

**THE ASSESSMENT OF MARKET POWER AND MARKET BOUNDARIES
USING THE HYPOTHETICAL MONOPOLY TEST**

by

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ABSTRACT

Over the last two decades or so, the ‘Hypothetical Monopoly Test’ has become central to the assessment of market boundaries and market power (an essential preliminary step in many forms of regulatory enquiry). However, to date, there has been no clear-cut statement of how the test should be operationalised in a general context (the test has usually been implemented in either a heuristic way, or in simple contexts in which cross-price effects are unimportant). This paper presents a set of results which clarify how the test can be operationalised – along with an algorithm for conducting the test in cases where there is adequate market data (price cost data and cross price elasticities).

1. INTRODUCTION

Competition and regulatory policy decisions often depend critically on assessments of market power, and of what is interpreted as the ‘relevant market’. For example, a wider interpretation implies a larger ‘market’, smaller market shares for individual firms and lower implied levels of market dominance. Following the publication of the US Department of Justice Merger Guidelines (DOJ [1984]), the concept of a ‘hypothetical monopoly test’ (HMT) has become a cornerstone for assessing market power and for defining the ‘relevant market’. In practice, the tendency has been to use intuitive judgement in deciding whether this test is passed or not (US DoJ [1984], OFT [1998], NERA [1992]). Whilst there has been some econometric work seeking to test the boundaries of trading markets by looking at price correlations across market segments (e.g. Stigler and Sherwin [1985], Slade [1986]) and on using demand elasticity information to help predict the consequences of merger when firms practice Bertrand-Nash competition (Hausman, Leonard and Zona [1994]), only recently has there been some work concerned with the direct implementation of the HMT - and this has been primarily concerned with the assessment of critical elasticity or critical sales loss for the single product case or for cases of price discrimination where cross price elasticities are zero (see e.g. Werden [1998], Hausman, Sidak and Singer [2001], Massey [2001], Langenfeld and Li [2001]). To our knowledge, there has been little formal analysis of how cross-price elasticity information should be used in applying the HMT. This motivates the present paper, which focuses on this general case. Sections 2 and 3 establish *necessary and sufficient conditions* associated with the HM test and examine how these can be translated into a computational algorithm for the study of market power and market boundaries. Section 4 follows this by

illustrating the approach using numerical examples (UK letters business; US low-alcohol beer market; a further study featuring chains of substitution between markets). Section 4 also illustrates the value of the approach for sensitivity analysis. Since there are always uncertainties associated with the data inputs, robustness to parameter variation is studied (and ways of establishing confidence intervals, through simulation, are also briefly discussed). Section 5 concludes.

The rest of this introduction presents the basic definition of the HMT, along with discussion of some operational issues that arise with respect to it. We do not review the intrinsic logic, or the strengths or weaknesses associated with the test as these have been rehearsed elsewhere (see e.g. Hausman, Leonard, Velluro [1996], Morris and Mosteller [1991], NERA [1992], Evans and Schmalensee [2001]). Section 2 develops the algorithm for identifying a ‘relevant market’, and section 3 discusses non-uniqueness arising from the definition and related matters. Section 4 gives some numerical illustrations and section 5 draws together the conclusions.

The literature has broadly distinguished three types of market; the trading market, the strategic market and the anti-trust market (Geroski [1997]). The *strategic market* is defined as the smallest viable area of economic activity; this is of interest to business decision makers who are rationally concerned with developing and marketing products and services, with defining and shaping ‘the business they are in’. The traditional economist’s definition of a market is the *trading market*, identified by the fact that the goods traded in it are sufficiently homogenous that the ‘law of one price’ holds. The third concept of a market, the *anti-trust market*, is motivated by regulatory and competition authority interest in identifying sets of potentially

differentiated products over which firms might be able to exert monopoly power. As previously discussed, the focus of the present paper is on the last of these, and on the now widely used '*hypothetical monopoly test*' (HMT) as a mechanism for identifying the market boundary. This test is explained in the US department of Justice horizontal merger guidelines (DOJ [1984]) as follows:

“Formally, a market is defined as a product or a group of products and a geographical area in which it is sold such that a hypothetical, profit maximising firm, not subject to price regulation, that was the only present and future seller of those products in that area would impose a '*small but significant and non-transitory*' increase in price above prevailing or likely future levels.” (My italics)

The HMT is often referred to, acronymically, as a SSNIP test (SSNIP deriving from the words in italics in the above definition). This definition suggests that, for the set of products or services in the market, it is profitable to raise prices. The guidelines clarify this basic statement by further proposing that the test should be a 5% SSNIP test of whether the hypothetical monopoly could profitably raise prices by 5% across the board. The guidelines further specify that 'non-transitory' should be interpreted as a period of at least a year and also recognise that the above definition may give rise to non-uniqueness over what constitutes the relevant market; regarding the latter problem, it is suggested that the relevant antitrust market should be defined as the “narrowest set of goods or services which could be successfully monopolised.” This point will be returned to in section 3 below.

