MODELING WELFARE LOSS ASYMMETRIES ARISING FROM UNCERTAINTY IN THE REGULATORY COST OF FINANCE

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ABSTRACT

The allowed rate of return (*AROR*) is a critical input in the regulatory assessment of revenue requirements, price caps and other controls. Errors in estimating *AROR* impact on investment incentives and price setting and hence can induce welfare loss. It is often suggested that the welfare losses that arise from under-estimation of the *AROR* may be significantly greater than arise from over-estimation. However, to date, this proposition has not been examined in any detail. This paper assesses the extent of welfare loss asymmetry and its implications for the choice of *AROR*.

1. Introduction

The allowed rate of return (*AROR*) is a critical input when setting price caps and controls. Regulators typically base the *AROR* on their view of the firm's weighted average cost of capital, the '*WACC*' (see e.g. Brennan and Schwartz [1982]). This paper focuses on the consequences of uncertainty in *WACC* estimation for the regulator's choice of *AROR*. It has often been argued by regulators and consultants that the welfare losses arising from 'under-investment' when setting too low an *AROR* are likely to be greater than from some degree of 'over-pricing' when setting it 'too high', and that this motivates setting an *AROR* above the mean value of the *WACC* distribution.[1] The qualitative argument is that the welfare impact of 'too low' an *AROR* on investment is likely to be significantly greater than the impact of 'over-pricing' if it is set 'too high'. However, whilst this may be plausible, there has been little work done to date on the likely *quantitative* impact, and hence an attendant difficulty in deciding on the extent of 'uplift' required in *AROR*. [2]

The only contribution to attempt an explicit analysis appears to be a report submitted by Wright, Mason and Miles [2003] to the UK Department of Trade and Industry and the Competition Commission; chapter 5 of that report discusses a single period 'now or never' or 'non-deferrable' investment, and the consequences of errors in estimation of the cost of finance (the latter modeled via a uniform distribution). In this simple model, if the actual cost of finance turns out to be higher than the allowed rate of return, the investment is not undertaken and there is neither profit to the firm nor consumer surplus; only if the cost of finance turns out to be less than the *AROR* is the project undertaken. The present paper can be viewed as a significant extension of this exploratory work in that it examines long lived investments within the context of a sequence of regulatory

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[1] Regulators who have accepted the argument include Ofcom [2005], CAA [2007], the Competition Commission [2007], NZCC [2004], and ACCC [2005]. Various consultancy reports suggest the use of higher percentiles of the *WACC* distribution (the 75th , 85th, even 95th): See Bowman [2004, 2005], SFG [2005], PWC [2006] and Dobbs [2008].

[2] Other sources for potential error in valuation include uncertainty concerning the scope for cost reductions through technical progress and learning by doing, uncertainty concerning developments in the level of future demand and so on. Whilst these other sources of uncertainty may also be important, it is worth noting that uncertainty over financing costs, the focus of the present work, is pervasive.

review periods, it allows a general distribution for the *WACC*, and considers already sunk investments and deferrable investments in addition to non-deferrable investments (it can be argued that the non-deferrable case is the least important category empirically, since most new investment is in fact deferrable).

The basic framework involves the regulator setting an *AROR* and then determining price controls which hold throughout a fixed regulatory review period (*RRP*).[3] The price controls aim to allow a revenue stream that will just compensate the firm for the costs it incurs, including the cost of capital (see Leland [1974], Brennan and Schwartz [1982], Guthrie [2006]). The actual cost of finance subsequently faced by the firm is uncertain and may thus deviate from the *AROR* set by the regulator. This means that the firm, once it knows its cost of finance, can decide on whether to invest or not, and where investments are deferrable, it may choose to defer the decision to the next *RRP*. However, once a service is launched, a ‘universal service obligation’ applies, such that the firm is required to satisfy the demand for the service through all future time (see Evans and Guthrie [2005]).

It is shown that for sunk investment, there is little argument for up-lift in *AROR*, whilst for both new non-deferrable and new deferrable investment, there is a strong case for uplift in *AROR*. This is for two reasons; firstly, because the *AROR* that maximizes economic welfare is likely to be well in excess of the mean of the *WACC* distribution, and secondly, because there is inevitably uncertainty over the exact location of the optimum, and the errors that arise from setting the *AROR* too high are much less than those associated with setting it too low. When price cap regulation is applied to a mix of the old and the new, it is further shown that even a quite small component of new investment can motivate a significant increase in the *AROR*.

Before proceeding, it is perhaps worth emphasizing that the argument that the *AROR* should be set above the expected value of the *WACC* distribution arises from two key

[3] This is standard practice for incentive regulation regimes. By contrast, rate of return regulation typically featured endogenous timing for the *RRP*, wherein either the regulator or the regulatee could initiate a request for a regulatory hearing (see Guthrie [2006]).

‘second best’ features. Firstly, the assumption that the firm has control over new investment and its timing, and secondly, the assumption that the *AROR* and the associated price caps and controls are fixed for the duration of the regulatory review period (*RRP*). The strength of the argument might be somewhat mitigated through the use of *AROR* adjustment triggers, so long as these feed through into price (cap) adjustments within the *RRP*.[4] Such an approach has not yet found any regulatory favor, perhaps because price uncertainty and tariff adjustment costs impact on both consumers and the firm. Alternatively, incentives for new investment might be tackled on a project specific basis, through assessments of likely impacts of real option effects or through adjustments to assumptions concerning future cash flows. However this is a long way from ‘light touch regulation’ as it would involve detailed second guessing of the circumstances surrounding each and every project specific proposal. It can be argued that building incentivization into the assessment of the *AROR* for each given class of investment is a relatively light touch approach and one which offers perhaps the simplest and clearest signal to the firm. Accordingly, the above alternatives are not considered further in what follows.

In what follows, section 2 presents the basic analytic framework and establishes solutions for the allowed rate of return for the three categories of investment. Section 3 then presents numerical solutions and sensitivity analysis. Section 4 examines robustness to variations in the model specification and section 5 concludes

2. The Analytic Framework

Three categories of investment are examined: category 1 is existing business (an already sunk investment); category 2 is new investment that can be undertaken, if at all, only in the up-coming regulatory review period (*RRP*); category 3 is investment which can be implemented in the *RRP* – or which can be deferred to a future *RRP*. In most cases, given the relatively short period of regulatory review (typically 5 years or less – and rarely

[4] Just as airlines implement price adjustments based on a ‘fuel price adjustment clause’, this would involve a ‘trigger’ that adjusts the *AROR*, contingent on events such as changes in the underlying level for the risk free rate of interest. See First Economics [2007] and Brealey and Franks [2009] for a discussion of the pros and cons of such schemes.

more than 7 years), a large part of the business is likely to be category 1 investment, followed by category 3. However, category 2 investment is included as a useful benchmark and starting point for formal analysis.

The base model assumes investment features a constant physical depreciation rate, a constant rate of expected demand growth, a constant elasticity of demand, no technical progress, and where investment can be deferred, no risk that the option will expire at a later date. Common knowledge is assumed by both the regulator and regulatee concerning these parameters.[5] The robustness of results to variations in assumptions is discussed in section 4 below. In particular, extensions to the case where there are economies of scale in investment and where there is the possibility that options expire are explored.

For the i^{th} category of investment, c_i denotes marginal operating cost; k_i, per unit capacity cost[6]; ε_i, demand elasticity; γ_i, the rate of physical depreciation; and α_i, the rate of demand growth. It is assumed that if there is falling demand over time ($\alpha_i < 0$), it falls at a rate less than that of physical depreciation (such that $\alpha + \gamma > 0$), so there is continuing demand for investment, and that demand is elastic ($\varepsilon_i < -1$).[7] The demand curve at time *t* is

$$q_{it} = B_i e^{\alpha_i t} p_{it}^{\varepsilon_i} . \tag{1}$$

Since each category of investment is dealt with separately, to reduce unnecessary notational clutter, the '*i*' subscript is omitted where this does not affect intelligibility.

The sequence of decisions is as follows:

1. Time 0 – The regulator sets the *AROR*, $\hat{r}$ and calculates the price cap, $\hat{p}$ for each category of investment. The actual cost of finance *r* is then drawn

[5] See Lewis and Sappington [1988]. The only source of asymmetric information in the present model arises from the fact that, after the regulatory price cap determination, the cost of finance becomes known to the firm cf. Baron and Myerson [1982].
[6] Assumed constant in the benchmark model - the effect of allowing economies of scale are analyzed in section 4.
[7] With constant elasticity demand, this ensures consumer surplus is finite in the subsequent welfare analysis.

from a fixed distribution with known density function $\phi(r)$ on support $[r_l, r_u]$.

2. Time 0 – For category 1 (sunk) investment, the firm has no decision to take. Investment is required to service demand and cover physical depreciation. For category 2 (non-deferrable) investment, the firm chooses to accept or reject the opportunity. If accepted, the firm must service demand over time at the set price. For category 3 (deferrable) investment, the firm chooses to accept now, or takes the option to defer to the next regulatory review period. Again, if accepted, the firm is required to service demand over time at the set price.

3. Time *T*, 2*T*, 3*T*… – The regulatory cycle repeats. For category 3 investment, if it was deferred at the previous *RRP*, then, if the investment opportunity still exists,[8] the decision on whether to invest is reconsidered again.

The firm's *WACC*, denoted *r*, is a random variable with known density function $\phi(r)$ assumed invariant from one *RRP* to the next.[9] The regulator knows this distribution, and sets a fixed *AROR*, denoted $\hat{r}$, and then price caps, $\hat{p}$ so as to reduce project value to zero (i.e. to just break even). Given stationarity, there is no reason for the regulator to set anything other than a time invariant price cap. At the next *RRP*, all new investment made in previous *RRP*s becomes category 1 sunk investment. For simplicity, in the above specification, the uncertainty in *r* is resolved immediately after the price caps are set, with the decision to defer only available to the beginning of each *RRP*.[10]

No distinction is drawn between private and social rates of time preference and it is also assumed that the different categories of investment can be valued at the firm's cost of capital. There are, of course, many examples of a regulator distinguishing the level of

[8] The case where the opportunity only exists with a fixed probability is examined in section 4. The solution naturally converges on that for category 2 investment as the probability $\rho \to 0$.

