On-line Estimation of Magnetizing Inductance and Rotor Resistance in Extended Kalman-Filter for Induction Machines

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***Abstract*—This paper proposes an on-time estimation of induction machine parameters based on the extended Kalman-filter. So far, the real time performance of Extended Kalman-filter algorithms has not been validated in the variation of motor parameters. Furthermore, the conventional parameter estimations in extended Kalman-filter has not been developed accurately based on the correct non-linear state space model. This paper proposes a new state space model of induction machine included motor parameters with less dependency to other model variables. This leads to achieve more accurate estimation results even in situation where a sudden variation of motor parameters happens. This paper describes the proposed method analytically. Simulation results for an induction machine are presented and discussed.**

***Index Terms*— Extended Kalman filter, induction motors parameters, on-line estimation, induction motor derives**

I. INTRODUCTION

It is essential to know the accurate value of rotor resistance (r_r) and magnetizing inductance (L_m) in induction motor (IM) drives performing based on the Field oriented control (FOC). Using this control method leads to improve the dynamic performance of IM with having the control on the flux in the air gap [1]. As it is obvious, the aim goal behind using FOC is aligning the d-axis of synchronous frame on the rotor flux vector. This purpose cannot be achieved without accurate slip calculation in indirect vector control method [2]. The accuracy of slip calculation depends on the actual magnetizing inductance and rotor resistance. These parameters can be estimated by applying the Kalman filter algorithm, which is considering the effect of the measured and process noises, for state estimation of AC motors.

The deviation of motor parameters is the result of the changing the temperature inside the motor, variation of fundamental frequency and saturation of inductances [3, 4]. Adapting the motor parameters based on the machine data sheet [5], high-frequency signal injection and on-line or off-line estimation or determination are the main approaches of parameters detection for electrical machines applications [3]. In online estimation methods, the parameters of the electrical machine are adapting when the drive is operating.

In [6, 7], the r_r or the rotor time constant (τ_r) are estimated by applying the extended Kalman filter (EKF) method. It considers these parameters as the additional states in the state space model of IM. As the variation of magnetizing inductance is not considered in these estimation methods, so the r_r estimation effected by inaccurate inductance considered in EKF. In [5], the off-line estimation of L_m is added to algorithm which already estimates r_r. The on-line estimation method needs to use the microcontroller with high memory capacity. This is required as the reason of heavy computation of Kalman filter in discrete form [8]. In [9], the constraint for estimated parameters are considered in implementing of EKF. In this method, the quadratic programming technique adjusts the EKF loops as the considered constraints are not be satisfied. However, the performance of proposed method has not been validated as the parameters of machine vary. In [10], the unscented Kalman filter (UKF) is utilized to estimate the states and parameter of IM. In this estimation method, the derivation section for linearization of non-linearity of state space model in conventional EKF is not implemented. So, the continues nonlinear dynamic equation are used without discretization and linearization in Jacobian matrix. The single point linearization is achieved in conventional EKF based on Jacobian matrix however, the minimal selected samples of a non-linear system in propagating Guassian random variables (GRV) is provided by UKF. So, the mean estimate and covariance are defined more accurately in sigma sample points method (which analyses GR variables close to second order of Taylor series) in UKF comparison with EKF. As declare in [1], the computation time for EKF and UKF is very close and accuracy of estimated has not been improved considerably. In [11], the Expectation Maximization algorithm is develop to improve the selection of initial values for covariance matrixes including of process and measurement matrix. This method adds more computation process to the EKF which already suffer from long computation process. In [7], the measurement and process noise covariance matrixes has been calculated based on proposed filtering. In this estimation method, the average of three continues points of the captured stator voltage and current data is considered as the main signal. Then, the deviation of the data from main signal is noticed as noise. The low-pass filter is not the suitable choice for removing the noise from measured data including the white Gaussian noise. As it is obvious, this type of noise includes of all frequency and it is not only high frequency domain. The sample rate, resolution of the sensors and quantization error effect on the accuracy of parameter identification in off-line or on-line parameter determination or estimation method [12].