There are a variety of practical issues concerning how the HMT might be implemented (a useful survey is given in NERA [1992]). In particular, the following are fairly critical:

1. Why a 5% price increase? Why not more, or less?
2. What base or benchmark prices should be used?

3. Should the test be based on prices - or price cost margins?
4. Why choose a period of one year duration?

The choice of 5% is pragmatic, since some market power exists if there is any alternative price vector in which prices are above the benchmark prices for which profitability is higher. This point is considered further in sections 2 and 3 below which examine in detail how it is possible to establish a **ZERO%SSNIP test** and an **α %SSNIP test** (where $\alpha > 0$ can be freely chosen). The ZERO%SSNIP test merely requires that it is profitable to raise prices by some (possibly arbitrarily small) amount; the α %SSNIP test, by contrast requires that it is profitable to raise prices by at least α %.

The choice of benchmark prices is also critical. Consider for example the case of a single product monopolist; if the price is currently the monopoly price, then no further price increase will increase profitability. It follows that, if the prices of the set of products under consideration already reflect monopoly power, the scope for further price increases will clearly be reduced. In such a case, testing market boundaries at existing prices as the benchmark may be inappropriate. Thus, in enquiries considering monopoly or anti-competitive practices, existing prices are a suspect benchmark (and some assessment of the competitive level for prices may be required). By contrast, in considering mergers, a benchmark of existing prices is reasonably logical, since the concern is one of assessing whether the merger would lead to an increase in monopoly power. Whilst the choice of benchmark is clearly an issue, the concern of the present paper lies in developing a computational algorithm which can be used to define market boundaries with respect to *any* initial choice of benchmark prices.

The DOJ guidelines apply the HM test to gross prices. An obvious difficulty with this definition is that it treats asymmetrically firms that are more, or less, vertically integrated. Some commentators have suggested that it might be better to apply the test to price-cost margins (e.g. NERA 1992]). It is straightforward to adapt the analysis presented below to provide an algorithm for this form of the test. Finally, duration is not explicitly modelled, except in so far as it is intrinsically captured in the estimation of demand elasticities. That is, the econometric analysis of demand might be developed in a way which usefully distinguishes short run and long run demand elasticities. Clearly, the choice of duration could alter estimates of elasticities, and this in turn would affect the assessment regarding the size of the market (the shorter the choice of duration, the narrower the market will appear to be).

The above discussion serves to clarify the essential fact that market delineation is intrinsically dependent on the underlying purpose of the enquiry (whether it be concerned with merger or some other particular form of anti-competitive practice).

The next section develops the implications of the ZERO%SSNIP test and provides the basis for a computational algorithm for establishing 'what the relevant market is', conditional on information on demand elasticities, benchmark prices and marginal costs. Section 4 then extends this analysis to the case of the α %SSNIP test (where $\alpha > 0$).

2. IMPLICATIONS OF THE HYPOTHETICAL MONOPOLY TEST

A major initial assumption is that it is possible to identify the set of products which is subject to investigation. This is not as straightforward as it might appear; for

example, a market which might at first sight appear to be ‘homogenous’ may in fact be segmentable in various ways (for example, by geography or customer type).

However, it is hard to imagine how one could proceed to an analysis of whether different products belong to the same market or not if one is unable to distinguish them in some way in the first place. It is also assumed that, in the process of ‘grouping’ a set of products in order to test whether there is market power with respect to that set, no change is allowed in the way these products are manufactured and distributed; that is, cost structures are unaffected by the process of ‘grouping’.¹

The products under investigation are labelled $1, \dots, n$, such that $N = \{1, \dots, n\}$ denotes the set of product identifiers (hereafter referred to simply as a ‘set of products’).

Conceptually, the demand for each product can be thought of as a function of all the prices $\mathbf{p}' = (p_1, \dots, p_n)$, so that the i^{th} demand can be written as $q_i(\mathbf{p})$ for $i \in N$. In what follows, it is assumed that these demand functions are twice continuously differentiable.

