Electric drives estimation and denoising schemes based on wavelet transforms

Damian Giaouris
School of Electrical, Electronic and Computer Engineering
University of Newcastle upon Tyne
Newcastle upon Tyne, NE1-7RU, UK
Damian.Giaouris@ncl.ac.uk

John W Finch
School of Electrical, Electronic and Computer Engineering
University of Newcastle upon Tyne
Newcastle upon Tyne, NE1-7RU, UK
j.w.finch@ncl.ac.uk

Abstract – **This paper presents the application of the wavelet transform in electric drives schemes. Wavelets can significantly improve the behavior of a Kalman filter used for speed estimation. Furthermore an adaptive multiresolution wavelet analysis method is proposed for current denoising especially targeted for electric drives applications that have a V/f structure. This adaptive method compensates for the delay problem that every wavelet based denoising scheme imposes.**

I. INTRODUCTION

In electric drives applications the use of signal transformations is very common. Among others, the Laplace and the z-transforms are used for designing controllers while the Fourier transform is primarily used for designing filters that will remove unwanted noise components from useful measurements. A relatively new technique in this is the wavelet transform with most applications in fault detection and condition monitoring [1-3]. It is found that the use of wavelets provides additional tools to monitor electric drives applications, with considerable advantages comparing to convectional detection techniques, which for example measure the $\Delta i/\Delta t$ of the phase currents. It is well known, in the digital signal processing community, that wavelets revolutionized data compression applications by offering compression rates which other methods could not achieve [4]. Another application similar to compression is wavelet shrinkage, which allows the denoising of useful signals without focusing on specific frequency coefficients [5]. As was reported in [5] simple drives denoising schemes (based on FIR filters) produced similar results to those of wavelets. Hence wavelets should not be applied for these applications since they are more complex than simple FIR filters. Wavelets can prove [5] to be more successful in speed sensorless applications, [6]. In these cases the useful information (rotor speed) is modulated by an injected high frequency signal and demodulating methods that are based on wavelets produce better results.

Another equally popular method to estimate the rotor speed is to use a Kalman filter [7] which has good estimation characteristics but its correct use is heavily correlated with good estimate of the noise properties of the current sensors (as also be proved here). An incorrect value of the covariance of the measurement noise will significantly downgrade the overall performance of the drive.

This paper addresses these two very important issues in the control of electrical machines, correct denoising and proper estimation of the noise components in the current measurements. The paper is organized as follows, initially the wavelet transform and the Kalman filter are briefly described in a way suited to drives applications and then the problems of the Kalman filter are investigated. A solution based on wavelets is proposed which greatly improves the behavior of the estimation technique. The second part of this paper presents a significance improvement in denoising schemes that are applied to electric drives and is based on wavelets. This is achieved by using an adaptive technique which changes the level of decomposition depending on the rotor speed.

II. WAVELET TRANSFORM

According to Strang [8] a transform is nothing more than another way to view a signal (or a vector). A transform, breaks a signal, f , into numerous fundamental components whose processing may help to reveal or remove specific characteristics of the signal. This breaking is accomplished by finding the correlation of the signal under investigation and the fundamental components $x_i, i = 0,1...$ The correlation of continuous time signals is expressed by an integral: $c_i = \int_{-\infty}^{+\infty} f \cdot x\, dt$. This is similar to the inner product of two vectors if it is assumed that the values of the two signals are "stored" in a vector with infinite entries. It is well known from vector theory that when the inner product of two vectors is zero then the vectors are orthogonal. By extending the same concept to signals, if the correlation of two signals is zero then they are orthogonal:

$$\int_{-\infty}^{+\infty} f \cdot g\, dt = 0 \Rightarrow f \perp g \tag{1}$$

In the classical Fourier transform the fundamental components are complex exponentials, $e^{-j\omega t}$ that extend from $-\infty$ to $+\infty$ which can be proved to be mutually perpendicular (orthogonal) to each other. These infinite complex exponentials form a basis where all signals can be decomposed and hence studied. The Fourier transform can be written as:

