Compensating Wage Differentials and the Value of Life

by

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ABSTRACT

This paper examines the relationship between Willingness-to-Pay and Compensating Wage Differential measures of the Value of a Statistical Life. It resolves some recently noted discrepancies between such measures.

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KEYWORDS: Willingness to pay, Compensating wage differential, Value of life.

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I. INTRODUCTION

Whilst the willingness to pay (WTP) approach to the value of statistical life is now well established (see e.g. Jones-Lee [1974, 1989]), perhaps the major source of empirical valuations emanates from the literature on Compensating Wage Differentials (CWD) (see e.g. Marin and Psacharopoulos [1982], Rosen [1986]). The traditional view has been that the trade off between wage and risk (the gradient of the wage risk function) provides an estimate of the value of life (Thaler and Rosen [1976]). However, if individuals live for more than one period, recent work by Ford, Pattanaik and Wei [1995] (FPW hereafter) suggests that the CWD approach does not correctly estimate the VOSL.[1] If this were true, it would be a result of some significance, given the volume of research utilizing the CWD approach. The object of this paper is to re-examine their argument. We show that, whilst the FPW analysis is internally consistent, it is really better viewed as an analysis of alternative willingness to pay questions (out of wealth and out of income). In particular, because there is no explicit modeling of how individuals optimize job risk, their conclusion, that the CWD measure is defective, is not established. The strategy of the present paper is to incorporate this element into the FPW 2-period model (along with the possibility of life insurance) and to examine the consequences of this reformulation.

The key result obtained is that allowing actuarially fair insurance resolves the discrepancies in the VOSL noted by FPW regarding WTP out of wealth vis a vis WTP out of income, but it does not always resolve the discrepancy between these measures and that associated with CWD.

II. THE MODEL

The framework used throughout the paper assumes individuals are Von Neumann-Morgenstern (VNM) expected utility maximizes with the individual's utility function having two arguments, namely present value wealth and longevity or time of death (a functional form popularized in the work of Jones-Lee; see e.g. [1974], [1989]). Ford, Pattanaik and Wei [1995] use this to effect a comparison of WTP and CWD measures of the VOSL. However, the FPW analysis really only compares two different types of willingness to pay question, namely

(i) "What would you be willing to pay out of initial wealth" (WTP) and

(ii) "What would you be willing to pay out of future wage income" (FPW take this as the CWD measure).

By contrast, the CWD methodology assumes that individuals optimize over the wage-risk opportunities available in choosing jobs, that an equilibrium wage-risk frontier arises out of the demand/supply of jobs of varying degrees of riskiness, and that the gradient of the wage-risk function at a given level of risk provides an estimate of the VOSL at that level of risk (see e.g. Thaler and Rosen [1976]). Accordingly, it is assumed in this section that there is a continuum of jobs with different risks and that the equilibrium wage risk trade off is defined via a continuously differentiable, strictly increasing wage function, $w(p)$. Thus $w'(p)>0$ is viewed as the CWD measure of the VOSL for an individual who chooses job risk p.[2]

FPW show that it is possible for the answers to questions (i) and (ii) above to be inconsistent. However, it is also the case that what they regard as the CWD valuation (that is, (ii) above) need not be equal to what is conventionally understood to be the CWD valuation of statistical life, namely $w'(p)$. This section explores these points and also examines the impact of admitting actuarially fair insurance.

Workers have an identical VNM utility function, denoted $U(W,t)$ where t denotes the age to which an individual lives (a random variable) and W denotes (present value) wealth accumulated by that time. Initial wealth is denoted W_0. There are just 2 periods (0 and 1). FPW discuss 2 cases: these are examined in turn. In each case, we consider the impact of introducing the wage-risk function $w(p)$ and also the possibility of actuarially fair insurance.

Case 1

The individual chooses a job with job risk p ; immediately following this, the uncertainty is resolved, the individual either lives, with probability $(1-p)$*, and is paid a wage* $w(p)$ *(at time 0), or dies (with probability p), and receives no reward.*

Let V_W denote the value of life as computed using willingness to pay out of initial wealth and V_w the value when calculated using willingness to pay out of wage income. Hence $V_W = dW_0 / dp$ and, since the expected incremental payment out of wage income is $(1-p)dw$, it follows that $V_w = (1-p)(dw/dp)$.

Expected utility for such an individual is

$$EU = pU(W_0,0) + (1-p)U(W_0 + w(p),1). \qquad (1)$$

It is assumed that the individual has optimized job selection. The first order necessary condition is that

$$\begin{aligned}\partial EU / \partial p = U(W_0,0) - U(W_0 + w(p),1) \\ + (1-p)U_1(W_0 + w(p),1)w'(p) = 0\end{aligned}. \qquad (2)$$