[9] The distribution for *r* can be derived using Monte Carlo simulation, based on assumptions concerning the processes that generate the underlying *WACC* components (see e.g. NZCC [2004], ACCC[2005], Bowman [2004, 2005], SFG [2005], Competition Commission [2007]).

[10] A more general formulation would involve the cost of finance evolving as a stochastic process and the possibility of projects being deferred within *RRP*s as well as across *RRP*s. Gaming issues or commitment problems are not addressed (notably, the problem that, once investment is sunk, the regulator has an incentive to reneg on the regulatory compact). See e.g. Besanko and Spulber [1992]. Newbery [1999],gives a useful review of the issues that can arise. The reasons why the regulator chooses to commit to a fixed *AROR* and fixed price caps ex ante (with no subsequent adjustment) also lie outside the model.

risk, and hence the cost of finance, by line of business (*LOB*); in the UK, Ofcom [2005] does this for BT, as does the CAA [2007] in its recent determination of financing costs for Heathrow and Gatwick airports. However, a mix of investment types still remains within each *LOB*, and the analysis presented in this paper can be viewed as applying to each *LOB* separately. The welfare measure used in what follows is the unweighted discounted sum of firm profits and consumers surplus, where the actual cost of finance r is used as the discount rate.

Closed form solutions are not available for the optimization problems faced by the firm or the regulator, even if $\phi(r)$ is assumed to take a simple form (such as the uniform distribution). A numerical optimization approach is therefore adopted, based on a simulation.[11] In recent years, regulators and other interested parties have often utilized the simulation approach for establishing the *WACC* distribution; whatever the details of the model used for the *WACC,* the ultimate outcome under the simulation approach is an empirical frequency distribution for r. That is, the simulation generates a set of drawings for the cost of finance $r(j),\ j = 1,..,n$ where n is chosen to be a large enough number to ensure stability of relevant statistics (such as the percentiles of the distribution).[12]

2.1 Category 2: Non deferrable New Investment

This type of investment is likely to be of less importance than category 1 and category 3 investments, simply because new investment is rarely a 'now or never' decision - there is usually an option to defer. However, it is useful to start here as it facilitates the somewhat more complex analysis of category 3 deferrable investment. This category 2 non-deferrable investment is similar in spirit to the exploratory work undertaken in Wright et al [2003]. The principle difference lies in the fact that this earlier work dealt only with a single *RRP* and a single period new 'now or never' investment problem. In the present model, there is an infinite program of ongoing investment (if the firm chooses

[11] The 'neoclassical' economics approach typically works with minimal qualitative structure. The value of such an approach is limited when the trade offs cannot be qualitatively signed (as here). In such circumstances, a computational approach can help to provide *quantitative* appreciation on how things interact (see Judd [1999]).

[12] In fact $n = 10^6$ is used; for the base case, this gives percentiles with standard errors of around 0.005% for the 1st and 99th, and of generally less than 0.003% in between.

to instigate the project) with possible growth in demand and physical depreciation. There is, in addition, an infinite sequence of *RRP*s and uncertainty concerning the cost of finance in future *RRP*s. Finally, both sunk and deferrable investments are examined in addition to non-deferrable category (since the latter is, empirically, the least substantive category).

The firm invests only if it calculates *NPV*>0. Given price is fixed at $\hat{p}$, demand[13] is

$$q_t = Be^{\alpha t}\hat{p}^{\varepsilon}. \qquad (2)$$

where the constant *B* determines the level of demand and α denotes the rate of growth in demand. Thus initial capacity $Q_0 = B\hat{p}^{\varepsilon}$ is required. With new investment in capacity at time *t*, denoted I_t, the growth in capacity is $\dot{Q}_t = I_t - \gamma Q_t$. Since capacity at time *t* must equal demand, it has to grow at the rate $\dot{Q}_t = \alpha Q_t$. Hence

$$I_t = (\alpha + \gamma)Q_t. \qquad (3)$$

That is, instantaneous investment covers depreciation of existing stock and the addition of new stock in order to satisfy demand at the regulated price.

Instantaneous profit at time *t* is

$$\begin{aligned}\pi_t &= (\hat{p} - c)q_t - kI_t = \left(\hat{p} - c - (\alpha+\gamma)k\right)q_t \\ &= \left(\hat{p} - c - (\alpha+\gamma)k\right)Be^{\alpha t}\hat{p}^{\varepsilon}\end{aligned} \qquad (4)$$

Value from the regulator's perspective is then

$$\begin{aligned}V &= \int_0^{\infty}\pi_t e^{-\hat{r}t}dt - kQ_0 = \int_0^{\infty}B\hat{p}^{\varepsilon}\left(\hat{p} - c - (\alpha+\gamma)k\right)e^{(\alpha-\hat{r})t}dt - kQ_0 = \\ &= B\hat{p}^{\varepsilon}\left(\frac{\hat{p} - c - (\alpha+\gamma)k}{\hat{r} - \alpha} - k\right)\end{aligned} \qquad (5)$$

For simplicity, parameter ranges are set such that regulatory price caps are always binding (that is, the unconstrained monopoly price is never below the price cap). The regulator uses $\hat{r}$ as the discount rate and aims to choose the price $\hat{p}$ so as to reduce discounted profit over the project life to zero; given (5), this implies choosing $\hat{p}$ so that

[13] The riskiness of the future cash flows in not explicitly modeled. Demand is best interpreted as 'expected demand', with the 'cost of finance', *r*, being the appropriate cost of finance for cash flows of this level of 'riskiness'.

$$\left(\frac{\hat{p} - c - (\alpha + \gamma)k}{\hat{r} - \alpha} - k \right) = 0, \tag{6}$$

so

$$\hat{p} = c + (\alpha + \gamma)k + \left(\hat{r} - \alpha\right)k = c + (\hat{r} + \gamma)k\ . \tag{7}$$

The price is thus set to long run marginal cost, based on the regulator's view of what constitutes an appropriate cost of finance, $\hat{r}$.

The actual cost of finance, r which holds on $(0,T)$ is now drawn (the cost of finance on the second and subsequent *RRP*s remain random variables from the perspective of the firm). In view of (7), (4) can be written as

$$\begin{aligned}\pi_t &= B\hat{p}^{\varepsilon}\left(\hat{p} - c - (\alpha + \gamma)k\right)e^{\alpha t} \\ &= B\hat{p}^{\varepsilon}(\hat{r} - \alpha)ke^{\alpha t} \qquad \text{for} \quad t \in [0, \infty)\ .\end{aligned} \tag{8}$$

Denote initial profitability as

$$\pi_0 = B\hat{p}^{\varepsilon}(\hat{r} - \alpha)k\ . \tag{9}$$

Then, value from the firm's perspective can be written as

$$\begin{aligned}V_f &= \int_0^T \pi_0 e^{(\alpha - r)t}dt + e^{-rT}\int_0^T \int_{r_l}^{r_u} \pi_0 e^{(\alpha - \tilde{r})t}\phi(\tilde{r}_2)d\tilde{r}_2 dt \\ &\quad + .. + e^{-rT}\int_{mT}^{(m+1)T}\int_{r_l}^{r_u} \pi_0 e^{(\alpha - \tilde{r})t}\phi(\tilde{r}_m)d\tilde{r}_m dt + ... - kQ_0 \\ &= \int_0^T \pi_0 e^{(\alpha - r)t}dt + e^{-rT}\int_0^{\infty}\int_{r_l}^{r_u} \pi_0 e^{(\alpha - \tilde{r})t}\phi(\tilde{r})d\tilde{r}dt - kQ_0 \qquad .\end{aligned} \tag{10}$$

The first line of (10) emphasizes that value to the firm is the discounted profits on $(0,T)$ based on the realized discount rate r, plus the discounted profits on each subsequent *RRP*, each of which depends on what the cost of finance is in that period. Viewed from time zero, the cost of finance r in the second and subsequent *RRP*'s are independent random variables, denoted as $\tilde{r}_t, t = 2,3,...$ in (10). The second term in the last line of (10) can be written as:

$$\int_0^{\infty}\int_{r_l}^{r_u} e^{(\alpha - \tilde{r})t}\phi(\tilde{r})d\tilde{r}dt = \int_{r_l}^{r_u}\left(\tilde{r} - \alpha\right)^{-1}\phi(\tilde{r})d\tilde{r}\ . \tag{11}$$

It proves useful to define an 'expected discount rate', denoted r_{ed}, that satisfies

$$\left(r_{ed} - \alpha\right)^{-1} = \int_{r_l}^{r_u}\left(\tilde{r} - \alpha\right)^{-1}\phi(\tilde{r})d\tilde{r}\ . \tag{12}$$

This can be solved for r_{ed} , given the *RHS* can be estimated numerically as

$$\int_{r_l}^{r_u} \left(\tilde{r}-\alpha\right)^{-1} \phi(\tilde{r}) d\tilde{r} \approx (1/n)\sum_{i=1}^{n}\left(r(i)-\alpha\right)^{-1}. \tag{13}$$

(recall the Monte Carlo simulation involves *n* runs, with $r(j)$ denoting the *WACC* that resulted from the *j*th simulation run). Note that r_{ed} depends on α [14] and that $r_{ed} < E(\tilde{r})$;[15] this follows from Jensen's inequality, since $\left(\tilde{r}-\alpha\right)^{-1}$ is a convex function of $\tilde{r}$. [16] The value to the firm, $V_f(r)$, conditional on the realized value *r* in the first *RRP*, can now be written as

$$\begin{aligned} V_f(r) &= \int_0^T \pi_0 e^{(\alpha-r)t} dt + e^{-rT} \int_0^\infty \int_{r_l}^{r_u} \pi_0 e^{(\alpha-\tilde{r})t} \phi(\tilde{r}) d\tilde{r} dt - kQ_0 \\ &= \left\{ \frac{1-e^{(\alpha-r)T}}{r-\alpha} + \frac{e^{(\alpha-r)T}}{r_{ed}-\alpha} \right\} \pi_0 - kQ_0 \end{aligned} \quad . \tag{14}$$

Define the present value factor as

$$A(r) \equiv \left\{ \frac{1-e^{(\alpha-r)T}}{r-\alpha} + \frac{e^{(\alpha-r)T}}{r_{ed}-\alpha} \right\} \quad . \tag{15}$$

and note that $Q_0 = B\hat{p}^{\varepsilon}$. So

$$V_f(r) = \left\{ \frac{1-e^{(\alpha-r)T}}{r-\alpha} + \frac{e^{(\alpha-r)T}}{r_{ed}-\alpha} \right\} \pi_0 - kQ_0 = B\hat{p}^{\varepsilon} k \left[A(r)(\hat{r}-\alpha)-1 \right]. \tag{16}$$

Thus the firm will only invest if value $V_f(r)$ is positive (for this 'all or nothing' project). That is, from (16), if

$$A(r)(\hat{r}-\alpha)-1 > 0 . \tag{17}$$

Given an estimate for r_{ed} from (12), equation (17) can be numerically solved to find the critical interest rate, denoted r_a , which will induce investment. That is, investment occurs if

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[14] To ensure convergence, values considered for the parameter α are restricted to satisfy $\alpha < r_l$, so that $\alpha < Min_i \;\; r\left((i)\right)$.