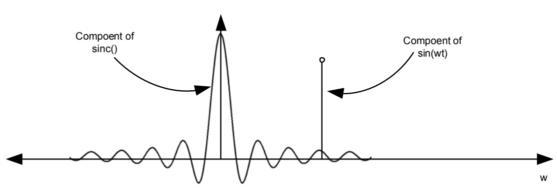
$$F(\omega)=\int_{-\infty}^{+\infty} f\, e^{-j\omega t}\, dt \qquad (2)$$

The correlation with one of these exponentials will produce a value and this will be the frequency component of the signal at that frequency. By using all the exponentials and their correlations with the signal *f*, the frequency spectrum can be derived. For example if the signal under consideration is a pure sine wave then the frequency spectrum will be a Dirac pulse at the frequency of the sine wave, Fig. 1a.

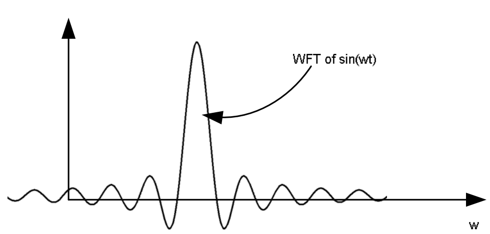
Unfortunately in reality and more importantly in real time applications it is impossible to study signals that extend from $-\infty$ to $+\infty$. Also there are applications (like fault detection and high frequency injection) where it is desirable to see when specific components appear in the signal. Hence the signal has to be truncated; i.e. only a small portion of the signal can be studied at each time. This effectively means that the fundamental component is multiplied by a window function $w(t)$ (often a rectangular window) which is continuously shifted to cover the signal under study; this is the windowed Fourier transform:

$$WF(\omega,\tau)=\int_{-\infty}^{+\infty} f(t)\,w(t-\tau)\,e^{-j\omega t}\, dt \qquad (3)$$

The effect of using windows is to smear and leak the frequency components of the signal. For example in the previous case with the sine wave the frequency spectrum will not be a pure Dirac pulse but it will be the convolution of the Dirac pulse with the $\operatorname{sinc}(\cdot)$ function (Fourier transform of the rectangular window), Fig. 1b. Hence if there are two frequency components that are close they may be shadowed by the main lobe of the $\operatorname{sinc}(\cdot)$ function and hence falsely believe that there is only one frequency component. To reduce the width of the main lobe the length of the time window must be extended but then it is possible that the two sine waves may not exist at the same time; hence the frequency spectrum will give an inaccurate representation of the signal. The time information is not lost in the frequency spectrum but it is well hidden under a series of subharmonics.



a:



b:

Fig. 1 Fourier transform of pure sine wave, of rectangular window, and windowed Fourier transform of pure sine wave.

The goal in most applications is to be able to identify when an event takes place (time resolution) and what is its frequency (frequency resolution). From the previous analysis it is obvious that it is not possible to have perfect frequency and time at the same time. To achieve this, the transformation must include windows whose size can vary and this is not possible with the windowed Fourier transform. To solve this problem the wavelet transform is forcing the window to have a logarithmic coverage of the frequency spectrum by imposing that the frequency width of the window is $\Delta f / f = \text{constant}$. This is achieved by using a version of the windowed Fourier transform over and over again for various lengths of the window. Furthermore the fundamental components of the decomposition are not truncated and shifted exponentials but other asymmetric and irregular small waves, i.e. wavelets. The transformation now will include not only the shifts on the wavelet but also their scale:

$$c(a,b)=\int x(t)\overline{\psi(at+b)}dt=|a|^{-1/2}\int x(t)\overline{\psi\left(\frac{t-b}{a}\right)}dt \qquad (4)$$

The asymmetric function ψ is called the mother wavelet and it is shifted, scaled and compared (correlation) with the original signal. Hence the wavelets achieve a logarithmic coverage of the time-frequency plane, Fig. 2, and they have arbitrary good frequency resolution for low frequency components and arbitrary good time resolution for high frequency components.