The individual's choice of p thus satisfies

$$w'(p) = \frac{U(W_0 + w(p),1) - U(W_0,0)}{(1-p)U_1(W_0 + w(p),1)}. \tag{3}$$

First, consider WTP out of initial wealth to secure a risk reduction: this is computed by asking what variation dW_0 will just compensate for a change in risk dp (such that expected utility remains constant). It is assumed that the individual is not allowed to re-optimize job-risk. The alternative assumption, that individuals can also simultaneously re-optimize job risk, is not explicitly considered, for the simple reason that, given the first order conditions for job selection, the additional terms sum to zero and have no effect. Thus

$$\begin{aligned} dEU &= \left[(p+dp)U(W_0 + dW_0,0) + (1-p-dp)U(W_0 + w(p) + dW_0,1)\right] \\ &\quad - \left[pU(W_0,0) + (1-p)U(W_0 + w(p),1)\right] \\ &= \left[U(W_0,0) - U(W_0 + w(p),1)\right]dp \\ &\quad + \left[pU_1(W_0,0) + (1-p)U_1(W_0 + w(p),1)\right]dW_0 = 0 \end{aligned} \tag{4}$$

Rearranging, this implies that

$$V_W = \frac{dW_0}{dp} = \frac{U(W_0 + w(p),1) - U(W_0,0)}{pU_1(W_0,0) + (1-p)U_1(W_0 + w(p),1)} \tag{5}$$

Now consider the individual's willingness to pay out of wage income, *dw*. [3]

$$\begin{aligned} dEU &= \left[(p+dp)U(W_0,0) + (1-p-dp)U(W_0 + w(p) + dw,1)\right] \\ &\quad - \left[pU(W_0,0) + (1-p)U(W_0 + w(p),1)\right] \\ &= \left[U(W_0,0) - U(W_0 + w(p),1)\right]dp \\ &\quad + (1-p)U_1(W_0 + w(p),1)dw = 0 \end{aligned} \tag{6}$$

It thus follows that

$$V_w = (1-p)\frac{dw}{dp} = \frac{U(W_0 + w(p),1) - U(W_0,0)}{U_1(W_0 + w(p),1)} \tag{7}$$

Hence, from (3), (5), and (7), since by assumption $p \in (0,1)$,

$$w'(p) = dw/dp > V_W > V_w. \tag{8}$$

Thus FPW were correct in noting a discrepancy, although the discrepancy they noted was between the two alternative WTP measures V_W and V_w; this holds true here, but interestingly, both these measures diverge from the CWD measure, $w'(p)$. The discrepancy between the WTP measures arises because the utility function is both time dependent and features risk aversion - if no actuarially fair insurance is available, this naturally leads to divergences in the valuation based on wealth *vis a vis* that based on income.[4] The discrepancy between these measures and the CWD measure arises simply because of the non-payment of wage in the event of death at time 0.

Of course, for this single period model, it is worth noting that the discrepancies, for small levels of risk, are of order $100p\%$, so for risks of less than 10^{-2}, the discrepancies are in magnitude less than 1%. Thus one might wish to argue that they are of no practical concern. However, it turns out that in the general multi-period or continuous time context, the magnitude of the discrepancy between the WTP measures and the CWD measure can be much more substantial (see Dobbs [1997]).

The divergence simplifies if actuarially fair insurance is available (in a way which clarifies the source of the divergence). The insurance pays a sum on the occurrence of death. It follows the individual can adjust, through such insurance, the present value of wealth in the two states; suppose these wealth levels are denoted W^d when the individual dies at time zero and W^a if the individual lives through the period.[5] Then, to be actuarially fair it must be that

$$pW^d + (1-p)W^a - W_0 - (1-p)w(p) = 0. \tag{9}$$

Expected utility is

$$EU = pU\left(W^d, 0\right) + (1-p)U\left(W^a, 1\right). \tag{10}$$

Optimal job selection implies the choice of p maximizes (10) subject to (9). Forming a Lagrangian and denoting the multiplier as λ, the first order necessary conditions are that

$$\partial L / \partial W^d = pU_1\left(W^d, 0\right) + p\lambda = 0, \tag{11}$$

$$\partial L / \partial W^a = (1-p)U_1\left(W^a, 1\right) + (1-p)\lambda = 0, \tag{12}$$

$$\partial L / \partial p = U\left(W^d,0\right) + U\left(W^a,1\right) \\ + \lambda\left[W^d - W^a + w(p) - (1-p)w'(p)\right] = 0 \qquad (13)$$

From (11), (12),

$$U_1\left(W^d,0\right) = U_1\left(W^a,1\right) = -\lambda. \qquad (14)$$

Using (9) and (14), equation (13) can be written as

$$w'(p) = \frac{U\left(W^a,1\right) - U\left(W^d,0\right) + U_1\left(W^a,1\right)\left(W^d - W^a + w(p)\right)}{(1-p)U_1\left(W^a,1\right)} \qquad (15)$$

Now, as before, consider variations in risk (dp), initial wealth $\left(dW_0\right)$ and wage $\left(dw\right)$ which maintain the same level of expected utility. In so doing, no re-optimization of job risk is allowed[6] but re-balancing of insurance is allowed; the variations dW^a, dW^d are not constrained to be equal, only to be actuarially fair. Thus, the variations must satisfy

$$\left(p+dp\right)\left(W^d + dW^d\right) + \left(1-p-dp\right)\left(W^a + dW^a\right) - W_0 \\ - dW_0 - \left(1-p-dp\right)\left(w+dw\right) = 0 \qquad (16)$$

where variations in wealth dW_0 and income, dw, are considered at the same time for brevity. So, using (9), equation (16) implies

$$\left(W^d - W^a + w\right)dp + \left(pW^d + (1-p)dW^a\right) \\ - dW_0 - \left(1-p\right)dw = 0 \qquad (17)$$