[15] Given the assumption that $\alpha < Min_i \;\; r\left((i)\right)$, empirically, the effect is small – that is, r_{ed} is close to $E(\tilde{r})$.

[16] The impact of uncertainty on discount factors is studied in Butler and Schachter [1989].

$$r < r_a, \tag{18}$$

and not otherwise, where r_a is the solution to

$$A(r_a)(\hat{r} - \alpha) - 1 = 0. \tag{19}$$

Note that, as $T \to \infty$, so $A(r) \to \left(r - \alpha\right)^{-1}$, and $r_a \to \hat{r}$, and the firm invests only if $r < \hat{r}$. However, in practice, *T* is finite, and it can be shown in this case that $r_a > \hat{r}$ and that r_a is an increasing function of $\hat{r}$; the higher the *AROR*, the higher the threshold for *r* above which investment ceases.

Consumer surplus at time *t* is

$$CS = \int_{\hat{p}}^{\infty} q(p)dp = \int_{\hat{p}}^{\infty} Be^{\alpha t} p^{\varepsilon} dp = \left[Be^{\alpha t} p^{\varepsilon+1} \big/ (\varepsilon + 1) \right]_{\hat{p}}^{\infty} = -Be^{\alpha t} \hat{p}^{\varepsilon+1} \big/ (\varepsilon + 1), \tag{20}$$

so, adding this to instantaneous profit, instantaneous economic welfare can be written as

$$\begin{aligned} w_t &= \left(-Be^{\alpha t} \hat{p}^{\varepsilon+1} \big/ (\varepsilon + 1)\right) + \left(Be^{\alpha t} \hat{p}^{\varepsilon}\right)\left[\left(\hat{p} - c - \left(\alpha + \gamma\right) k\right)\right] \\ &= Be^{\alpha t} \hat{p}^{\varepsilon} \left\{ \frac{\varepsilon}{\varepsilon + 1} \hat{p} - c - \left(\alpha + \gamma\right) k \right\} \end{aligned}, \tag{21}$$

if there is investment; that is, if $r < r_a$ (and zero otherwise). Note that w_t does not depend on *r*, although it does depend on $\hat{r}$ via $\hat{p}$.

Writing initial instantaneous welfare as

$$w_0 = B\hat{p}^{\varepsilon} \left\{ \frac{\varepsilon}{\varepsilon + 1} \hat{p} - c - \left(\alpha + \gamma\right) k \right\}, \tag{22}$$

so that $w_t = w_0 e^{\alpha t}$, the net present value for welfare, for a given realization *r* in the first *RRP* is

$$\begin{aligned} W(r, \hat{r}) &= \int_0^T w_0 e^{(\alpha - r)t} dt + e^{-rT} \int_0^{\infty} \int_{r_l}^{r_u} w_0 e^{(\alpha - \tilde{r})t} \phi(\tilde{r}) d\tilde{r} dt - kQ_0 \\ &= \left\{ \frac{1 - e^{(\alpha - r)T}}{r - \alpha} + \frac{e^{(\alpha - r)T}}{r_{ed} - \alpha} \right\} w_0 - kQ_0 = A(r) w_0 - kQ_0 \end{aligned} \tag{23}$$

when investment occurs (and zero when it does not). Welfare *W* is written as $W(r, \hat{r})$ in (23) to emphasize the fact that it is a function of both the realized cost of finance *r*, and also the *AROR*, $\hat{r}$; the latter dependency arises because both w_0, Q_0 in (23) depend on $\hat{p}$

which in turn depends on $\hat{r}$ through (7). Thus, expected *NPV* welfare, denoted EW_2, is given as

$$EW_2 = \int_{r_l}^{r_a(\hat{r})} W(r,\hat{r})\phi(r)dr \quad , \tag{24}$$

where $r_a(\hat{r})$ is implicitly defined by (19).

To summarize: The regulator first chooses $\hat{r}$. Given this discount rate, the regulator then calculates the price cap $\hat{p}$ that reduces its view of project value to zero. The firm then observes the actual cost of finance *r* and decides whether to undertake the investment. It does so if the realized return *r* is below the threshold rate $r_a(\hat{r})$. Only if $r < r_a(\hat{r})$ does the firm view the project as having positive value, and only if it undertakes the investment is there any economic welfare gained from it. One can now ask the question – what choice of *AROR* would actually maximize expected economic welfare? This is denoted $\hat{r}_2*$ (the subscript identifying the category of investment under consideration) and is the solution to the problem

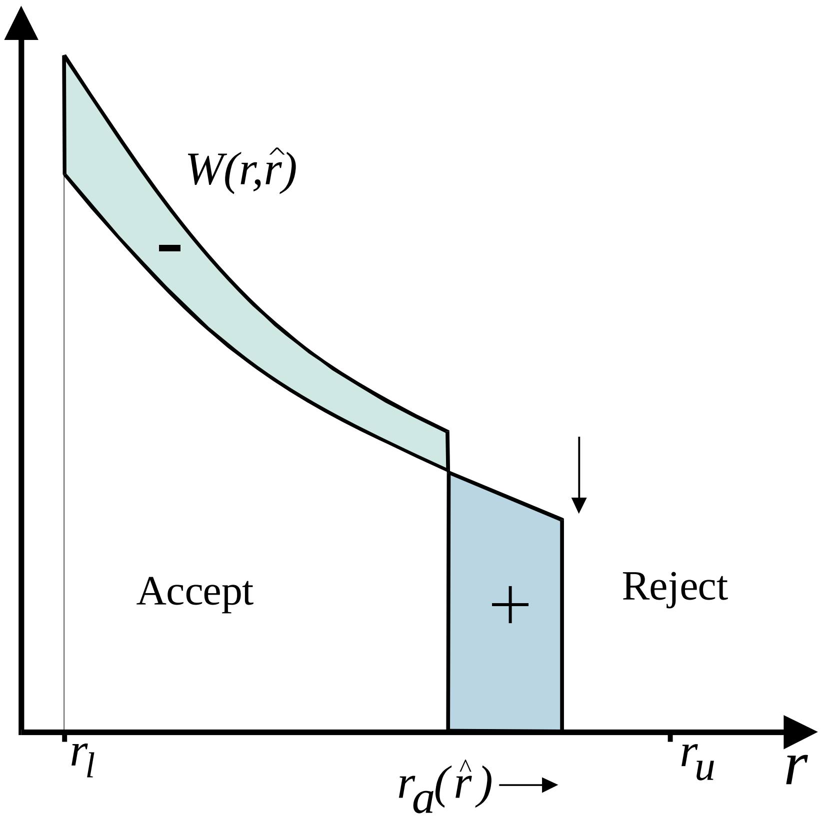
$$\hat{r}_2* = \arg\max_{\hat{r}} \int_{r_l}^{r_a(\hat{r})} W(r,\hat{r})\phi(r)dr \,. \tag{25}$$

Mathematically, the solution can be shown to lie *anywhere* in the interval $[r_l, r_u]$ (see appendix). However, unless demand is very elastic, the solution will lie above the mean of the *WACC* distribution, and usually significantly above it. With some algebraic effort, it can be shown that $\partial W(r,\hat{r})/\partial r < 0, \partial^2 W(r,\hat{r})/\partial r^2 > 0$ and that $\partial W(r,\hat{r})/\partial \hat{r} \gtrless 0$ as $r \gtrless r_a(\hat{r})$ (proof omitted). So, for $r < r_a(\hat{r})$, *W* is positive, decreasing, and convex in *r*, whilst for $r > r_a(\hat{r})$, the firm chooses not to invest and the welfare contribution is zero. Figure 1 illustrates the function $W(r,\hat{r})$, and shows how there are two opposing forces affecting EW_2; firstly, an increase in $\hat{r}$ increases $r_a(\hat{r})$ and this increases the range of *r* on which projects are accepted (and hence adds economic welfare); secondly, the increase in $\hat{r}$ increases the price cap and so increases price above the socially optimal level, resulting in a decrease in economic welfare at all levels of *r*, given that $r < r_a(\hat{r})$.

Clearly, at low values for $\hat{r}$ and hence lower values for $r_a(\hat{r})$, it is the welfare gain from increasing investment that dominates; as $\hat{r}$ and hence $r_a(\hat{r})$ is increased, this is eventually balanced by welfare losses arising from 'over pricing' as the price cap is relaxed.

Figure 1: The $W(r,\hat{r})$ function:

Effect of an increase in $\hat{r}$ leading to an increase in $r_a(\hat{r})$



Expected economic welfare EW_2 can be evaluated numerically. The simulation generates *n* realizations for *r*; the calculation of (24) is thus given simply as

$$EW_2 \approx (1/n)\sum_{\forall i\ s.t.\ r(i)<r_a(\hat{r})} W\left(r(i),\hat{r}\right). \qquad (26)$$

Numerically, the optimization in (25) is achieved by setting $\hat{r}$ equal to each of the percentiles of the *WACC* distribution and selecting the percentile value that yields a maximum value for EW_2.