The on-line estimation of r_r and L_m is presented in [13]. However, the defined variables ($\frac{\frac{Lm^2}{Lr}r_r}{Lr}, \frac{1}{(\frac{Lm^2}{Lr})}$) as the additional states in state space model are not defined correctly. Based on the defined variables in this paper, some parts of index in Jacobian matrix considered constant which is not the true scenario. This causes that the variation of estimated parameters effect on those variables which their value are considered constant. This leads to the updated Jacobian matrix in each interrupt service routine will not be correct. So, the state prediction and correction algorithm in EKF is not updating with the correct Jacobian matrix. Therefore, in this paper the constant variables such as σL_s, ($\frac{-1+\sigma}{\sigma L_m}$) and $\frac{L_m}{\sigma L_s L_r}$ are considered in Jacobian matrix. These variables are approximately constant as the L_m and r_r are varying. The simulation results have been captured to validate the performance of on-line estimation method.

This paper organized as follow. Section II describes the analytical model of IM, KF and proposed EKF. Finally, section III and IV present the simulation results and work conclusion.

II. MATHEMATICAL MODEL DEVELOPMENT

A. Induction Machine Model

The state space model of induction machine in stationary frame can be derived as follow:

$$\frac{d}{dt}\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} =$$

$$\begin{bmatrix} a & 0 & c & b \\ 0 & a & -b & c \\ \frac{L_m}{\tau_r} & 0 & \frac{-1}{\tau_r} & -w_r \\ 0 & \frac{L_m}{\tau_r} & w_r & \frac{-1}{\tau_r} \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ds}^s \\ V_{ds}^s \end{bmatrix} \quad (1)$$

$$a = \frac{-r_s}{\sigma L_s} - \frac{1-\sigma}{\sigma}, b = \frac{L_m}{\sigma L_s L_r} w_r, c = \frac{L_m}{\sigma L_s L_r \tau_r}$$

In (1), the d-q axis of stator currents, rotor flux and input voltages are described by $i_{dq}^s, \lambda_{dqr}^s$ and V_{dq}^srespectively.

For practical case, the estimated stator current are considered as output signal ($y(t)$) which can be compared with measured stator signal (2).

$$y(t) = Hx(t) \to y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{ds}^s \\ i_{qs}^s \\ \lambda_{dr}^s \\ \lambda_{qr}^s \end{bmatrix} \quad (2)$$

By considering the discrete sampling interval (t_s), the linear discrete time varying of the machine model, described in (1), can be defined as follow:

$$x(k+1) = F(k)x(k) + G(k)u(k) \quad (3)$$

$$y(k) = Hx(k) \quad (4)$$

Where:

$$F(k) = t_s \begin{bmatrix} \frac{-r_s}{\sigma L_s} - \frac{1-\sigma}{\sigma} & 0 & \frac{L_m}{\sigma L_s L_r \tau_r} & \frac{L_m}{\sigma L_s L_r} w_r \\ 0 & \frac{-r_s}{\sigma L_s} - \frac{1-\sigma}{\sigma} & \frac{L_m}{\sigma L_s L_r} w_r & \frac{L_m}{\sigma L_s L_r \tau_r} \\ \frac{L_m}{\tau_r} & 0 & \frac{-1}{\tau_r} & -w_r \\ 0 & \frac{L_m}{\tau_r} & w_r & \frac{-1}{\tau_r} \end{bmatrix} + [I]_{4\times4}$$

$$\sigma = 1 - \frac{L_m^2}{L_r L_s}$$

To make the model close to the practical nature, the disturbances needs to be model. So, the stochastic state space model which contained of the process noise ($w(k)$) and measurement noise ($v(k)$) can be defined as follow:

$$E\left\{ w(k)\ w(j)^T \right\} = Q\delta_{kj} \quad Q \geq 0 \quad (5)$$

$$E\left\{ v(k)\ v(j)^T \right\} = R\delta_{kj} \quad R \geq 0 \quad (6)$$

R, Q and δ_{kj} are the constant scaler variance and Kronecker Delta respectively. To solve the stochastic state space model, the Gaussian distribution should be applied. This solution needs the calculation of the state error covariance matrix ($P(k)$) and the output error covariance matrix ($S(k)$). Calculation of these two matrix are the initial steps of solving the Gaussian distribution. The distribution mean vectors of state and output ($\hat{x}, \hat{y}$) are two answers of the Gaussian distribution solution. The distribution of $\hat{x}$ *and* $\hat{y}$ are based on the probability of the below distribution at defined discrete sequence k:

$$N(\hat{x}(k), P(k)) \quad (7)$$

$$N(\hat{y}(k), S(k)) \quad (8)$$

B. Kalman-Filter

The distribution mean vector of state and output $(\hat{x}, \hat{y})$ are the estimated variables in Kalman-filter. The state prediction of Kalman-filter algorithm can be defined as follow:

$$\hat{x}(k+1/k) = F(k)\hat{x}(k/k) + G(k)u(k) \tag{9}$$

$$P(k+1/k) = F(k)P(k/k)F(k)^T + Q \tag{10}$$

and the state correction equations are:

$$K(k+1) = P(k+1/k)H(k+1)^T\left[H(k)P(k+1/k)H(k)^T + R\right]^{-1} \tag{11}$$

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + K(k+1)\left[y(k+1) - H(k+1)\hat{x}(k+1/k)\right] \tag{12}$$

$$P(k+1/k+1) = P(k+1/k) - K(k+1)H(k+1)P(k+1/k) \tag{13}$$

It should be noticed that $\hat{x}(k/k)$ and $\hat{x}(k+1/k)$ are the estimation of the state in discrete sequence of k which is based on data available up to and including k and $k+1$ sequence respectively. The Kalman gain is defined by $K(k+1)$.

C. Extended Kalman-filter

The parameters of induction machine can be considered as augmented state in state vector to be estimated by extended Kalman-filter (EKF). In proposed EKF, the rotor resistance and magnetizing inductance are considered as the new states which behave such as a time-varying parameters. It should be noticed that the variation of the parameters is much slower than the stator and rotor flux states in induction machine. By considering the parameters as new states, the state space model becomes none-linear as result of states multiplication. To solve the none-linear state estimation problem, the Extended Kalman-filter as the nonlinear state estimator is used. In this algorithm, the time varying parameters, which considered as additional state, is defined as follow:

$$\theta(k+1) = \theta(k) + n(k) \tag{14}$$

In (14), the unknown random disturbance is introduced by $n(k)$. Then, the extended state space model can be derived as follow:

$$x(k+1) = F(\theta(k),k)x(k) + G(\theta(k),k)u(k) + w(k) \tag{15}$$

and by considering the new state vector as:

$$z(k) = \begin{bmatrix} x(k) \\ \theta(\mathrm{k}) \end{bmatrix} \tag{16}$$

the augmented state model is presented such as:

$$z(k+1) = f(z(k), u(k)) + w'(k) \tag{17}$$

$$f(z(k),u(k)) = \begin{bmatrix} F(\theta(k),k)x(k) + G(\theta(k),k)u(k) \\ \theta(\mathrm{k}) \end{bmatrix}$$

$$w'(k) = \begin{bmatrix} w(k) \\ \mathrm{n(k)} \end{bmatrix}$$

The output vector, which is not depend on $\theta(k)$ can be explained as follow:

$$y(k) = Hz(k) + v(k) \tag{18}$$

It should be noticed that the mean value of the random vector (white Gaussian noise) are assumed to be zero. This assumption is based on intending of implementing the KF algorithm. The covariance of measurement noise and proses noise in EKF is given as follow:

$$E\left\{v(k)v(j)^T\right\} = R\delta_{kj} \tag{19}$$

$$E\left\{\begin{bmatrix} w(k) \\ \mathrm{n(k)} \end{bmatrix}\begin{bmatrix} w(j) \\ \mathrm{n(j)} \end{bmatrix}^T\right\} = \tag{20}$$

$$E\begin{Bmatrix} w(k)w(j)^T & w(k)n(j)^T \\ n(k)w(j)^T & n(k)n(j)^T \end{Bmatrix} = \begin{Bmatrix} Q_{ww} & Q_{wn} \\ Q_{nw} & Q_{nn} \end{Bmatrix}\delta_{kj}$$

The covariance of natural state uncertainty and covariance of parameters disturbance vector are symbolled by Q_{ww} and Q_{nn}. As the result of no correlation between the natural and parameters states, $Q_{nw} = Q_{wn} = 0$, the initial state error of the covariance matrix can be defined as follow:

$$P(0) = \begin{bmatrix} P_{ww} & P_{wn} \\ P_{nw} & P_{nn} \end{bmatrix} \tag{21}$$

To describe the quality of prior information of the natural and parameter states, P_{ww} and P_{nn}needs to be correctly defined.

