a new type of fuzzy model-based control strategy to suppress unwanted instabilities in the buck converter and other nonsmooth dynamical systems.

[1]The proof of this theorem is out of the scope of this paper and it will present in later publications.

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