Suppose there is an arbitrary initial price vector, \mathbf{p} at which the concept of a market is to be tested. In a given application this might take the form of extant prices (if the assessment is concerned with a merger analysis) or with some estimation of what prices would be if the market was to an extent ‘competitive’ (for example, marginal cost prices, Nash Bertrand, Cournot etc.). The hypothetical monopoly test looks at the smallest sub-set of products for which a hypothetical monopolist, if in control of that

¹ Thus the possibility that grouping products might alter the way these products are produced - and hence the costs, say because of economies of scale or scope, is ignored.

sub-set, could profitably proportionately raise prices. The focus of the present section is on this standard form of the test (the case where price changes are not restricted to proportionate increases is discussed in the appendix). Since the focus is on relatively small price changes, it is reasonable to assume that, for the range of outputs under consideration, the marginal cost c_j of producing an additional unit of output of each product indexed $j \in N$ does not vary with output.

Suppose that an index subset $K \subseteq N$ identifies the set of products which does indeed satisfy the HM test. For K to be the “market” according to the hypothetical monopoly test, it must be the case that a proportionate rise in prices for this subset of products increases profitability. Thus, consider a price change from \mathbf{p} to $\bar{\mathbf{p}}$ such that

$$\bar{p}_j = (1+z)p_j, \quad j \in K, \text{ and some } z > 0 \quad (1)$$

This induces a change in quantities in the “market”; specifically, taking an exact Taylor series expansion with respect to z ,

$$q_j(\bar{\mathbf{p}}) = q_j(\mathbf{p}) + \left(\sum_{m \in K} \frac{\partial q_j(\mathbf{p})}{\partial p_m} p_m \right) z + O(z), \text{ for } j \in K \quad (2)$$

where $O(z)$ is the remainder term having the property that $O(z)/z \rightarrow 0$ as $z \downarrow 0$.

Assuming the marginal costs of production are constant in the relevant region, it is useful to define the ‘quasi’ profit function as

$$\Pi^K = \sum_{k \in K} (p_k - c_k) q_k(\mathbf{p}). \quad (3)$$

This function can be used to accurately measure *changes* in profit when the price vector is changed from \mathbf{p} to $\bar{\mathbf{p}}$ as defined by (1).² Thus, again taking an exact Taylor series expansion with respect to z , the overall change in profit can be written as (see appendix)

$$\Delta\Pi^K(z) = z \sum_{k \in K} \left(p_k q_k(\mathbf{p}) + (p_k - c_k) \sum_{m \in K} \frac{\partial q_k(\mathbf{p})}{\partial p_m} p_m \right) + O(z). \quad (4)$$

It is useful, in what follows, to define cross price elasticities, at the current price vector \mathbf{p} , using the standard notation $\eta_{ab} = \frac{\partial q_a(\mathbf{p})}{\partial p_b} \frac{p_b}{q_a(\mathbf{p})}$ for all $a, b \in N$ so that

$q_k(\mathbf{p})\eta_{km} = \frac{\partial q_k(\mathbf{p})}{\partial p_m} p_m$, and to define the price cost mark-up on the k^{th} product as

$$m_k \equiv (p_k - c_k) / p_k \quad (5)$$

The ZERO%SSNIP test

This sub-section presents some basic results for the ZERO%SSNIP test; this is of interest for two reasons; firstly, because it may be unclear what percentage level should be chosen in the test – and secondly, because the results can be obtained under slightly weaker assumptions than those required to establish a test for a positive percent level. Define:

$$\Delta_K^0 \equiv \sum_{k \in K} p_k q_k \left(1 + m_k \sum_{m \in K} \eta_{km} \right). \quad (6)$$

This function plays the key role in operationalising the ZERO%SSNIP test (the superscript ‘0’ identifies this as relevant for a ZERO%SSNIP test; the notation Δ_K^α then being used for the case of the α %SSNIP test below). It follows from (4)-(6) that

² A general nonlinear total cost function for the products in set K , of the form $c(\mathbf{q})$, which is linear on a region $Q \subset \mathbb{R}^n$, can be written as $c(\mathbf{q}) = F + \sum_{k \in K} c_k q_k$ for $q \in Q$. Since the concern is

$$\Delta\Pi^K(z) = \Delta_K^0 z + O(z). \quad (7)$$

Clearly, $\Delta\Pi^K(0) = 0$ and for sufficiently large z , $\Delta\Pi^K(z) < 0$; that is, at the initial price vector, by assumption, there are positive profits, whilst if prices are raised sufficiently high, nothing will be sold.