D. State predication and state correction in EKF

To solve the EKF estimating the parameters of IM, the state prediction and state correction equations need to be implemented. The sate estimator equation in state estimation of EKF can be defined as follow:

$$f(z(k),u(k)) = \begin{bmatrix} F(\theta(k),k)x(k) + G(\theta(k),k)u(k) \\ \theta(\mathrm{k}) \end{bmatrix} \tag{22}$$

In above equation, the $\frac{1}{L_m}$ and $\frac{{L_m}^2}{L_r}r_r$ are considered as fifth and sixth states for state space model of Induction machine. The state error covariance vector in state prediction and state prediction equations can be defined based on the partial derivation or Jacobian matrix which is:

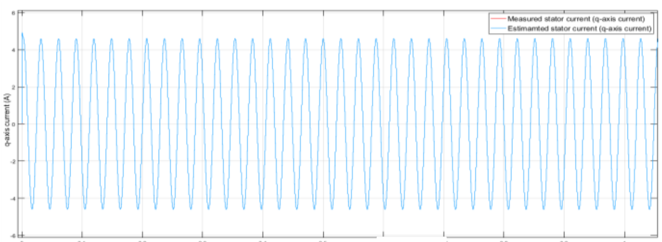
$$F(k) = \frac{\partial f(.)}{\partial z(k)} \tag{23}$$

Based on the Table. 1, the initial state values for $\frac{1}{L_m}$ and $\frac{{L_m}^2}{L_r}r_r$ are 3.6914 and 0.6473. The measurement and process noise covariance are also defined in Appendix.

As the $\begin{bmatrix} V_{ds}^{s} \\ V_{ds}^{s} \end{bmatrix}$ is the voltage input matrix of the Extended Kalman filter, this can be provided by using the input voltage of the SVM. However, as the input of SVM are normalized, these d-q stator voltage in stationary frame need to multiplied in $\frac{2V_{dc}}{3}$.

III. MATLAB SIMULATION RESULTS

In first step, the EKF is implemented in Matlab simulation in the switching frequency of 20KHz. The reference d-q axis currents (in synchronous frame) are considered as 2.3A and 3.98A respectively. The estimation is validated as the induction machine is rotating with the mechanical speed of 100 rad/sec. The estimated and measured d-q axis of stator flux in stationary frame are demonstrated in Fig.1. As shown in Fig.1, the estimated d-q axis currents are exactly compliance with the measured values. The estimated d-q axis of rotor flux are also compared with the actual rotor flux which used in making the model of machine (Fig.1 (c, d)).



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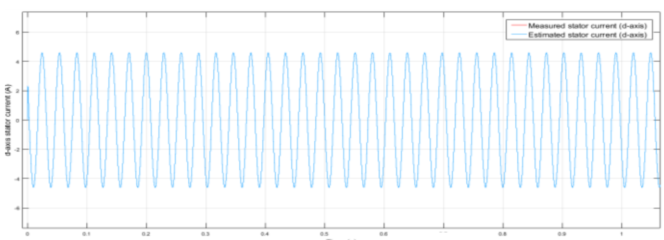

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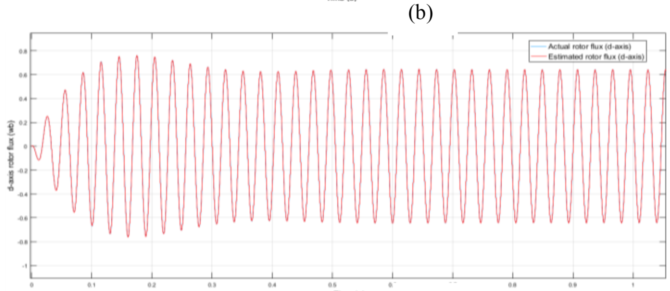

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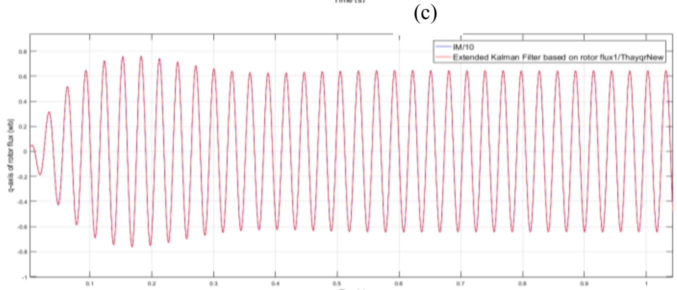