2.2 Category 3: Deferrable Investment

The price cap is set as in (7). Once r is known, the firm accepts or defers the project to the next *RRP*. The optimal decision rule involves setting a 'critical value', denoted r_c, such that investment is accepted if $r \le r_c$ and deferred if $r > r_c$. The firm's objective is to choose r_c to maximize expected firm value. Denote this expected value, prior to the drawing for *r*, as *EV*. If the firm defers to the next regulatory review, given the stationary environment, the expected value at that time is again *EV*. Writing *EV* as a function of r_c this implies

$$EV(r_c) = \int_{r_l}^{r_c} V_f(r)\phi(r)dr + \int_{r_c}^{r_u} e^{-rT} EV(r_c)\phi(r)dr \ . \tag{27}$$

Firm value $V_f(r)$ is given by (16), so the first integral represents the expected value contribution when investment is undertaken in the first *RRP* (when $r \le r_c$). The second integral represents the expected value contribution when investment is deferred (when $r > r_c$). Re-arranging (27),

$$EV(r_c) = \int_{r_l}^{r_c} V_f(r)\phi(r)dr \Big/ \left\{1 - \int_{r_c}^{r_u} e^{-rT}\phi(r)dr\right\}. \tag{28}$$

The optimal strategy is to maximize (28) with respect to the choice variable r_c. The optimal choice for r_c is

$$r_c{*} = \arg\max_{r_c} \int_{r_l}^{r_c} V_f(r)\phi(r)dr \Big/ \left\{1 - \int_{r_c}^{r_u} e^{-rT}\phi(r)dr\right\} \qquad . \tag{29}$$

Computationally, this optimization can be achieved as part of the Monte Carlo simulation. In the current implementation, it is optimized accurate to the nearest percentile value of the *WACC* distribution. That is, setting r_c to each percentile value of the *WACC* distribution, equation (28) is evaluated and the solution is the percentile value that gives maximum expected value. The key elements in (28) to be evaluated are:

$$\int_{r_l}^{r_c} V_f(r)\phi(r)dr \approx (1/n)\sum_{\forall i\ s.t.\ r(i)<r_c} B\hat{p}^{\varepsilon} k\left[A\big(r(i)\big)(\hat{r}-\alpha)-1\right] \tag{30}$$

and

$$\int_{r_c}^{r_u} e^{-rT}\phi(r)dr \approx (1/n)\sum\nolimits_{\forall i\ s.t.\ r(i)\geq r_c} e^{-r(i)T} \ . \tag{31}$$

The optimal solution for r_c* is an increasing function of $\hat{r}$ such that $r_c*(\hat{r}) > \hat{r}$.

Expected economic welfare for this category 3 investment, denoted by EW_3, is

$$EW_3(\hat{r}) = \int_{r_l}^{r_c*(\hat{r})} W(r,\hat{r})\phi(r)dr \Big/ \left\{1 - \int_{r_c*(\hat{r})}^{r_u} e^{-rT}\phi(r)dr\right\}, \tag{32}$$

where $W(r,\hat{r})$ is given by (23).

The denominator in (32) is determined as per (31) and the numerator as

$$\int_{r_l}^{r_c*(\hat{r})} W(r,\hat{r})\phi(r)dr \approx (1/n)\sum\nolimits_{\forall i\ s.t.\ r(i)<r_c*(\hat{r})} W\left(r(i),\hat{r}\right). \tag{33}$$

Thus EW_3 can be evaluated numerically for any given value set for $\hat{r}$. The optimal choice for *AROR* for this category 3 investment is thus

$$\hat{r}_3* = \arg\max_{\hat{r}} \int_{r_l}^{r_c*(\hat{r})} W(r,\hat{r})\phi(r)dr \Big/ \left\{1 - \int_{r_c*(\hat{r})}^{r_u} e^{-rT}\phi(r)dr\right\}. \tag{34}$$

The optimization (accurate to the nearest *WACC* percentile value) is found by setting $\hat{r}$ to each *WACC* percentile value in turn and evaluating (32), the optimum being that which yields the maximum value.

The trigger value $r_c*(\hat{r})$ for category 3 is naturally lower than the trigger value $r_a(\hat{r})$ for category 2. That is, the cost of finance *r* has to be even lower for it to be viewed as worth investing and not deferring.[17] However, note that for category 3 investment, and in contrast to category 2, economic welfare is not completely lost when *r* is less than the trigger level that induces investment, since although the investment is deferred, it is not abandoned 'for ever'.

[17] This option value effect has long been recognized; see e.g. McDonald and Siegel [1986], Pindyck [1988], Dixit and Pindyck [1994], and in the context of regulatory price caps, Dobbs [2004].

2.3 Category 1: Existing capacity

Denote the level of existing capacity as Q_e. For a given regulated price $\hat{p}$, it is possible that this is above or below the initially required level of capacity, $Q_0 = B\hat{p}^{\varepsilon}$. If $Q_e < Q_0$, this means that an initial pulse of investment is required, followed by ongoing investment to cover depreciation in existing assets and future demand growth.[18] The alternative possibility is that there is excess installed capacity at time 0. In this case, there is an interval on which there is zero investment when price is set at short run marginal cost, followed by an interval in which the regulated price rises in order to choke demand to existing capacity, until a time is reached at which price reaches long run marginal cost, and new investment commences. It is possible, but rather intricate, to model this. In any case, the firm is a going concern and given the service obligation, will have had to meet demand at the regulated price in the past, so it is not unreasonable to assume that existing capacity Q_e is well adjusted, such that $Q_e \approx Q_0$. The solution for the case where $Q_e = Q_0$ is in fact the same as that when $Q_e < Q_0$ so in what follows it is assumed that $Q_e \leq Q_0$. The analysis parallels that for category 2 investment, although the key difference is that existing capacity Q_e is already sunk.

The long run marginal cost regulated price $\hat{p}$ applies, as in (7); economic welfare is then similar to that in (23) except that part of capacity, Q_e, is already sunk. Initial investment in capacity is $Q_0 - Q_e$. Thus economic welfare conditional on the value of *r* in the first *RRP* is

$$W_1(r,\hat{r}) = \int_0^T w_0 e^{(\alpha-r)t} dt + e^{-rT}\int_0^{\infty}\int_{r_l}^{r_u} w_0 e^{(\alpha-\tilde{r})t}\phi(\tilde{r})d\tilde{r}dt - k\left(Q_0 - Q_e\right)$$
$$= \left\{\frac{1-e^{(\alpha-r)T}}{r-\alpha} + \frac{e^{(\alpha-r)T}}{r_{ed}-\alpha}\right\} w_0 - k\left(Q_0 - Q_e\right) = A(r)w_0 - kQ_0 + kQ_e \qquad (35)$$

(recall $W_1(r,\hat{r})$ is a function of $\hat{r}$ because both w_0 and Q_0 are functions of $\hat{r}$). This equation applies for all realizations $r \in \left[r_l, r_u\right]$. Thus

[18] Recall that it is assumed that even if demand growth is negative, the rate of depreciation of physical capacity is assumed to be greater, so there is continuing need for investment over time.

$$EW_1 = \int_{r_l}^{r_u} W_1(r,\hat{r})\phi(r)dr \,. \tag{36}$$

Optimal *AROR* for this category 1 investment is then given as

$$\hat{r}_1{*} = \arg\max_{\hat{r}} \int_{r_l}^{r_u} W_1(r,\hat{r})\phi(r)dr \tag{37}$$

Notice that the term kQ_e in (35) is a constant, and so has no effect on the optimization (when choosing the best value for the *AROR*). That is, no specific choice for Q_e is required and, without loss of generality, it can be set to zero when conducting the ensuing analysis. The numerical calculation of EW_1 is simply

$$EW_1 \approx (1/n)\sum_{i=1}^{n} W_1\left(r(i),\hat{r}\right) \tag{38}$$

It can be shown mathematically that the optimal value for $\hat{r}$ for this category lies below the mean value of the *WACC* distribution, although empirically, it generally lies quite close to the mean value (see section 3).

2.4 The Optimal Allowed rate of return

In addition to determining the optimal *AROR* for each category of investment separately, it is also possible to compute the *AROR* when there is a mix of the three different types of investment. Denote the set of key parameters as $\mathbf{x} = (c,\ K,\ \gamma,\ \alpha,\varepsilon,T)$. Expected economic welfare, $EW_i, i = 1,2,3$ is a function of these key parameters, the strength of demand parameter B_i and the choice of *AROR*, $\hat{r}$. It is straightforward to establish that expected welfare $EW_i, i = 1,2,3$ is linearly homogenous in the strength of demand parameters $B_i, i = 1,2,3$, using equations (22)-(36). Thus, denoting the functional dependence by writing $EW_i(B_i,\mathbf{x}_i,\hat{r})$, this can be written as

$$EW_i(B_i,\mathbf{x}_i,\hat{r}) = B_i EW_i(1,\mathbf{x}_i,\hat{r}),$$

where $\mathbf{x}_i$ denotes the parameter vector associated with the i^{th} category of investment.

Computationally, the computer program which runs the Monte Carlo simulation is set up to solve for $EW_i(1,\mathbf{x}_i,\hat{r})$, i=1,2,3 for each percentile value for $\hat{r}$. The optimization problem when there is a mix of the three categories of investment is then set as

$$Max_{\hat{r}}\ \ EW(\hat{r}) = \sum_{i=1}^{i=3} B_i EW_i(1, \mathbf{x}_i, \hat{r}) \quad (39)$$

where the parameters $B_i, \mathbf{x}_i$, i=1-3 are pre-set and the B_i are chosen such that $\sum_i B_i = 1$. The importance of each category of investment in (39) is determined by the choice of value for the strength of demand parameter B_i, i=1,2,3. In much of the scenario analysis, $\mathbf{x}_i$ does not vary with i; in this case it is also possible to interpret the values for B_i, i=1-3 as the relative size of these types of business in terms of initial revenue shares.[19]

The optimization in (39) gives $\hat{r}*$ accurate to the nearest percentile, in that $EW_i(1, \mathbf{x}_i, \hat{r})$, i=1-3 and hence $EW(\hat{r})$ are evaluated for all percentile values of $\hat{r}$ in order to determine that which yields the maximum value.