Given the assumptions regarding the demand functions $q_i(\mathbf{p})$, the function $\Delta\Pi^K(z)$ will be continuously differentiable; however, without imposing further restrictions on demands, fairly clearly the function $\Delta\Pi^K(z)$ can be non-linear in a way that is hard to characterise. This can pose problems for SSNIP tests, as illustrated in figure 1.

Figure 1 here

Panels (a) and (b) illustrate problematic cases for $\alpha\%$ SSNIP tests ($\alpha > 0$). In panel (a) profitability falls for small increases (such as 5%) but profit can increase for larger price increases (such as 10%). Thus there is no market power under a 0% or 5% SSNIP test, but there is under a 10% test. Panel (b) illustrates the case where a small price increase is profitable but larger increases may or may not be (a 0% and a 10% test show market power, whilst a 5% test does not). In what follows, the aim is to establish a range of results (by tightening the underlying assumptions regarding the demand system) which should prove helpful in the conduct of market power assessments.

A sufficient condition for there to exist some market power can be based on the ZERO% SSNIP test. From (7), the gradient of $\Delta\Pi^K$ at $z=0$ is

$$d\Delta\Pi^K(0)/dz = \Delta_K^0 \quad (8)$$

solely with changes in profit, the ‘fixed’ cost F arising from inframarginal product can be ignored.

so the sign of Δ_K^0 indicates whether an initial raising of prices is profitable or not.

This implies the following sufficient condition:

Proposition 1: A hypothetical monopolist can profitably raise prices for a product set K if $\Delta_K^0 > 0$ (the ZERO%SSNIP test).

Proof: Since $\Delta\Pi_K(0) = 0$ and $d\Delta\Pi_K(0)/dz = \Delta_K^0$, if $\Delta_K^0 > 0$, then there exists an interval $(0, \delta)$ for some $\delta > 0$ on which $\Delta\Pi_K(z) > 0$

Unfortunately, finding $\Delta_K^0 < 0$ does not preclude the possibility that higher increases in price might be profitable (as in panel (a) in figure 1). This does not appear unduly promising - but consider panels (c) and (d), where $\Delta\Pi_K(z)$ is depicted as a concave function of z - it is clear that, under concavity, the sign of Δ_K^0 will suffice to identify uniquely whether market power exists or not. In what follows, it is argued that concavity is typically a reasonable assumption to make for the function $\Delta\Pi_K(z)$.

A fairly common assumption in the mathematical analysis of multi-product pricing is that the profit function is concave in prices. This assumption implies that, when the first order conditions are satisfied, they do indeed identify a global maximum solution. Clearly, if it is assumed that the profit function (3) is concave in prices, then this also implies that the function $\Delta\Pi_K(z)$ is concave in z (a proof is given in the appendix). It is also possible to point to special cases where the profit function is indeed concave. For example, if the multi-product linear demand system is linear in the relevant region around the extant price vector, then it can be represented as

$$\mathbf{q} = \mathbf{B} + \mathbf{A}\mathbf{p}, \quad (9)$$

where \mathbf{B} is a column vector and \mathbf{A} , a square matrix of constants. It is then straightforward to show that the profit function Π_K will be concave in prices if the matrix $\mathbf{A} + \mathbf{A}'$ is negative semi-definite. A sufficient condition for this to be the case is that the matrix \mathbf{A} has a row and column dominant diagonal; that is, if own price effects outweigh, in absolute value, the sum of the other cross price effects. This latter assumption is fairly common in theoretical analysis, and is usually satisfied in empirical applications.³ SSNIP tests typically involve relatively small changes in price (5 or 10 percent at most); thus equation (9) will often prove to be a good local approximation for the demand system in this neighbourhood of the benchmark price vector \mathbf{p} ; the further assumption that own price effects outweigh cross price effects then entails that this system will indeed be concave in prices. This in turn entails that $\Delta\Pi_K(z)$ is concave in z (see appendix).

The assumption of concavity allows the following necessary and sufficient conditions to be stated for the ZERO%SSNIP HM test:

Proposition 2: If profit functions are concave in prices, then a set of products K is a ‘relevant market’ according to the ZERO%SSNIP hypothetical monopoly test if and only if

- (i) $\Delta_K^0 > 0$, and
- (ii) $\Delta_L^0 \leq 0$ for all $L \subset K$

Proof: See appendix

³ In practice, the demand system here will comprise only a (probably fairly small) sub-set of ‘all products’. This is so because any regulatory or anti-trust investigation will be concerned with some ‘industry grouping’ (such as ‘telecommunications’). It follows that properties which arise at the individual level (notably that demands are homogenous of degree zero in money and prices) imply no useful restrictions on the structure of the above functions.