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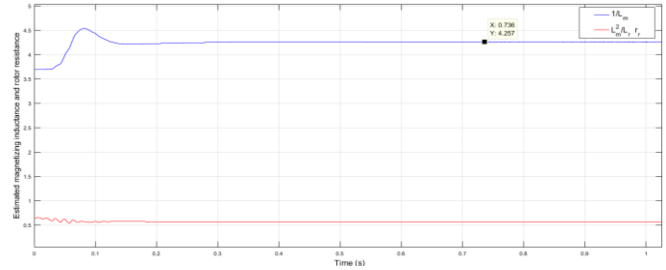
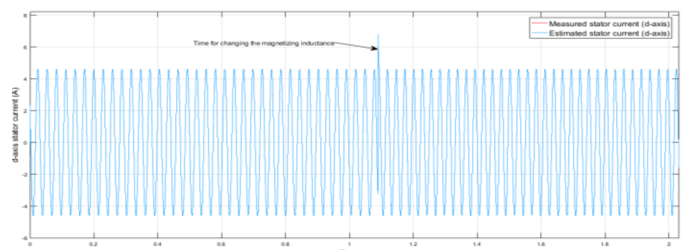


(e)

Fig. 1. Estimated states in stationary frame for applied $i_{ds}^{e*} = 2.3A$, $i_{qs}^{e*} = 3.98\ A$. (a) q-axis stator current (b) d-axis stator current (c) d-axis rotor flux (d) q-axis rotor flux (e) estimated parameters ($\frac{1}{L_m}$ and $\frac{{L_m}^2}{L_r} r_r$).

($\frac{1}{L_m}$ and $\frac{{L_m}^2}{L_r} r_r$) states in EKF.

As shown in Fig.1 (e), the estimated magnetizing inductance is $\frac{1}{4.257} = 0.235$ H. The actual magnetizing inductance is 0.2709. This shows that the error of estimation is lower than 12%. The estimated rotor resistance can be calculated from estimated state $\frac{{L_m}^2}{L_r} r_r$ which has the value of 0.565. So by considering constant leakage rotor inductance ($L_{lr} = 0.0133\ H$) and estimated L_m, the value of the estimated rotor resistance is 2.1880Ω. As the simulation is running, the value of the L_m in machine modelled changed to the half and the process of the EKF is analyzed as shown in Fig.2 . As the reference current signal are not changed to the new value, so the estimated stator currents sensor do not have



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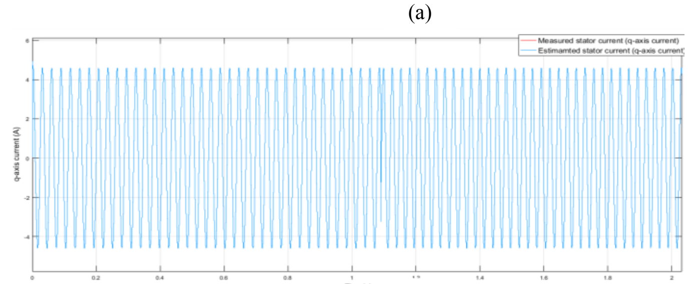

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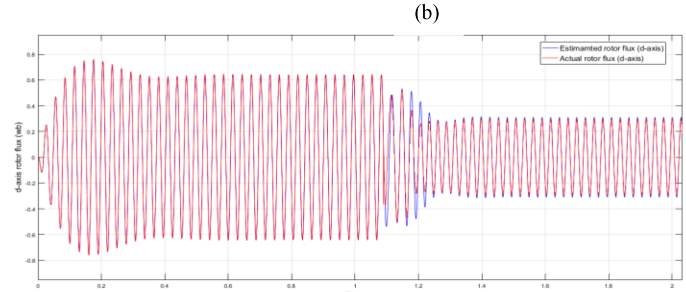