3. Determining the optimal allowed rate of return using Monte Carlo Simulation

This section provides a numerical analysis of how the optimal value for the allowed rate of return depends on key parameters. Clearly this analysis is contingent on the distribution assumed for the firm's *WACC*. For concreteness, the benchmark empirical distribution for the *WACC* is modeled as a truncated normal distribution with mean of $\mu = 10\%$ and a standard deviation $\sigma = 1.5\%$ and a range $[r_l, r_u] = [5\%, 15\%]$.[20] It is then straightforward to examine through sensitivity analysis the impact of adjustments to μ, σ, r_l, r_u. Although there are issues concerning how best to estimate the distribution for *WACC*, the focus of the present paper concerns the 'follow up stage' of how, given the

---

[19] Given $\hat{p}_i = c_i + (\hat{r} + \gamma_i)k_i$ from (7), if $\mathbf{x}_i$ does not vary with i then neither does $\hat{p}_i$ so this can be written as $\hat{p}$. Revenue share of the ith category of investment at time zero can then be written as
$$R_{i0} \Big/ \sum_{j=1}^{j=3} R_{j0} = \hat{p}q_{i0} \Big/ \sum_{j=1}^{j=3} \hat{p}q_{j0} = q_{i0} \Big/ \sum_{j=1}^{j=3} q_{j0} = B_i \hat{p}^{\varepsilon} \Big/ \sum_{j=1}^{j=3} B_j \hat{p}^{\varepsilon} = B_i \Big/ \sum_{j=1}^{j=3} B_j = B_i.$$

[20] Given the wide range, there is very little truncation in practice. Nominal *WACC*s of around 10% are fairly typical in the UK in the last decade for regulated business. For example, a mean of 10% and standard deviation of 1.5% approximates to the distribution generated in a case study of BT plc reported in Dobbs [2008], based on central parameter estimates in Ofcom [2005]. The lower limit is truncated at 5% to allow comparative statics analysis of growth rates in demand up to the 5% level.

Table 1: Selected Percentiles

Mean	10.00%
Percentile	
30	9.19%
40	9.60%
50	9.98%
55	10.17%
60	10.36%
65	10.56%
70	10.76%
75	10.99%
80	11.23%
85	11.52%
90	11.88%
95	12.40%
99	13.29%

assessment of the *WACC* distribution, a simulation approach can be used to explore the dependence of the optimal *AROR* on key parameters affecting welfare loss asymmetry.

In what follows, the benchmark case uses parameter values $c=1,\ k=10,\ \gamma=0.1,\ \alpha=0,\ \varepsilon=-3, T=5$. The impact on the optimal choice for *AROR* of variations in these parameters is then examined. The motivation for the particular choice of benchmark parameter values, with the exception of that for *RRP* at *T*=5 years, is not particularly strong. For this reason, the impact of a fairly wide range of parameter variations is studied in the ensuing sensitivity analysis. The benchmark values for *c*,*k* give equal weight to operating and capital costs in *LRMC* (at a discount rate of 10%); however, bringing operating costs down to zero or increasing capital costs 10 fold are then considered. Depreciation is set at 10% (giving circa a 7 year half life), but setting it to zero is also considered. Variations in demand growth from -5% through to +5%, and for demand elasticity from -1.5 through to -6, are also considered. The basic simulation involved taking *n*=1 million drawings $r(i)=1,..,n$ from the normal distribution with $\mu=10\%, \sigma=1.5\%$, discarding those outcomes lying outside the specified range. For each realization, the *WACC* value is computed. A frequency distribution is then computed for the *WACC* from which it is then possible to compute percentiles, along with the measures of welfare loss developed in section 2 above. Table 1 gives selected percentiles for the *WACC* distribution,

Figure 2 is based on the benchmark parameter values. It illustrates

(i) The relative symmetry of the welfare function for category 1 sunk investment, with the maximum at around 9.8%, just below the mean value of the WACC distribution of 10%. This is because for sunk investment the only impact on welfare is from under- or over-pricing, and this impact is relatively symmetric.

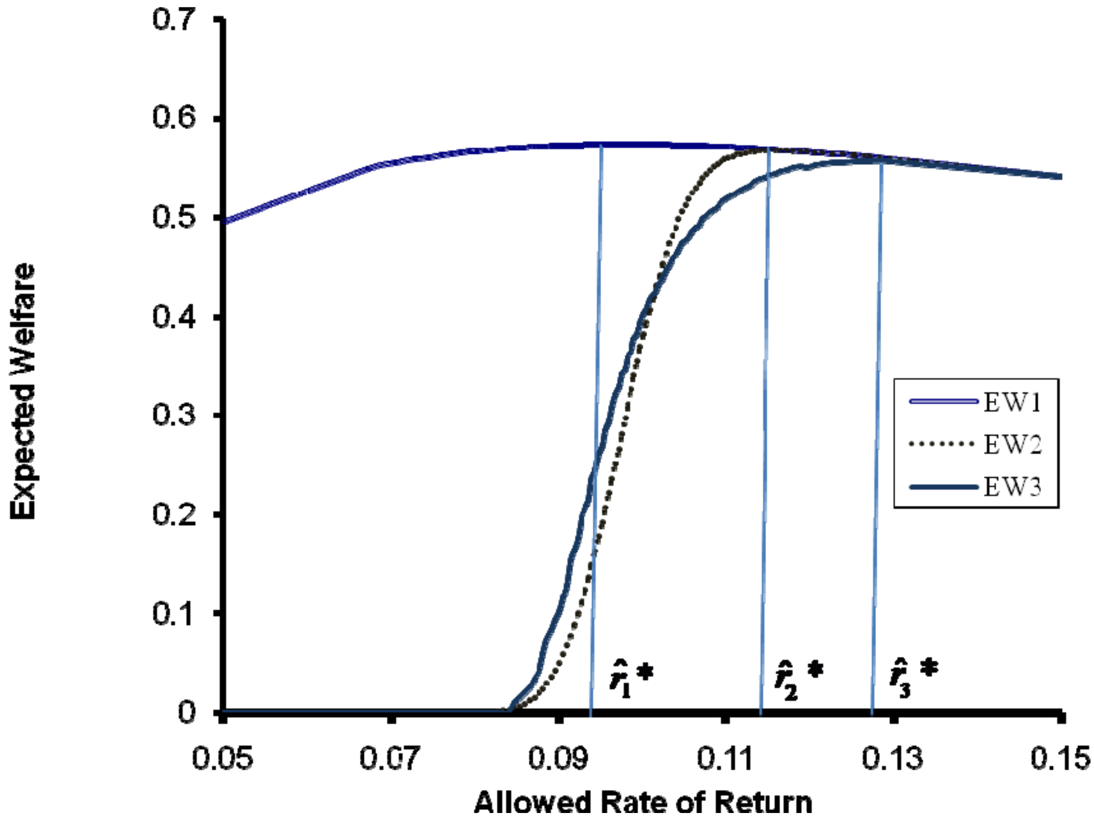
(ii) The significant asymmetry in expected economic welfare for both category 2 (non-deferrable) and category 3 (deferrable) investments as a function of the choice of *AROR*.

(a) around the mean value for *WACC*: there is significant increase in expected economic welfare by increasing *AROR* above the mean of the *WACC* distribution

(b) around the optimum choice for *AROR*. Although at sufficiently high values for *AROR* there is some fall away in economic welfare, setting too high an *AROR* is still likely to be less welfare costly than setting too low a rate.

(iii) In so far as the precise value for the optimal *AROR* is uncertain, it makes sense to err on the side of setting *AROR* too high rather than too low.

Figure 2 : Expected Welfare at Benchmark Values



The key feature for *new* investment in Figure 2, whether it is deferrable (with expected economic welfare EW_3) or not (with EW_2), is that when a low value for *AROR* is set, the investment rarely takes place and so economic welfare gain is small. Expected welfare then increases rapidly as *AROR* is increased up to and above the *WACC* mean value, eventually tailing off at high values for *AROR*. This is directly a result of the trade off between the welfare benefits of incentivizing investment versus the welfare costs associated with potential 'overpricing'. The latter only becomes significant at higher levels for the *AROR*. The other point to note is the similar structure for category 2 and 3 investments; welfare gain in both cases climbs rapidly once the *AROR* rises above its mean value. The welfare gain for non-deferrable investment naturally tends to peak at a lower *AROR* than that for deferrable investment, this reflecting the option value 'gain from waiting'.

Table 2: Optimal Values for the Allowed Rate of Return as a function of various parameters. *WACC* distribution mean 10%, standard deviation 1.5% Benchmark parameter values: $c=1,\ k=10,\ \gamma=0.1,\ \alpha=0,\ \varepsilon=-3, T=5$

Case	*Marginal Cost* c	*Capital Cost* k	*Depreciation* γ	*Demand Growth Rate* α	*Demand Elasticity* ε	*Regulatory Review Period* T	*Existing (Sunk) Investment Category 1*	*New Non-deferrable Investment Category 2*	*New Deferrable Investment Category 3*
1	1	10	0.1	0	-3	5	9.8 (45th)	11.6 (86th)	12.7 (97th)
2	0	10	0.1	0	-3	5	9.8 (45th)	11.4 (83rd)	12.4 (95th)
3	1	20	0.1	0	-3	5	9.7 (43rd)	11.5 (84th)	12.5 (96th)
4	1	100	0.1	0	-3	5	9.8 (44th)	11.4 (83rd)	12.4 (95th)
5	1	10	0	0	-3	5	9.8 (45th)	11.4 (83rd)	12.4 (95th)
6	1	10	0.1	0	-1.5	5	9.8 (45th)	11.8 (89th)	13.0 (98th)
7	1	10	0.1	0	-6	5	9.8 (45th)	11.2 (79th)	11.7 (88th)
8	1	10	0.1	0	-3	3	9.7 (43rd)	11.0 (75th)	12.3 (94th)
9	1	10	0.1	0	-3	7	9.8 (45th)	12.0 (91st)	12.7 (97th)
10	1	10	0.1	0.05	-1.5	5	9.8 (44th)	11.1 (77th)	12.0 (91st)
11	1	10	0.1	0.05	-3	5	9.8 (44th)	10.9 (73rd)	12.0 (91st)
12	1	10	0.1	0.05	-6	5	9.7 (43rd)	10.7 (68th)	11.2 (79th)

Table 2 reports on how the optimal *AROR* for the 3 categories of investment varies as a function of the parameters[21] $c, k, \gamma, \alpha, \varepsilon, T$. The key general features in this table are

(i) It shows that the *AROR* should be set slightly below the mean of the *WACC* distribution for sunk (category 1) investment and that parameter variations have little effect on this conclusion.