The logic for proposition 2 can easily be seen from figure 1 panels (c) and (d). Under concavity, it is clearly a necessary and sufficient condition for there to be *market power* for the set of products K that the gradient of $\Delta\Pi_K(z)$ at $z=0$ be positive (as in panel (c)) whilst, for K to be a *smallest subset*, we need to rule out price increases being profitable (for *any* level of price increase) for *any* non-null proper sub-set of K . Referring to figure 1 panel (d), condition (ii) does this; if $\Delta_L^0 \leq 0$, then $\Delta\Pi_K(z) \leq 0$ for *all* $z>0$. Note, as a corollary, that if $\Delta_L^0 > 0$ for any non-empty set $L \subset K$, then this is a sufficient condition to rule out K as a smallest sub-set of products for which there is market power - naturally, L then becomes a candidate for being a smallest subset and hence a market under the ZERO%SSNIP test.

Conditions (i) and (ii) in proposition 2 thus provide a method for computing market boundaries. Note also that even if no assumptions are made regarding the behaviour of the function $\Delta\Pi_K(z)$, the sufficient condition for market power outlined in proposition 1 should still prove useful in the exploration of market boundaries.

Given benchmark prices, quantities, estimates of marginal costs and the matrix of cross price elasticities, it is possible to check proposition 2 conditions (i) and (ii), and hence to establish market boundaries according to the HMT. The main drawback is that, as the number of products/services/regions increases, the number of potential markets increases ‘factorially’, and hence the computational burden can become significant when a large number of products is involved. However, in practice, the availability of the necessary quantitative information (in particular elasticity

information) is more likely to be the binding constraint when a large number of products are involved.

A computational algorithm⁴ was developed to operationalise the use of proposition 2 conditions (i) and (ii). The program involves computing Δ_K^0 using (6) for all possible subsets of the total product set N - namely $\{1\}, \dots, \{n\}, \{1,2\}, \{1,3\}, \dots, \{1,n\}, \{2,3\}, \{2,4\}, \dots, \{2,n\}, \dots, \{1,2,3\}, \{1,2,4\}, \dots, \{1,2, \dots, n\}$. Those sub-sets for which $\Delta_K^0 > 0$ are then identified - and finally, the smallest subsets are calculated; that is, if the subsets which have $\Delta^0 > 0$ are denoted $L_i, i = 1, 2, \dots, m$, then all sets i for which $L_i \supset L_j$ for some $j \in 1, \dots, m$ are dropped. Some numerical examples based on this algorithm are discussed in section 4 below.

The α %SSNIP Test

The above analysis was solely concerned with whether a grouping of products constituted a relevant market under the zero%SSNIP HMT, where this merely requires that it is profitable to raise prices. No condition was placed on the extent of the price increase to be imposed. Given that in practice, regulators and Judges often wish to focus on some particular magnitude of proportional price increase (notably 5%), it is natural to consider whether the above approach can be adjusted to provide such a test. With general non-linear demand functions, it is naturally difficult to establish a simple form of test. However, if it can be assumed that demand functions are linear in the appropriate region on which the price change is being considered, it

⁴ Developed as a FORTRAN program, for both zero and α %SSNIP tests. The code for this is available from the author on request.

turns out to be possible to establish an appealing simple criterion. First, define the functions

$$\psi_K \equiv \sum_{k \in K} (q_k P_k \sum_{i \in K} \eta_{ki}) \quad (10)$$

and

$$\Delta_K^\alpha \equiv \Delta_K^0 + \alpha \psi_K. \quad (11)$$

where, for example, $\alpha = 0.05$ for a 5% SSNIP test. The following is then a parallel result to that obtained for the zero% case:

Proposition 3: If profit functions are concave in prices and demand functions are linear in the region of the price change, then a set of products K is a ‘relevant market’ according to an $\alpha\%$ SSNIP hypothetical monopoly test if and only if

- (i) $\Delta_K^\alpha > 0$
- (ii) $\Delta_L^\alpha \leq 0$ for all subsets $L \subset K$,

Proof: See appendix

The proof is based on the fact that, given the assumptions, the function $\Delta \Pi^K(z)$ is quadratic in z and can in fact be written as

$$\Delta \Pi^K(z) = \Delta_K^0 z + \psi_K z^2. \quad (12)$$

Thus, setting $z = \alpha (>0)$, clearly, $\Delta \Pi^K(\alpha) \geq 0$ as $\Delta_K^\alpha \equiv \Delta_K^0 + \alpha \psi_K \geq 0$, from which proposition 3 then immediately follows. A final point worth remarking (see appendix) is that, for the profit function to be concave when demands are linear in the relevant price region, it must be that $\psi_K \leq 0$ (and strictly negative, for strict concavity). This condition is worth checking prior to any application of the test outlined in the above propositions.