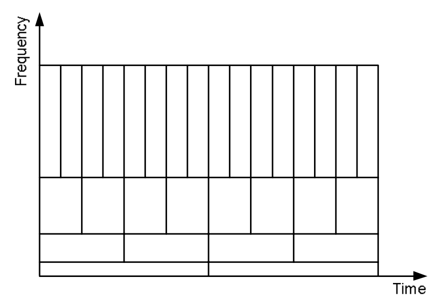


Fig. 2 Time frequency plane for the wavelet transform

An obvious consequence of this continuous scaling and shifting is that the wavelet transform involves “two times” infinite number of coefficients and hence it is unappealing to on-line applications, i.e. it does not constitute an orthogonal transformation. In [9] Mallat proposed the fast wavelet transform which uses only a finite number of scales and shifts through successive high and low pass filtering. Each scale is represented by a dyadic filter bank. The outputs of the high pass filter are called details and the outputs of the low pass filter are called approximations. Then the approximations from the current scale are filtered again by another set of 2 filters. The successive filtering of the approximations at each scale accomplishes the fast wavelet transform, which is an orthogonal transformation. The synthesis or the inverse wavelet transform is accomplished by using a similar process. Before the synthesis bank the approximations and details can be process for example to remove unwanted noisy components.

Since the wavelet transform is a linear transformation then the details and approximations of two different signals (a current measurement and the sensor noise) can be added together to produce the details and approximations that the sum of the two signals would produce (output of sensor). Also it can be assumed that a pure noise signal will have coefficients with small absolute value. Hence before the synthesis bank a threshold can be applied to the coefficients and if they are below a specific value they will be disregarded. This is an irreversible operation and will also influence the useful signal but since that has more coefficients with high values the final result will be a slightly distorted, almost noised free, signal.

III. KALMAN FILTER

The (digital) Kalman Filter (KF) is a stochastic posteriori estimator whose estimating gain (**K**) is continuously adapted to minimize the covariance of the error (**P**) between the real and the estimated state vector. The system is described in the normal discrete state space form with the addition of two extra vectors that contain noisy signals, Figs. 3 & 4:

$$\begin{aligned} \mathbf{X}(k+1) &= \mathbf{\Phi}(k)\mathbf{X}(k) + \mathbf{\Gamma}(k)\mathbf{U}(k) + \mathbf{W}(k) \\ \mathbf{Y}(k) &= \mathbf{H}(k)\mathbf{X}(k) + \mathbf{V}(k) \end{aligned} \qquad (4)$$

The vector **W** represents the effect of the unmodelled uncertainties and **V** represents the noise added at the (current) measurements by the sensors.

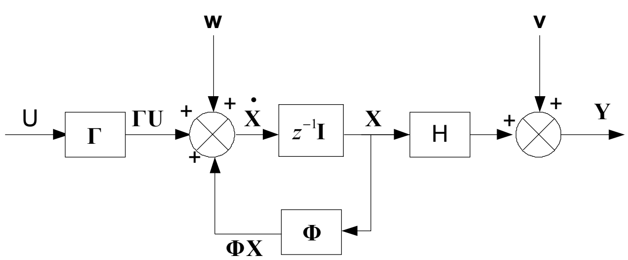


Fig. 3 Stochastic state space model

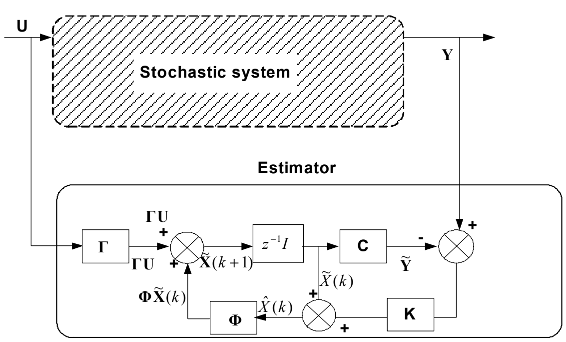


Fig. 4 Posteriori estimator

The basic steps in designing a KF are:

- $\mathbf{P}(k+1/k) = \mathbf{\Phi}(k)\mathbf{P}(k/k)\mathbf{\Phi}^T(k) + \mathbf{Q}(k)$
- $\hat{\mathbf{X}}(k+1/k) = \mathbf{\Phi}(k)\hat{\mathbf{X}}(k/k) + \mathbf{\Gamma}(k)\mathbf{U}(k)$
- $\mathbf{K}(k+1) = \mathbf{P}(k+1/k)\mathbf{H}^T(k+1)\times \left[\mathbf{H}(k+1)\mathbf{P}(k+1/k)\mathbf{H}^T(k+1) + \mathbf{R}(k+1)\right]^{-1}$
- $\hat{\mathbf{X}}(k+1) = (1-\mathbf{K}(k+1)\mathbf{H}(k+1))\hat{\mathbf{X}}(k+1/k) + \mathbf{K}(k+1)\mathbf{Y}(k+1)$
- $\mathbf{P}(k+1/k+1) = [\mathbf{I}-\mathbf{K}(k+1)\mathbf{H}(k+1)]\mathbf{P}(k+1/k)$

where **R** and **Q** are the covariances of **V** and **W** respectively.

Hence initially the error covariance is calculated and then the new state vector is predicted. Based on these the optimum KF gain is calculated and then through that the best state vector and error covariance are updated.

The induction machine is modelled with dq quantities (in the stator reference frame) and the signal **W** represents the changes in the rotor and stator inductances and resistances. This happens due to temperature increases or magnetic nonlinearities (Eddy currents, saturation…). The signal **V** again represents the noise that is added to the stator current measurements; the rotor currents cannot be measured and hence have to be estimated. Therefore the KF will have to smooth the two stator currents and based on the stochastic model, the input signal and the noisy stator measurements to estimate the two rotor currents expressed at the stator reference frame. The previous 5 equations show the estimated state vector heavily depends on the values of **R** and **Q**. A large value of **R** (or small **Q**) means that the model is more accurate and hence the filter is focused on predicting the state vector by the first 2 equations. If **R** is decreased (or **Q** is increased) then the KF assumes that the noise that is added to the measurements has a smaller effect than the model uncertainty and will focus more on correcting the predicted values of the state vector, i.e. will increase the gain **K**.

A. Practical problems by using a Kalman filter

As indicated, the KF can estimate all four dq currents (stator and rotor) but this imposes a heavy computational effort on the overall drive. Also, as for all drives estimating schemes, eventually it cannot cope with low speed operation since the induction machine then behaves like a resistance. Another practical problem is the correct choice of the measurement and process noise covariance. To test this sensitivity of the KF a simple V/f scheme, [10], was used where it was assumed that the applied torque on the rotor shaft can be measured. Hence the KF will estimate the four dq currents and through that the electromechanical toque. Using the value of the load torque and assuming negligible friction it is thus possible to estimate the rotor speed. This is an idealised case which is found only in simplified studies, but this paper is focused on minimising the sensitivity of the KF rather than introducing a new scalar control or estimation scheme.

B. Kalman filter and wavelets

In Fig. 5i the error between the estimated and the real speed response of such a scheme can be seen. This uses a 2 pole induction machine with a constant load of 10Nm, a desired speed of 40Hz and an acceleration of 50rad/s^2. The sampling frequency was 10kHz, the initial error covariance 10**I**, the process noise 0.0001**I** and the measured noise r**I**. For fuller motor details see table I.

If the KF had the wrong information (or simply the noise characteristics change) the estimating error would diverge to infinity with catastrophic results for the drive. The result of this is shown in Fig. 5iii where it is clear that the overall behaviour of the KF is downgraded. A possible solution to that problem is to use wavelets and their denoising properties but in reverse. In this case all the coefficients with high values are disregarded and the synthesis bank works only with the noise components. This will effectively remove any dc component of the signal and the variance of the noise can be calculated on-line by using the formula:

$$Var = \frac{1}{N}\sum_{k=1}^{N}(h(k))^2 \tag{5}$$

where h is signal after the synthesis bank.