(ii) It shows that the optimal *AROR* for deferrable (category 3) and non-deferrable (category 2) new investment is considerably higher (generally around the 85th-95th percentiles).

(iii) That the optimal percentile for deferrable new investment is generally higher than that for non-deferrable new investment.

Parameter specific effects for new investment show

(iv) That the more inelastic demand is, the higher the *AROR*.[22]

(v) That increasing capital to operating cost tends to reduce slightly the optimal *AROR*

(vi) That depreciation and operating cost have the same impact on *AROR* (since they both solely affect the long run marginal cost).

(vii) That a higher growth rate in demand tends to reduce slightly the optimal *AROR*.

(viii) The shorter the *RRP*, the lower the *AROR*.[23]

The main feature, also manifest in figure 1, is that the optimal *AROR* for category 2 and 3 investment occurs at fairly high percentiles of the *WACC* distribution. For the most

[21] Variation in mean and standard deviation of the *WACC* distribution is discussed later.

[22] An anonymous referee pointed out that in adjusting elasticity, this affects position as well as curvature, so it might be useful to examine elasticity impact via a 'compensated demand curve' in which demand at the price cap is held constant as elasticity is varied. This makes some sense, particularly when demand is not growing over time. It involves simultaneously adjusting the strength of demand parameter *B* along with ε such that $B_1 \hat{p}_1^{\varepsilon_1} = B\hat{p}^{\varepsilon}$ when elasticity is changed from $\varepsilon \rightarrow \varepsilon_1$. In fact, doing this leaves results for sunk investment exactly as they are, whilst it tends to make the optimal *AROR* somewhat more sensitive to demand elasticity than is reported in Table 2. For example, in Table 2, changing elasticity from -1.5 to -4.5 means category 2 optimal *AROR* changes from 11.2% to 11.8%, whilst changing elasticity and compensating for the demand shift effect, *AROR* changes from 11.1% to 11.9% . The impact is more substantial for deferrable investment; in this case the same change in elasticity in Table 2 resulted in a shift from 11.7 to 13.0%, whilst with the compensating demand shift, this becomes 11.3 to 13.3%.

[23] In a sense, it is not really appropriate to vary *T* because, as pointed out earlier, there are benefits (cost incentivization etc.) that arise from setting a particular duration for *RRP* are not explicitly modeled here. However, the effect is reported for completeness.

important category of new investment (category 3, deferrable), only if demand is fairly elastic and/or there is significant growth rate in demand is it reduced. For most parameter values in Table 2, the optimal percentile for category 3 investment is in the high 80s and 90s. The other feature, as previously remarked, is that the optimal choice of *AROR* for category 1 sunk investment does not vary much across parameter variations, at a value generally slightly below the mean of the *WACC* distribution (typically around the 45th percentile).

Figure 3 illustrates these comparative statics effects on category 2 non deferrable investment for changes in parameter values by $\pm 50\%$ on benchmark values (note the effects for alpha are for changes from 0% growth to -5% and +5% respectively – given the benchmark of zero for this parameter). The results for deferrable investment are similar and are hence not reported, whilst the results for sunk investment are all 'flat' (parameter changes induce little change in optimal *AROR* for sunk investment).

Figure 3 – Impact on the optimal *AROR* for non-deferrable Investment of $\pm 50\%$ variation in parameter values

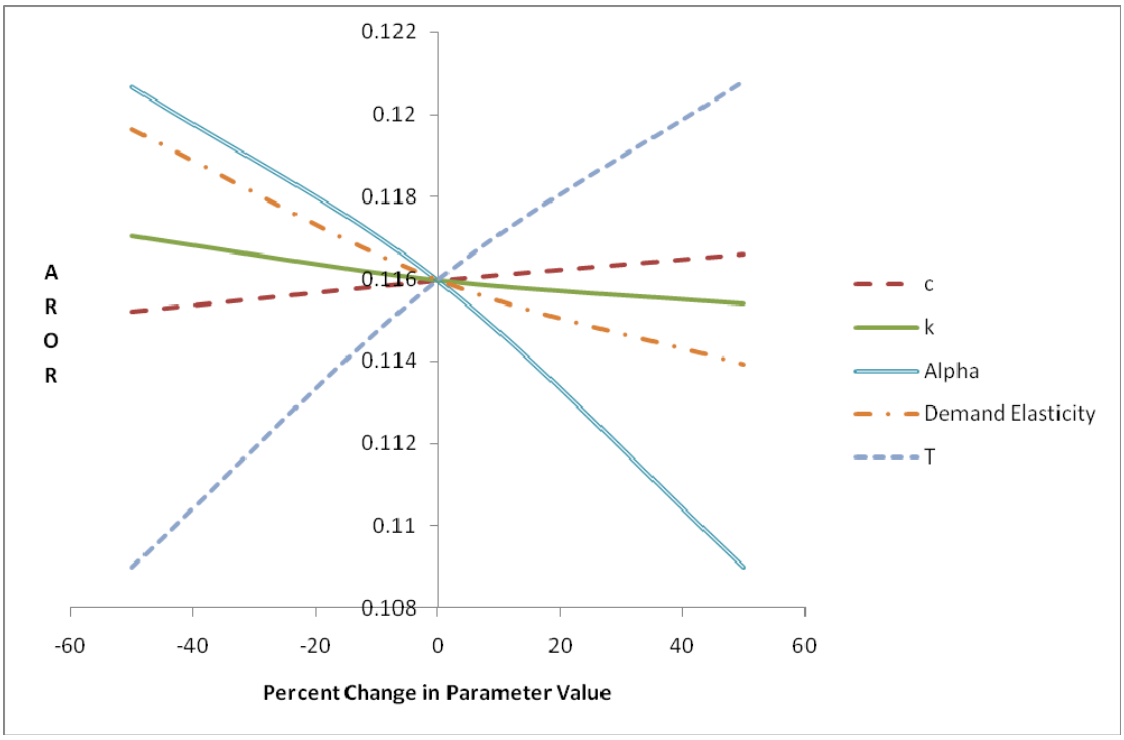


Table 3 : New Investment Deferrable: Optimal Values for the Allowed Rate of Return as a function of various Parameters
WACC distribution mean 10%, standard deviation 1.5%
Benchmark parameter values: $c = 1,\ k = 10,\ \gamma = 0.1,\ \alpha = 0, \varepsilon = -3, T = 5$.
Investment Mix: $A_2 = 0$ throughout, and $A_1 = 1 - A_3$.

Case	*Marginal Cost* c	*Capital Cost* k	*Depreciation* γ	*Demand Growth Rate* α	*Demand Elasticity* ε	*Regulatory Review Period* T	*Proportion of deferrable new investment to total investment* A_3	*Optimal Allowed Rate of Return* $\hat{r}*$	*Optimal Percentile* $perc(\hat{r}*)$
1	1	10	0.1	0	-3	5	0.05	10.9	74
2	1	10	0.1	0	-3	5	0.1	11.3	82
3	1	10	0.1	0	-3	5	0.2	11.7	88
4	1	10	0.1	0	-3	5	0.3	11.8	90
5	0	10	0.1	0	-3	5	0.1	10.9	74
6	1	20	0.1	0	-3	5	0.1	11	76
7	1	100	0.1	0	-3	5	0.1	10.9	74
8	1	10	0	0	-3	5	0.1	10.9	74
9	1	10	0.1	0	-1.5	5	0.1	11.9	90
10	1	10	0.1	0	-3	5	0.1	11.3	82
11	1	10	0.1	0	-6	5	0.1	10.5	63
12	1	10	0.1	0	-3	3	0.1	11.1	77
13	1	10	0.1	0	-3	7	0.1	11.4	83
14	1	10	0.1	0.05	-1.5	5	0.1	11.6	86
15	1	10	0.1	0.05	-3	5	0.1	11	75
16	1	10	0.1	0.05	-6	5	0.1	10.4	62
17	1	10	0.1	α_3=0.05	-3	5	0.1	11.2	79
18	1	10	0.1	α_3=0.05	-3	5	0.2	11.4	83
19	1	10	0.1	0	ε_3=-6	5	0.1	9.9	48
20	1	10	0.1	0	ε_3=-6	6	0.2	10.6	51
21	1	10	0.1	α_3=0.05	ε_3=-6	5	0.1	10.1	52
22	1	10	0.1	α_3=0.05	ε_3=-6	5	0.2	10.2	55

Table 3 provides a study of how variation in underlying parameters affects the optimal value for *AROR* for the case where there are is a mix of sunk and deferrable investment (for investment categories 1 and 3 respectively, whilst the extent of non-deferrable category 2 investment is set to zero). Give outcomes in Table 2, the results for the case where there is a mix of sunk and non-deferrable investment (categories 1 and 2 respectively, whilst the extent of deferrable category 3 investment is set to zero) are naturally very similar to those reported in Table 3 and so are omitted.

In table 3, rows 1-4 illustrate the impact of increasing the proportion of new deferrable investment relative to existing, sunk, investment. In row 1, it is 5% and this is then increased through 10, 20 to 30% new deferrable investment (setting $B_2 = 0$ and varying $B_3 = 0.05, 0.1, 0.2, 0.3$, implying $B_1 = 0.95, 0.9, 0.8, 0.7$ respectively). As a consequence, the optimal *AROR* increases from the 74th percentile at 5% new investment through to the 90th percentile when there is 30% new investment. These first four rows illustrate a fairly general point that, even with new investment being small relative to existing business, its impact on the optimal choice of *AROR* can be substantial. Even with only 5% potentially new business investment, the 74th percentile is optimal (at benchmark parameter values).

Row 5 shows the effect of reducing the marginal operating cost to zero. This reduces the *AROR* from its benchmark value of the 82nd percentile to the 74th percentile. A similar impact occurs if one reduces the rate of depreciation (γ) to zero, as per row 8. This is logical since both affect the long run margin cost equally. Rows 6 and 7 also show the impact of increasing *k* relative to *c* and hence parallel that in row 5 (where *c* is reduced to zero). These results show that greater capital to operating cost across all types of investment) tends to reduce somewhat the percentile choice for *AROR*, but that the *AROR* always remains above the 70th percentile. Rows 9-11 illustrate the importance of demand elasticity; the more elastic the demand (across all types of investment), the lower the optimal *AROR* will be. Elasticity appears to have this impact primarily because of its effect on economic welfare arising from existing business. Notice in Figure 2 that EW_1, although peaking at around the 45th percentile, is fairly flat as a function of the choice of

AROR. However, as demand becomes more inelastic, the optimal *AROR* moves to the left slightly, but more tellingly, the curvature of the function increases. This means that EW_1 has a bigger impact in determining the maximum for the function *EW* in (39) (given that there is heavy weight, 80-95%, accorded to existing sunk investment in table 3).