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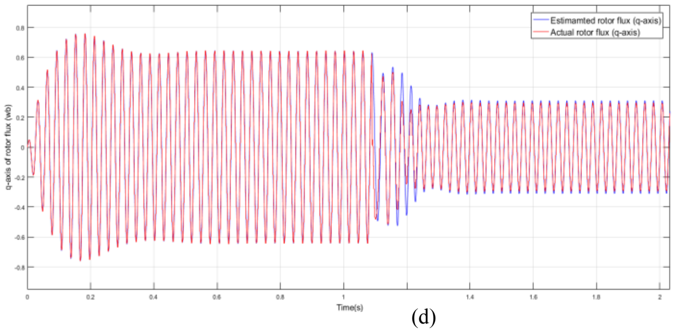

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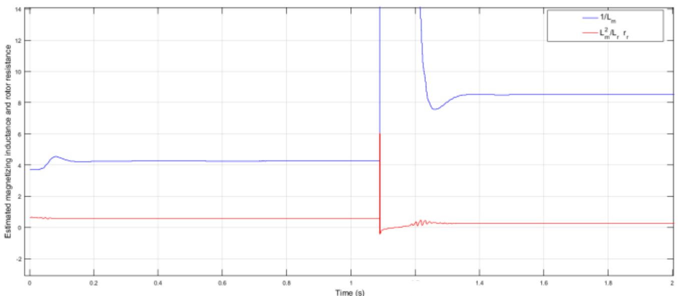


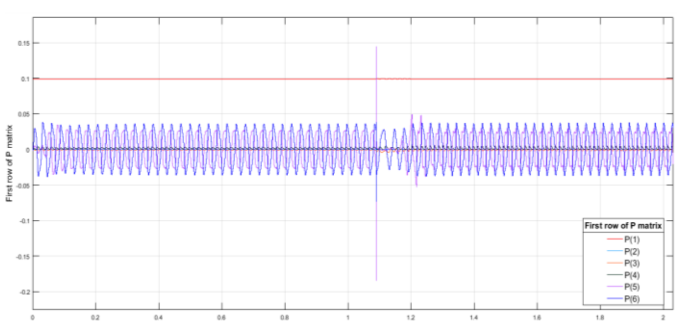
(e)

Fig. 2. Estimated states as magnetizing inductance changed to half (a) q-axis stator current (b) d-axis stator current (c) d-axis rotor flux (d) q-axis rotor flux (e) estimated parameters ($\frac{1}{Lm}$ and $\frac{Lm^2}{Lr} r_r$).

any variation in magnitude. As expected from rotor flux equations, the estimated rotor flux drops to half as the magnetizing inductances changed to half. As the value of the magnetizing current is suddenly changed to the half (which is not the case in practical as it changes gradually), the estimated values varied not accurately for few second and then converge to expected levels.

As shown in Fig. 2(f), the estimated magnetizing inductance is $\frac{1}{8.25} = 0.1220$ H. This estimated value is very close to half of the inductance (0.2709*.5=0.1350 H).

To demonstrate the observability of the defined non-liner Jacobian matrix in (25), the conversion of the state error covariance matrix (P) need to be proved. As shown in Fig.3, the conversion of each index of matrix demonstrated.



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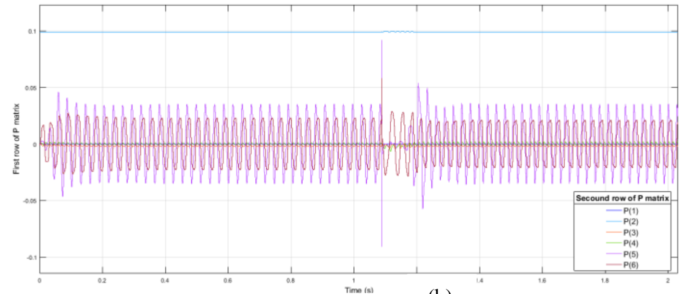

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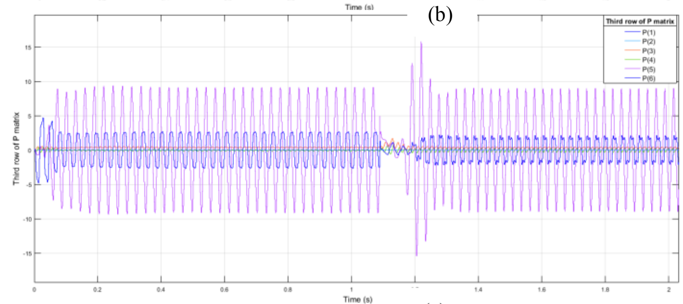