TABLE I

RATED VALUES FOR DELTA-CONNECTED SQUIRREL CAGE IM

Quantity	Value
Power	7.5 [kW]
Pole Pair Number, P	1
Rated Frequency	50 [Hz]
Rated Voltage	415 [Volts]
Rated Torque	25 [Nm]
Rated Speed	2860 [rpm] or 300 [rad/sec]
Rated Current	13.5 [A]
Stator Resistance, R_s	2.19 [Ω]
Rotor Resistance, R_r	1.04 [Ω]
Stator Leakage Inductance, l_s	17.59 [mH]
Rotor Leakage Inductance, l_r	17.59 [mH]
Mutual Inductance, L_m	0.55 [H]
Estimated Inertia, J	0.221 [kg m^2]

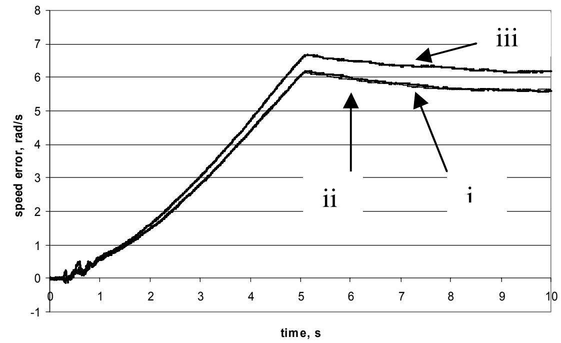


Fig. 5 Error between actual and estimated speed
i r=0.1 and the KF has the correct information
ii r=0.1 and the KF has correct information
iii r=0.1 and the KF assumes r=0.01

A similar result can be achieved by using the details directly and hence avoiding the extra computational effort that is required for the synthesis bank. To insure the system performance in the presence of non-white noise signal (which is commonly the case in real applications) the details of the first three levels (scales) were used and their variance was calculated on-line. Fig. 6 shows the mean value of the on-line calculated variance of the details for first three levels. The KF was continuously updated with that value and the improvement is clear in Fig. 5ii.

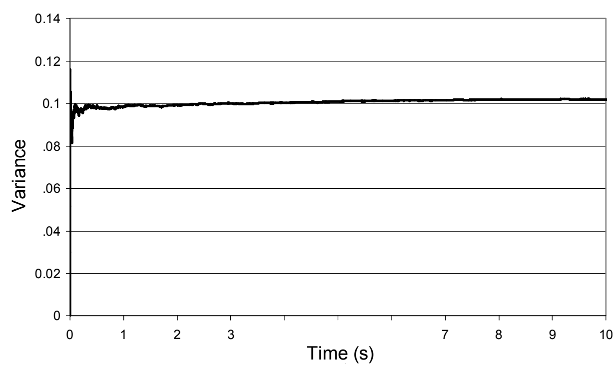


Fig. 6 Mean value of the variances of the 3 level details

IV. ADAPTIVE MULTILEVEL WAVELET ANALYSIS

It was shown experimentally in [5] that when the wavelets are used to denoise the stator currents in a V/f scheme the results are not encouraging and simple FIR schemes produced better results. This is due to the inherent delay that is caused by the alignment between the analysis and synthesis banks. The main problem is that there is no coherent methodology of how many levels of decomposition should be used and which wavelet is more appropriate. A usual wavelet choice is from the Daubechies family (DB1-DB43) which are compactly supported and has a high number of vanishing moments for a given support width. In previous work [5] it was found that the best wavelet was the DB2 or DB3 wavelet, but the main problem comes from the level of the analysis. More layers give better performance but bigger delay. This delay is $\left(2^{\text{number of filters}}-1\right)\times \text{Filter Order}$ for every level. This produced a peculiar relationship between the wavelets, level of decomposition and integral of squared error between the noise free and actual signal [5]. In IM drives the problem is complicated as the denoising process may be required on the stator currents. These do not have the simple relationship that the voltage must follow: small amplitude at low frequency and large amplitude when frequency is high (the voltage over Hertz ratio has to remain constant). In the low frequency region the delay is not very important since it can cause a small phase shift, but in this region the level noise that is present can greatly influence the overall behaviour by affecting the peak values produced. In the high frequency region the peak change is minor but the phase shift can now be more than a full cycle and hence to produce instability.