3. CRITICAL SALES LOSS, CRITICAL ELASTICITY, CRITICAL MARK-UP AND CRITICAL PRICE CHANGE

For the single product case, *Critical sales loss* is defined as the % change in sales that would make the $\alpha\%$ SSNIP test just marginal. Again in the single product case, the *critical elasticity* is the value of the own price elasticity of demand that would make the $\alpha\%$ SSNIP test just marginal. These concepts have been discussed and applied in recent work (see e.g. Werden [1998], Hausman, Sidak and Singer [2001], Massey [2001], Langenfeld and Li [2001]). On the same line of reasoning, it is worth also defining the critical mark-up and the critical price change. The *critical mark-up* being defined as the level of mark-up which would just make the test marginal, while the *critical price change* would be the value of α that just makes the test marginal.

These concepts reflect a standard and useful form of sensitivity analysis in which each parameter is unilaterally adjusted so as to make the result marginal. Such calculations are helpful in assessing the robustness of the result. In the case where there are several products being investigated, it is possible to undertake the same form of calculations for each parameter (including cross price elasticities etc.) although the number rapidly proliferates and the usefulness of the exercise diminishes. However, it is worth remarking that one variation is always worth exploring in both single and multi-product cases – and that is the *critical price increase* – the value of α that just makes the test marginal. The rest of this section is concerned with computing these critical values.

In the single product case, the $\alpha\%$ SSNIP test in proposition 3 above simplifies as follows. Firstly, the function Δ_K^0 in (6) simplifies to $\Delta^0 \equiv pq(1+m\eta)$ where p is the

price, q the sales, m the mark-up and η the own price elasticity for this single product, whilst the function ψ_k in (10) simplifies to $\psi = pq\eta$. Hence $\Delta\Pi^k(\alpha)$ simplifies to

$$\Delta\Pi(\alpha) = pq \left[(1 + m\eta)\alpha + \eta\alpha^2 \right]. \quad (13)$$

Critical Elasticity

The critical elasticity, denoted η^{crit} , is the value of the elasticity which just makes the test marginal ($\Delta\Pi = 0$). Hence

$$\Delta\Pi(\alpha) = pq \left[(1 + m\eta^{crit})\alpha + \eta^{crit}\alpha^2 \right] = 0 \quad (14)$$

and so

$$1 + m\eta^{crit} + \eta^{crit}\alpha = 0 \Rightarrow \eta^{crit} = \frac{-1}{m + \alpha} \quad (15)$$

Critical Sales Loss

The critical sales loss, denoted CSL , is that percentage of sales loss that would make the test marginal. Since $\eta^{crit} = CSL/\alpha$, it follows that

$$CSL = \frac{\alpha}{\eta^{crit}} = \frac{-\alpha}{m + \alpha}. \quad (16)$$

As previously remarked, these concepts have found practical application (Hausman et al [2001], Massey [2001], Langenfeld and Li [2001]). The other concepts have not been discussed in the previous literature, but are of equal interest – namely the ‘critical mark-up’ and the ‘critical percentage price increase’. This is particularly so because uncertainty is rarely confined solely to the elasticity estimates; marginal costs, and hence mark-ups are often of equal concern.

Critical Mark-up

The critical mark-up, m^{crit} , is defined by

$$\Delta\Pi(\alpha) = pq \left[(1 + m^{crit}\eta)\alpha + \eta\alpha^2 \right] = 0 \quad (17)$$

and hence

$$m^{crit} = -\left(\alpha + \frac{1}{\eta}\right) \quad (18)$$

Critical percentage price increase

The critical percentage price increase is defined by

$$\Delta\Pi(\alpha^{crit}) = pq \left[(1 + m\eta)\alpha^{crit} + \eta(\alpha^{crit})^2 \right] = 0$$

so that

$$\alpha^{crit} = -(m + 1/\eta) \quad (19)$$

The above critical values are readily computed and can be useful when assessing the robustness of results to variations in underlying parameter estimates. Whilst it is possible to define similar critical values in the multi-product case, the number quickly proliferates, and their relevance diminishes. However, the critical price change, α^{crit} , remains a useful number to compute for those cases where there is market power under the ZERO%SSNIP test (if there is market power under the 0% test, then it becomes meaningful to ask at what level it disappears). It is of particular interest because it is a number which can be usefully compared across different product groupings. From equation (11), α^{crit} is defined as

$$\Delta_K^{\alpha^{crit}} \equiv \Delta_K^0 + \alpha^{crit}\psi_K = 0 \Rightarrow \alpha^{crit} = -\Delta_K^0 / \psi_K \quad (20)$$