The impact of elasticity is even more notable if the demand associated with new investment is more elastic than that for existing business – this is illustrated in rows 19 and 20, and also 21 and 22; here, existing business has elasticity -3 whilst new (deferrable) business has elasticity set at -6. Even with 20% new / 80% existing business, the *AROR* is not much above its mean value. Elasticity plays an even bigger role in determining the optimal *AROR* here because of the impact elasticity has on the *relative* magnitude of economic welfare gain. That is, reducing elasticity from -3 to -6 reduces substantially the amount of consumer surplus that is added by new investment, and hence reduces the welfare impact relative to that of existing business (with elasticity -3).

Finally, rows 14-16 illustrate the impact of increasing the growth rate in demand from zero to 5% per annum (whilst also varying the elasticity of demand). The general effect of a higher growth rate is to reduce somewhat the optimal *AROR* (for any given mix of business and any given elasticity of demand etc.). Row 17 , 18, and 21, 22 then examine the case where there is a 5% growth rate in demand and higher elasticity (-6) for new investment compared to that for existing/sunk investment (which has 0% growth and elasticity -3). These results are consistent with those for the unilateral variations from benchmark values considered in Table 2 above.

Table 4 reports on the effects of varying the mean and standard deviation of the *WACC* distribution (holding other parameters at their benchmark values). In general, the higher the standard deviation relative to the mean, the lower the resulting optimal percentile for the *AROR* , although the impact on the absolute value of *AROR* is fairly limited. For example, in rows 6-8, the optimal *AROR* changes very little whilst the percentile drops from the 89th to the 73rd.

Table 4 : New Investment Deferrable: Optimal Values for the Allowed Rate of Return as a function of *WACC* distribution Parameters
Benchmark parameter values: $c = 1,\ k = 10,\ \gamma = 0.1,\ \alpha = 0,\ \varepsilon = -3, T = 5$
Weights: $A_1 = 0.9, A_2 = 0, A_3 = 0.1$ throughout.

Case	*Mean* μ *(%)*	*Standard Deviation* σ *(%)*	*Optimal Allowed Rate of Return* $\hat{r}*$	*Optimal Percentile* $perc(\hat{r}*)$
1	5	1.5	5.6	66
2	10	1.5	11.3	82
3	15	1.5	16.7	88
4	20	1.5	22	91
5	10	0.5	10.9	97
6	10	1.0	11.2	89
7	10	1.5	11.3	82
8	10	2.0	11.2	73

To summarize, the general conclusions are that

(i) if the *AROR* is distinguished by investment category, then it is reasonable to set a rate close to the mean value for the *WACC* distribution for existing (sunk) investments, but a much higher percentile (typically 85^{th} – 95^{th}) for new business.

(ii) where the *AROR* is applied to a mix of investment (sunk/new deferrable or non-deferrable), the choice of percentile depends on the characteristics of these investments. Even a small amount of potential new investment can induce a significant uprating in the choice of *AROR* and

(a) a higher percentile should be chosen the greater the likely amount of new investment in the *RRP*, although

(b) a lower percentile should be chosen the more elastic demand is likely to be – and particularly if demand associated with new investment is likely to be more elastic than that for existing sunk investment, and

(c) to some extent a lower percentile should be chosen the more capital intensive the investment, the more long lived the investment, and the higher the growth rate in future demand.

4. Robustness of Assumptions

The model presented in section 2 is of course stylized is various ways: it assumes constant returns to scale, a constant growth rate in demand and a constant elasticity of demand. Inevitably, any quantitative investigation requires some specification, and hence will be restrictive in one way or another. It can be argued that, whatever the details of demand structures and growth in demand over time, the essential asymmetry described in section 3 is likely to remain. For example, it is possible to construct a 'life cycle' model for demand in which there is growth, maturity and decline. However, this would not be expected to significantly alter the results obtained above. The assumption of constant elasticity demand likewise can be modified, but would not affect the essential message. A more important element that might attenuate the impact of *AROR* on economic welfare is if the scale of investment tends to reduce costs; this is addressed in more detail in section 4.2 below, whilst section 4.1 examines the possibility that options to defer are not necessarily perpetual.

4.1 Options may expire

One simple modeling extension for category 3 deferrable investment is to incorporate a probability that the option to defer investment expires in the next regulatory review period. If ρ denotes the probability that the option to invest is still available at the next *RRP*, then this modifies equation (27) to

$$EV(r_c) = \int_{r_l}^{r_c} V_f(r)\phi(r)dr + \rho\int_{r_c}^{r_u} e^{-rT} EV(r_c)\phi(r)dr \,, \tag{40}$$

and this then affects the optimal solution in equation (29), which becomes

$$r_c{*} = \arg\max_{r_c} \int_{r_l}^{r_c} V_f(r)\phi(r)dr \Big/ \left\{1-\rho\int_{r_c}^{r_u} e^{-rT}\phi(r)dr\right\}, \tag{41}$$

whilst (32) becomes

$$EW_3(r_c{*}) = \int_{r_l}^{r_c{*}} W(r)\phi(r)dr \Big/ \left\{1-\rho\int_{r_c{*}}^{r_u} e^{-rT}\phi(r)dr\right\}. \tag{42}$$

Clearly, if $\rho = 1$ the solution for category 3 is the same as that reported in section 3 above. By contrast, if $\rho = 0$, then there is no scope for deferment and the solution is as for category 2 investment. Thus as ρ is varied, the solution is intermediate those for category 2 and category 3 investment reported in section 3; indeed the relationship is fairly linear; that is, if ρ =0.5, the solution is roughly halfway between that for category 2 and category 3. For this reason a separate table is not presented for these results. The argument in section 3 was that whether investment was deferrable or not (whether it is category 3 or 2), there is a significant incentive to set *AROR* above the mean value. This is unaffected by this modeling extension.

4.2 Investment Scale effects

One might expect that falling average costs, for example arising from economies of scale in investment, might alter results.[24] For example, in a 1-period monopoly model, setting price equal to marginal cost gives not only zero profit but also a welfare optimum. By contrast, in the presence of fixed costs and/or when marginal cost falls with output, a zero profit price level lies above the welfare optimal level. Thus if average and/or marginal costs are declining, this may give some inducement toward a lower choice for the *AROR* to offset this ‘over-pricing’ impact. To explore this issue, two specifications are considered; the first introduces a simple fixed cost term, whilst in the second, there is a constant elasticity of scale (constant cost elasticity) for the initial level of investment. In this case, larger initial investment drives down marginal cost. These extensions make the price cap equation non-linear, requiring an iterative numerical solution. For this reason, in what follows only the simpler category 2 (non-deferrable) investment case is examined.[25]

(a) Scale Effect 1: Fixed costs

Here, initial investment cost is $F + kQ_0$ (cf. kQ_0 in the original analysis). This can also be interpreted as allowing some fall in marginal costs so long as this is ‘ infra-marginal’

[24] My thanks to an anonymous referee for raising this issue.
[25] There is no impact on category 1 sunk cost investment in any case, and the expected welfare function for category 3 investment is likely to continue to closely mimic that for category 2.

in the sense that the choice of Q_0 does not affect the subsequent marginal cost of follow-on investment. As *F* is increased, the regulator needs to increase $\hat{p}$ (to cover the additional costs) and this impacts on the subsequent decision by the firm on whether to invest or not. The consequence of increasing *F* is that it tends to reduce the optimal choice of *AROR*. Table 5 illustrates the impact for parameters at benchmark values and strength of demand parameter *A* set to 1000 – essentially, as *F* is increased, the optimal choice for *AROR* is driven down, eventually reaching the level for category 1 sunk investment (at benchmark parameter values, this was the 46th percentile). However as this level for $Perc(\hat{r})$ is approached, the firm is likely to not invest at all – in table 5, if *F* is raised much above 160, expected economic welfare collapses, as the firm chooses never to invest.

Table 5 –Non-deferrable Investment: Impact of fixed cost *F* on the optimal percentile for *AROR*, $Perc(\hat{r})$

F	EW_2	$Perc(\hat{r})$
0	566.16	86
10	554.06	85
50	500.59	82
100	415.38	77
150	263.7	61
160	206.31	53

(b) Scale Effect 2:- the constant cost elasticity case

This specification allows investment scale to impact on the subsequent marginal cost of follow on investment. The initial capital cost term, kQ_0, is replaced with the cost function $C(Q_0) = kQ_0^{\eta}$ where η is the cost elasticity (cf. Evans and Guthrie [2006]). The marginal cost after the initial installation is then assumed constant, and determined as the marginal cost of the last unit of initial investment (as $k\eta Q_0^{\eta-1}$).[26] Again focusing on category 2 investment, adjusting value equations and re-optimizing, a similar effect is found – that is, as the cost elasticity is reduced, so the optimal choice of *AROR* tends to

[26] So $\eta = 1$ corresponds to the constant unit capacity cost case.

reduce. However, in this case the effect is not monotonic – as the cost elasticity is reduced, eventually the effect reverses. The point at which the effect reverses depends on the level of demand (see Table 6).

Intuitively, as the cost elasticity is first reduced, this creates the same 'fixed cost effect' as in section (a) above, the same need to raise price above marginal cost in order for the firm to break even, with the attendant welfare loss from this level of 'over pricing'. The fact that the marginal cost of subsequent investment is falling also probably increases this effect. However, as the cost elasticity is further reduced, this diminishes the overall significance of capital costs, and also the impact on the subsequent marginal cost of capacity (since this is now close to zero). For these reasons, the optimal *AROR* returns to the level set where there are no significant returns to scale.