Thus a new scheme is needed. This scheme will adapt the level of the decomposition depending on the desired frequency of the signal. For example, if the frequency of the noisy signal is from 0 to 15 Hz then the 5th level will be used, if the frequency is from 15 to 30 Hz then the 4th, from 30 to 40 Hz the 3rd, from 40 to 50 Hz the 2nd, and finally from 50 and above the first level. The only problem that arises with this pseudo-adaptive method is the "optimal" choice of these breaking points. This is similar to the problem of gain scheduling in nonlinear control systems. Only "knowledge based methods" (Fuzzy Logic, Neuro-Fuzzy) can be used, or trial and error techniques. Here the changing points were found by trial and error methods. This method is called Adaptive Multilevel Wavelet Analysis (AMWA).

To test the AMWA denoising scheme a simple ramp acceleration of a V/f scheme was used, there is no low frequency voltage boost and the load torque is also zero. The motor parameters are shown in Table I. The acceleration was set to 20 rad/s and the V/f ratio is equal to 415/50= 8.3 V/Hz. The wavelet was the DB2 and the sampling period was set to 1ms. The sensor distortion used was a simple white noise signal with zero mean and variance of 1, Fig. 7. The AMWA breaking points were chosen to be at 10Hz, 20Hz, 30Hz, 40Hz, and 50Hz. The resulted denoising current is shown in Fig. 8 and the Integral of Time Squared Error (ITSE) is shown in Fig. 9. To compare with the classical wavelet denoising the 5th level decomposition was used alone and its ITSE is shown in Fig. 10.