This value is computed in the numerical work and sensitivity analysis reported in section 4 below. If the value is significantly above 5%, it gives an indication of the robustness of the 5% test. Of course, where α^{crit} is very large, although this does suggest some degree of robustness for the 5% test, not too much credence should be given to its precise numerical value – as clearly it is based on parameter estimates

which are assumed to be invariant over the price change – and on the assumption that multi-product demands are to good approximation linear for such a price perturbation.

The maximum profit proportionate percent price increase

Having identified the percent price increase, α_{crit} , below which price increase is profitable, it is also worth remarking that it is possible to also compute the level of proportionate price increase which would attain maximum profit. Defining this as

α_{max} , then

$$\alpha_{max} = \arg \max_{\alpha} \Delta \Pi^K(\alpha) = \Delta_K^0 / (2\psi_K) = 1/2 \alpha_{crit}. \quad (21)$$

The estimate of α_{max} is an estimate of the ‘cellophane ceiling’.⁵ For the single product case, α_{max} gives an estimate of how far away the current price is from that which would maximise profit. For the multi-product case, it is an estimate of how far away current prices are from the profit maximising point under the condition that prices are raised proportionately from existing levels. Naturally, a monopolist free to choose prices would generally wish to change prices by variable, non-proportionate, amounts. As an indicator of the ability to increase profit, it thus a lower bound estimate.

4. NON-UNIQUENESS AND RELATED CONSIDERATIONS

Non-Uniqueness

There has been some discussion in the literature (OFT [1992], NERA [1992], regarding the fact that the market definition test does not necessarily uniquely identify

⁵ When initial prices are used as a benchmark for evaluating the hypothetical monopoly test, it may be that the firm has exerted its monopoly power in setting these prices. If so, it will not be profitable to further raise prices. This potential difficulty in using existing prices was discussed in section 1 above; it often goes by the name of the ‘cellophane fallacy’, following a US supreme court case concerning the market for cellophane wrapping (US v E.I.duPont de Nemours&Co., 353 US 586 [1957]).

the boundaries of the relevant market. For this reason, the DOJ guidelines suggest that one should focus on the *smallest sub-set* of products for which there is market power. In practice, this does not get rid of the potential for non-uniqueness. To spell this out, consider a simple 3-product example.

Example: Suppose there are 3 products, labelled 1,2,3, and suppose that, at existing prices, calculations indicate that $\Delta_1^\alpha, \Delta_2^\alpha, \Delta_3^\alpha \leq 0$ whilst $\Delta_{\{1,2\}}^\alpha, \Delta_{\{1,3\}}^\alpha < 0, \Delta_{\{2,3\}}^\alpha > 0$ and $\Delta_{\{1,2,3\}}^\alpha > 0$. Then according to proposition 2, the market is $\{2,3\}$, the ‘smallest subset’ over which a hypothetical monopolist could exert power. This market is uniquely defined. However, if these results are kept the same except that now $\Delta_{\{1,2\}}^\alpha < 0, \Delta_{\{1,3\}}^\alpha, \Delta_{\{2,3\}}^\alpha > 0$, then in this case, both $\{1,3\}$ and $\{2,3\}$ constitute markets under the α % SSNIP test.

There is no problem with non-uniqueness, of course. In the above example, in the case where there are two product groupings which would confer additional market power at extant prices, it is useful to have identified both of them. This is clearly so in merger or collusive practice investigations since it directs the focus to those product groupings over which monopoly power *might* be exerted (for example, one would be less concerned with merger or collusion across products 1 and 3 in the above example). In practice, a merger or collusive practice investigation would normally be focused on a particular set of products, so the question simplifies to that of whether this constitutes a set which passes or fails the SSNIP test. Furthermore, in investigating anti-competitive practices, identifying all sets of products which would fail a SSNIP test is a useful precursor to any investigation, since it helps to direct scrutiny in the relevant directions.

Product substitutability as a necessary condition for a product grouping to be a market under the HMT

The following corollary to propositions 2 and 3 is also worth remarking:

Corollary: Any grouping of products K cannot be a ‘relevant market’ according to the α %SSNIP (or ZERO%SSNIP) HMT if the products within that grouping are independent and/or complements ($\eta_{ki} \leq 0$ for all $k, i \in K, k \neq i$).