Table 6: Non-deferrable Investment: Impact of cost elasticity η and level of demand *B* on the optimal percentile for *AROR*, $Perc(\hat{r})$

η	*B*=1000	*B*=100	*B*=50	*B*=20
1	86	86	86	86
0.9	84	83	82	82
0.8	84	81	81	78
0.7	87	81	78	74
0.6	91	83	79	69
0.5	94	86	83	70

To sum up, it appears that scale effects can reduce to some extent the optimal value for *AROR*. However, it is important to note that it does not eliminate at all the significant welfare loss asymmetry described in figure 2 above. That is, even when the optimal *AROR* is reduced somewhat, when there is uncertainty over its precise value, it remains the case that the welfare loss arising from setting an *AROR* above the optimal value too high is significantly less than that which can arise from setting it below.

5. Conclusions

In current UK and EU regulatory practice, the estimate of what the appropriate allowed rate of return, *AROR*, should be within the forthcoming regulatory review period plays an important role in determining price controls and revenue requirements. This paper focuses on the problem of setting a fixed allowed rate of return for the duration of a fixed regulatory review period, given that this is but the first of an ongoing sequence of review periods and that the allowed rate of return influences price caps and controls. When the *AROR* is set 'too low' relative to the welfare maximizing level, this tends to result in under investment and under pricing, whilst if it is set too high, this tends to give rise to over-investment and over pricing.

There are two reasons for setting an *AROR* above the mean value of the *WACC* distribution – firstly, because the value that maximizes economic welfare generally lies to the right of the mean of the *WACC* distribution – and secondly, because expected economic welfare is an asymmetric function; given the precise value of the optimal *AROR* is uncertain, for each percentage point the *AROR* is inadvertently set above the optimum, the welfare loss is less than that which arises from setting it an equal number of percentage points too low. It follows that the allowed rate of return on *new* investments should generally be set at a significantly higher percentile value of the *WACC* distribution – that is, at percentile values in the high 80s or 90s. Where the *AROR* is likely to be applied to business which involves a mix of both new and old assets, the proportions of sunk *vis a vis* new investment potential within the *RRP* will naturally influence the extent of uplift in the optimal choice of *AROR* compared to the *WACC* mean. However, the asymmetry in the welfare function for new investment (*vis a vis* that for sunk investment) is so strong that even if the proportions of potential new investment are quite small, this can still induce a significant uplift in the optimal choice for the *AROR* compared to the *WACC* mean.

References

Australian Consumer and Competition Commission, 2005, Assessment of Telstra's ULLS and LSS monthly charge undertakings, Draft decision, August 2005, Appendix C. Available at www.accc.gov.au

Baron D. and Myerson R., 1982, Regulating a monopolist with unknown cost, ***Econometrica***, 50, 1231-1262.

Besanko D. and Spulber D., 1992, Sequential equilibrium investment by regulated firms, ***Rand Journal of Economics***, 23(2) 153-170.

Bowman R.G., 2004. "Response to *WACC* issues in commerce – Commissioner's draft report on the Gas control Enquiry." Report prepared for PowerCo, June. Available at: www.comcom.govt.nz/RegulatoryControl/GasPipelines/ContentFiles/Documents/Submission%20-%20Powerco%20WACC%

Bowman R.G., 2005, Queensland Rail – Determination of regulated WACC, Report, available at www.qca.org.au/www/rail/Sub_QRattach7_2005%20DAU%20Draft.pdf

Brealey R. and Franks J., 2009, Indexation, investment and utility prices, ***Oxford review of economic policy***, 25(3) 435-450.

Brennan M.J. and Schwartz E.S., 1982, Consistent regulatory policy under uncertainty, ***Bell Journal of Economics***, 13(2): 506-521.

BT, 2005, Ofcom's approach to risk in the assessment of the cost of capital : BT's response to the Ofcom consultation document, 5/4/2005, available at http://www.btplc.com/responses.

Butler J.S. and Schachter B., 1989, The investment decision: Estimation risk and risk adjusted discount rates, ***Financial Management***, (4)13-22.

CAA, 2008, Economic Regulation of Heathrow and Gatwick Airports 2008 -2013 – CAA Decision, available at http://www.caa.co.uk/docs/5/ergdocs /heathrowgatwickdecision_mar08.pdf

Competition Commission, 2007, BAA Ltd : A report on the economic regulation of the London airports companies (Heathrow Airport Ltd and Gatwick Airport Ltd) available at http://www.competition-commission.org.uk/

Dixit A. and Pindyck R., 1994, ***Investment under Uncertainty***. Princeton: Princeton University Press.

Dobbs I.M., 2004, Intertemporal price cap regulation under uncertainty, ***Economic Journal***, 114, 421-440.

Dobbs I.M., 2008, Setting the regulatory allowed rate of return using simulation and loss functions – the case for standardizing procedures, ***Competition and Regulation of Network Industries***, 9(3), 229-246

Evans L.T. and Guthrie G.A., 2005, Risk, price regulation and irreversible investment, ***International Journal of Industrial Organization***, 23, 109-128.

Evans L.T. and Guthrie G.A., 2006, Incentive regulation of prices when costs are sunk, ***Journal of Regulatory Economics***, 29(3)239-64.

First Economics, 2007. "Automatic annual adjustment of the cost of capital: A discussion paper." 30 March 2007. Available at www.xfi.ex.ac.uk/ conferences/costofcapital/papers/earwaker_automatic_adjustment_ of_the_cost_of_capital.pdf

Guthrie G.A., 2006, Regulating infrastructure: the impact of risk and investment, ***Journal of Economic Literature***, 44, 925-972.

Judd K.L., 1999, ***Numerical methods in economics***, MIT press, Cambridge, Mass.

Leland H., 1974, Regulation of natural monopolies and the fair rate of return, ***Bell Journal of Economics and Management Science***, 5(1) 3-15.

Lewis T.R.and Sappington D.E., 1988, Regulating a monopolist with unknown demand and cost functions, ***Rand Journal of Economics***, 19, 438-457.

McDonald R. and Siegel D., 1986, 'The value of waiting to invest', ***Quarterly Journal of Economics***, vol. 101, pp. 707-728.

Newbery D., 1999, ***Privatization, Restructuring and Regulation of Network Industries***, MIT Press, Cambridge, Mass.

New Zealand Commerce Commission, 2004, Gas Control Inquiry Final Report, 29 Nov., 2004, available at www.med.govt.nz/ers/gas/final-report/final-report.pdf.

Ofcom, 2005, Ofcom's approach to risk in the assessment of the cost of capital, 26/1/2005. Available at: http://www.ofcom.org.uk/consult/condocs/cost_capital2/

Pindyck R.S., 1988, Irreversible investment, capacity choices and the value of the firm, ***American Economic Review***, 78, 969-985.

PWC, 2006, TenneT TSO Comparison study of the WACC, Report to TenneT, May 8, 2006. Available at http://www.dte.nl/images/Comparison%20study%20of%20the%20WACC-%20Mei%202006_tcm7-87013.pdf

SFG, 2005, A framework for quantifying estimation error in regulatory WACC. Report for Western Power in relation to the Economic Regulation Authority's 2005 Network Access Review, 19/5/2005. Available at: http://www.wpcorp.com.au/documents/AccessArrangement/2006/Revised_AAI_Appendix_4_WACC_SFG_MAY2005.pdf

Vickers J. and Yarrow G., 1988, ***Privatization: An economic analysis***, MIT Press, Cambridge, Mass.

Wright S., Mason R., Miles D., 2003, A study of certain aspects of the cost of capital for regulated utilities in the U.K., 13/2/2003. The Smithers &Co. report commissioned by the UK regulators and the office of Fair Trading. Available at: www.ofcom.org.uk/static/archive/oftel/publications/pricing/2003/capt0203.pdf .

APPENDIX

To show that the optimal solution for category 2 investment can lie *anywhere* in the interval $\left[r_l, r_u\right]$, it suffices to focus on a special case where $T \to \infty$, $\alpha = \gamma = c = 0$, and the density function is a uniform distribution. In this case, $r_a = \hat{r}$ and $\phi(r)$ is a constant so the first order condition for the optimization problem in (25) simplifies to $\left(\partial / \partial \hat{r}\right) \int_{r_l}^{\hat{r}} W(r, \hat{r}) dr = 0$. The optimal choice, $\hat{r}^*$ must satisfy the equation:

$$\frac{\partial}{\partial \hat{r}} \int_{r_l}^{\hat{r}} \left(\frac{B\hat{p}(\hat{r})^{\varepsilon}}{r} \left\{ \frac{\varepsilon \hat{p}(\hat{r})}{1+\varepsilon} \right\} - kB\hat{p}(\hat{r})^{\varepsilon} \right) dr = 0$$

Since $\hat{p} = \hat{r}k$ in this special case, this means

$$\frac{\partial}{\partial \hat{r}} \left\{ B\left(\hat{r}k\right)^{\varepsilon} \left[\frac{\varepsilon \hat{r}k}{1+\varepsilon} \ln(\hat{r}/r_l) - k(\hat{r}-r_l)k \right] \right\} = 0$$

$$\Rightarrow \quad \frac{\partial}{\partial \hat{r}} \left\{ \hat{r}^{\varepsilon} \left[\frac{\varepsilon \hat{r}}{1+\varepsilon} \ln(\hat{r}/r_l) - (\hat{r}-r_l) \right] \right\} = 0$$

$$\Rightarrow \quad \varepsilon \hat{r}^{\varepsilon-1} \left[\frac{\varepsilon \hat{r}}{1+\varepsilon} \ln(\hat{r}/r_l) - (\hat{r}-r_l) \right] + \hat{r}^{\varepsilon} \left[\frac{\varepsilon}{1+\varepsilon} \left[\ln(\hat{r}/r_l) + 1 \right] - 1 \right] = 0$$

This implies the optimal value $\hat{r}^*$ satisfies

$$\ln(\hat{r}^*/r_l) + \left(r_l/\hat{r}^*\right) - 1 = \frac{1}{\varepsilon\left(1+\varepsilon\right)}$$

Now, as demand elasticity $(\varepsilon < -1)$ is varied, the right hand side can vary from 0 (as $\varepsilon \to -\infty$) to $+\infty$ (as $\varepsilon \to -1$), so $\hat{r}^* \to r_l$ when demand becomes extremely elastic, whilst as it becomes more inelastic, the value for the RHS increases, and eventually, $\hat{r}^* \to r_u$, the upper limit.