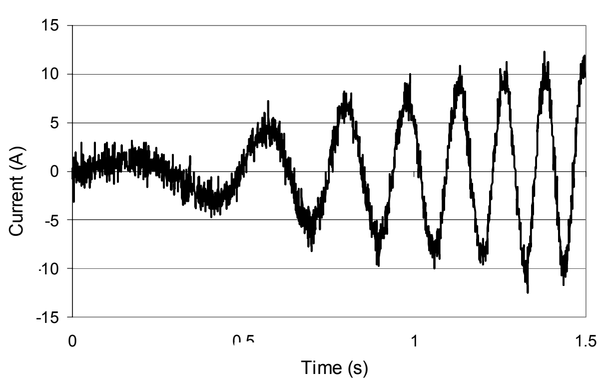


Fig. 7 Noisy stator current signal

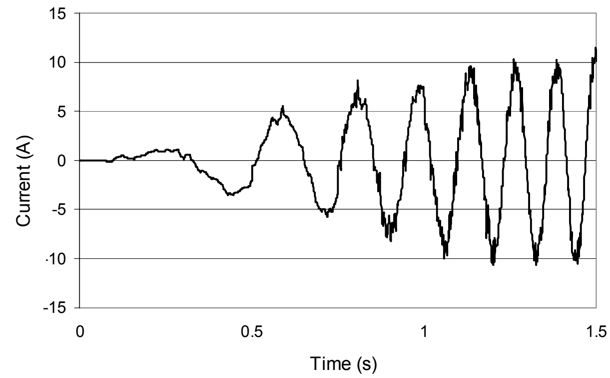


Fig. 8 Denoised stator current with the adaptive scheme

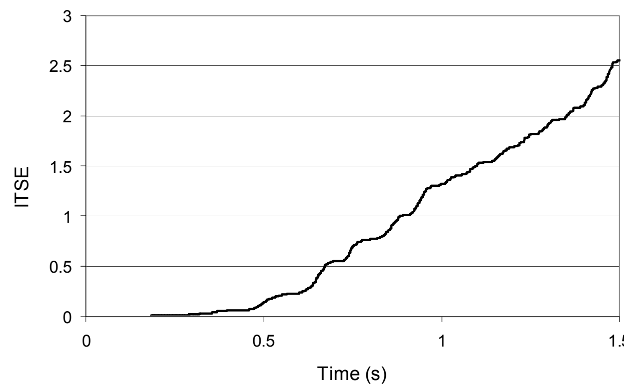


Fig. 9 ITSE for adaptive scheme

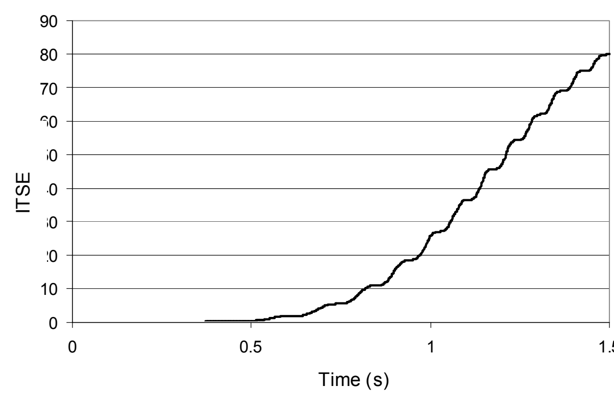


Fig. 10 ITSE for normal wavelet scheme

It is clear from the previous figures that AMWA gave much better results. Further tuning could improve the overall performance.

V. CONCLUSIONS

The use of wavelets in various applications of electric drives was presented and shown to provide satisfactory results. More specifically the wavelet transform can estimate the measurement noise covariance and through that update the KF, considerably improving the performance of that estimating method. An adaptive multiresolution scheme was also proposed based on wavelets which helps with the main problem of applying wavelet denoising methods in electric drives. The new scheme gave significantly better results than a normal wavelet scheme which makes application of wavelets in electric drives more attractive.

VI. ACKNOWLEDGEMENT

The authors wish to acknowledge the support of Control Techniques for this and related work.

VII. REFERENCES

[1] F. Briz, M.W. Degner, A. Diez and R.D. Lorenz, "Sensoless position control of permanent magnet synchronous machines without limitation at zero speed", *IEEE Trans Ind Apps*, vol. 38, no. 3, May/June 2003, pp. 670-678.

[2] L. Eren and M.J. Devaney, "Bearing damage detection via wavelet packet decomposition of the stator current", *IEEE Trans Instr and Meas*, vol. 53, no. 2, April 2004, pp. 431 – 436.

[3] L. Jianguo Liu, P. Pillay P and H. Douglas, "Wavelet modeling of motor drives applied to the calculation of motor terminal overvoltages", *IEEE Trans Ind Electronics*, vol. 51, no. 1, Feb. 2004, pp. 61 – 66.

[4] L.D. Donoho, "De-noising by soft-thresholding", *IEEE Trans Inf Th*, vol. 41, no. 3, May 1995, pp. 613 – 627.

[5] D. Giaouris and J.W. Finch: "Denoising using wavelets on electric drives applications", *Electric Power Systems Research*, submitted.

[6] M. Linke, R. Kennel and J. Holtz: "Static and dynamic behavior of saturation-induced saliencies and their effect on carrier-signal-based Sensorless AC drives", *IECON 2002*, vol. 1, 5-6 Nov 2002, pp. 674 – 679.

[7] D.J. Atkinson, P.P. Acarnley and J.W. Finch, "Observers for induction motor state and parameter estimation", *IEEE Trans Ind Apps*, vol. 27, no. 6, Nov/Dec 1991, pp. 1119 – 1127.

[8] G. Strang and T. Nguyen, *Wavelets and Filter Banks*, Wellesley-Cambridge: 1996.

[9] S.G. Mallat: "A theory for multiresolution signal decomposition: The wavelet representation", *IEEE Trans Patt Anal & Mach Intell*, vol. 11, no. 7, July 1989, pp 674 – 693.

[10] C.J. Francis and Z. De La Parra: "Stator resistance voltage-drop compensation for open loop AC Drives", *IEE Proc Electric Power Appl*, vol. 144, no. 1, January 1997, pp 21 – 26.