The variation in expected utility is

$$dEU = (p+dp)U\left(W^d + dW^d,0\right) + (1-p-dp)U\left(W^a + dW^a,1\right) \\ - pU\left(W^d,0\right) + (1-p)U\left(W^a,1\right) \\ = \left[U\left(W^d,0\right) - U\left(W^a,1\right)\right]dp \\ + \left[pU_1\left(W^d,0\right)dW^d + (1-p)U_1\left(W^a,1\right)dW^a\right] = 0 \qquad (18)$$

It thus follows that, using (13), (14) and (17), equation (18) can be simplified to yield

$$\left[\left(1-p\right)w'\left(p\right)\right]dp = dW_0 + \left(1-p\right)dw. \qquad (19)$$

Thus, setting $dw = 0$ to obtain dW_0 / dp and $dW_0 = 0$ to obtain dw/dp, (19) implies that

$$dW_0 / dp = \left(1-p\right)dw / dp = \left(1-p\right)w'\left(p\right). \qquad (20)$$

Hence, given that $V_W = dW_0 / dp$ and $V_w = \left(1-p\right)\left(dw / dp\right)$, it follows that

$$V_W = V_w = \left(1-p\right)w'\left(p\right) \qquad (21)$$

Thus, as one would expect, insurance resolves the discrepancy between wealth and income, but there remains a discrepancy between these measures and the CWD estimate of the VOSL, the divergence arising quite simply because of the non-payment of wage in one of the states (hence the factor $(1-p)$).

In the above analysis, it is straightforward to also allow the individual to simultaneously re-optimize the level of job-risk; however, given the first order conditions, the additional terms cancel out, as one would expect (see footnote 5). Note also that, if no optimal re-balancing of insurance is allowed (implying the restriction $dW^a = dW^d = dW_0$), this *does* make a material difference and the discrepancy between V_W, V_w also remains.

Case 2

The individual chooses a job; immediately following this the wage is paid and then the uncertainty resolved. The individual either lives to the end of the period (time 1, *with probability* 1-*p), or dies at the beginning (time* 0, *with probability p).*

The analysis can easily be repeated in this case, and, as one would expect, there is no discrepancy between any of the measures (either with or without insurance). That is

$$V_W = V_w = w'(p) \tag{22}$$

However, it is worth remarking that, in the presence of actuarially fair insurance, the individual must be permitted to re balance this. If the individual is viewed as choosing a job and insurance, and then, with ***both these fixed***, asked for willingness to pay out of initial wealth W_0, then the restriction on re-balancing in itself generates a discrepancy.

III. CONCLUDING COMMENTS

The object of the present paper has been to clarify earlier work on the comparison of compensating wage differential and willingness to pay measures of the value of life. It was argued that the analysis conducted by Ford, Pattanaik and Wei [1995] really identified a distinction between two willingness to pay questions (out of wage income, out of wealth) and did not directly address the compensating wage differential measure. The analysis in section 2 identified in what circumstances there are discrepancies between such measures.

This paper confined itself to analysis of the 2-period problem (and it was noted that discrepancies within such a model would tend to be small in practice). However, a more realistic analysis would allow that individuals live in continuous time and can optimize their selection of job, job risk and job duration. This is beyond the scope of this paper. However, work on this continuous time case constitutes the subject of another paper, and it is worth perhaps commenting on some of the results so far available from that.[7] The basic insight behind this more general model is that, if workers can costlessly optimize job risk, then the gradient of the equilibrium wage/risk function does give an appropriate estimate of the value of life of individuals in a job of given risk. However, when there are positive transactions costs, individuals will rationally change jobs only at discrete intervals; in this case, the CWD estimate of the value of life can be significantly different from the willingness to pay measure.

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FOOTNOTES

[1] Multi-period risk has been considered in the context of the WTP approach (Jones-Lee [1989], Shepard and Zeckhauser [1984]) and in the estimation of time preference rates (Viscusi and Moore [1989]), but the inter-relationship between WTP and CWD in the multi-period context seems to have been only recently considered.

[2]Note that, since $w''(p)$ is likely in general to be non-zero, the value of life on this interpretation may well vary with the risk faced.

[3]The remark regarding re-optimisation of job-risk p again applies.

[4]Thus adding fair insurance contradicts FPW's claim that "(t)he models can also be extended in a variety of ways, but the main conclusion seems to be that, in general, the two approaches do not yield the same valuation of human life." (p.230)

[5]Since there is no knowing until death how long one has to live, the adjustment to wealth is necessarily posthumous; the VNM utility function clearly does not discriminate between wealth to the individual, and wealth to his/her estate. See Jones-Lee [1989] for a discussion of this point.

[6]The remark regarding re-optimisation of job-risk p again applies.

[7] See Dobbs [1997] for preliminary results.