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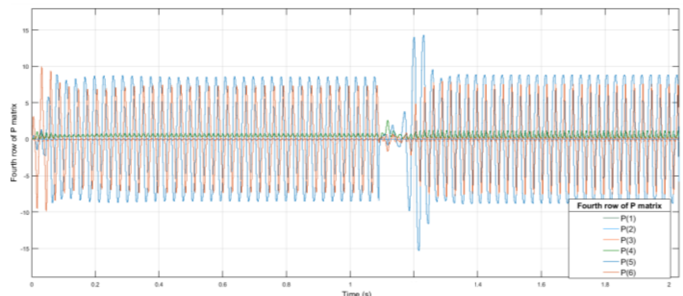

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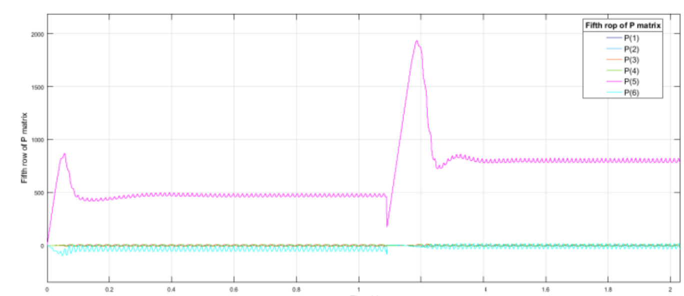

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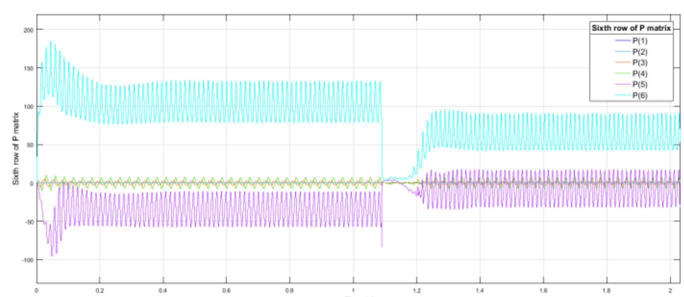
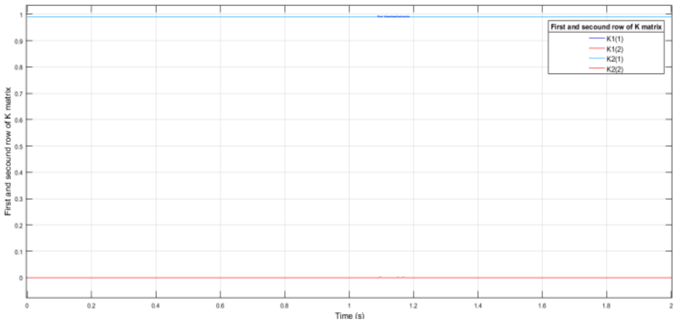


(f)

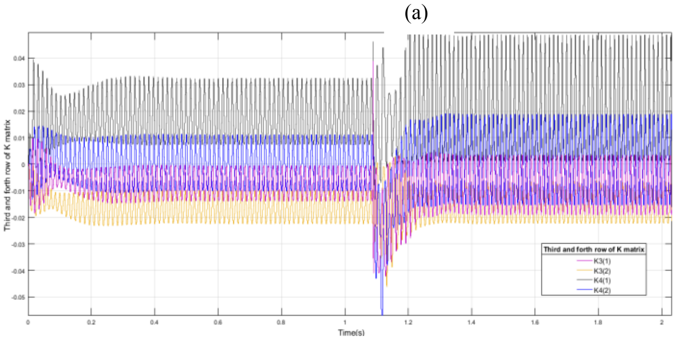
Fig. 3. State error covariance matrix (P). (a) First row (b) Second row (c) Third row (d) Fourth row (e) Fifth row (f) Sixth row

As shown in Fig.3, the indexes of first forth row of the P matrix oscillate around the zero. P1(1) and P2(2) are the only two index which have the constant level in these matrix rows. These indexes are related to estimated d-q axis stator currents which are the two measurable states in extended Kalman filter.

In two last row of P matrix, P5(5) and P6(6) change their value to double and half respectively. This is because of fifth and six state of the z matrix need to be changed to correctly estimate the parameters estimation. The convergence of the index of P matrix is essential for accurate estimation algorithms. If the P matrix diverge, it leads to the estimated states go to infinity and wrong value. To analyze the performance of the gain matrix (K), the first, second and third pairs of matrix row are demonstrated in Fig.4.



(a)



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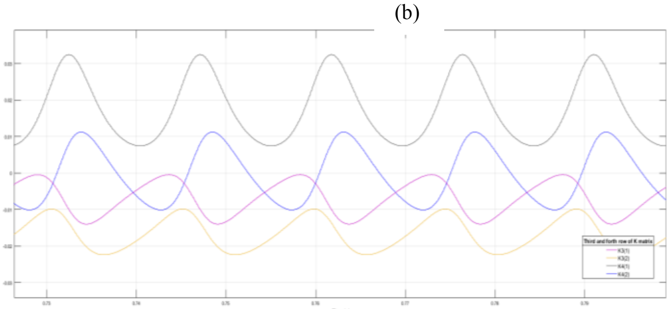

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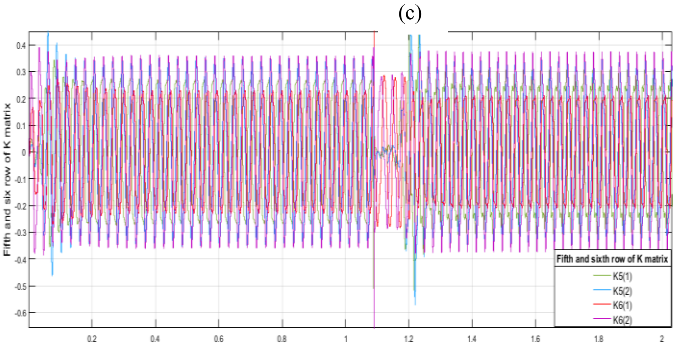

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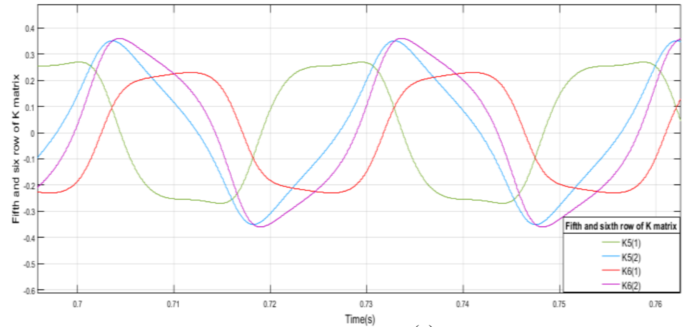

(e)

Fig. 4. Gain matrix ((K)) indexes. (a) First and second row (b) Third and fourth row (c) Third and fourth row (d) Fifth row (e) Sixth row

By considering the some approximation, K3(1)-K3(2) and K4(2)-K4(1) are the negetive of each other. As can be realized from above figure, the K5(2) and K6(1) are equal and K5(1) should be negative to produce K6(2). So, the Kalmna gain matrix can be provided by:

$$K(k) = \begin{bmatrix} k_{11} & k_{12} \\ k_{11} & k_{12} \\ k_{31} & k_{32} \\ -k_{32} & -k_{31} \\ k_{51} & k_{52} \\ k_{52} & -k_{51} \end{bmatrix} \qquad (24)$$

IV. CONCLUSION

In this pepper, the extended Kalman-filter based on the novel non-linear states space model of induction machine, included essential motor parameters, is implemented in Matlab Simulation. The performance of the algorithm is completely analyzed with considering the behavior of indexes in state error covariance matrix and Kalman gain matrix. The transient and steady state dynamic behavior of estimated states are also discussed and presented in case of motor parameter variation. Furthermore, the mathematical analyzation and simulation results are presented.

V. Appendix:

The specification of 4 Poles Induction machine is as follow:

Table I. Specification of Induction machine (1.1 kW)

r_s	r_r	V_{DC}	Current (peak)
2.291 Ω	2.5067 Ω	350 V	4.6 A
L_s	L_m	L_r	Base Speed
0.2842 H	0.2709 H	0.2842 H	1500 rpm

The measurement and process noise covariance matrix are defined as:

$$R=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}, Q=\begin{bmatrix}10 & 0 & 0 & 0 & 0 & 0\\ 0 & 10 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

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