Proof: It suffices to establish this for proposition 3 (that is, for arbitrary $\alpha \geq 0$). From proposition 3, for K to be a relevant market requires (i) $\Delta_K^\alpha > 0$ and (ii) $\Delta_L^\alpha \leq 0$ for all $L \subset K$. From (6),

$\Delta_K^\alpha \equiv \sum_{k \in K} p_k q_k (1 + (m_k + \alpha) \sum_{i \in K} \eta_{ki})$. If $\eta_{km} \leq 0$, for all $k \neq m$, then since $(m_k + \alpha) > 0$ for all k , it follows that $\Delta_K^\alpha \leq \sum_{k \in K} p_k q_k (1 + (m_k + \alpha) \eta_{kk})$.

However, $\Delta_{\{k\}}^\alpha = p_k q_k (1 + (m_k + \alpha) \eta_{kk})$ and for all $k \in K$, it must be that $\Delta_{\{k\}}^\alpha \leq 0$ from part (ii) of proposition 3. Hence $\Delta_K^\alpha \leq \sum_{k \in K} \Delta_{\{k\}}^\alpha \leq 0$ which contradicts the requirement that $\Delta_K^\alpha > 0$.

This makes sense, since there can be no increase in market power, associated with an enlargement in the set of products, unless the products included in the set are to some extent economic substitutes.

4. SOME NUMERICAL EXAMPLES

To illustrate the approach, this section first reports results for a two product case on which the relevant data are available - the UK letters business. Following this, two 5-product cases are discussed; the first of these utilises elasticity information from a recent study on US lite beers whilst the second is an artificial example which is

designed to illustrate the ‘*chains of substitution effect*’ being identified by the SSNIP test. Testing product groupings is computationally feasible using hand calculations, but for more than three products, this is singularly tedious, and a computer program or spreadsheet is needed to undertake the exploration for larger product groupings.

UK Letters Business

This example gives a simple illustration of the approach for the two-product case.

Table 1 gives estimates for the relevant parameters (taken from Cuthbertson and Dobbs [1996]).⁶ Table 1 panel (1c) reports Δ_K^0 , Δ_K^α for $\alpha = 0.05(5\%)$, and α^{crit} for each product grouping. These estimated own price elasticities are relatively low (in absolute terms) and so, not surprisingly, both first class and second class letters would be classified as being in a market of their own under a 5% SSNIP test. In terms of market power, there is a significant increase in moving to the 2-product grouping and this is reflected in the estimates of α^{crit} ; the calculations suggest market power for any %test up to 27% and 30% for 1st and 2nd class mail respectively, whilst for the joint market, this rises to a massive 556%. Of course, one would not take such estimates ‘literally’; they are valid only if demand functions remain linear on the interval of the price changes; this is unlikely to be a reasonable assumption for such large changes.⁷ However, the magnitude of the α^{crit} value remains a useful *indication* of the strength of market power at the current prices.

⁶ The original work is based on data 1983-1987, a period before major impacts from other communications media.

⁷ In particular, a unilateral rise in the second class price by 30% at that time would have raised the price very close to the price of first class mail; clearly, for such levels of price increase, second class mail would cease to exist!

Table 1 here**US Lite Beers**

To illustrate the application of the conditions in propositions 2 and 3 to a slightly more complex case, consider the numerical example presented in Table 2. The relevant information for the implementation of the test comprises the initial prices, quantities, marginal costs and the matrix of cross price elasticities. The cross price elasticity information used in Table 2 panel (2a) is taken from econometric work reported in Hausman, Leonard and Zona [1994]. Unfortunately, no price, cost, or volume data are reported in that study so a full analysis is not possible. In what follows, the case where prices, costs and volumes are all set equal is examined – the study then explores the impact of variations in the price cost margin on the assessment of market power and market groupings.

Table 2 here

Table 2 panel (2b) reports Δ_K^0 , Δ_K^α for $\alpha = 0.05(5\%)$, and α^{crit} (%) for all product groupings when prices are set at $p_i = 1.4$. Panel (2c) then gives the smallest sub-set product groupings for the zero% test whilst panel (2d) gives the result for the 5% test. Panel (2c) illustrates the fact that the focus on smallest sub-sets by no means eliminates non-uniqueness, and the comparison of results in (2c) with (2d) shows that the form of the test can make a significant difference to conclusions regarding market boundaries. It is for this reason that, in any practical application, conclusions should be studied for sensitivity/robustness to variations in the estimates of the key parameters involved (notably the percentage value chosen for α). The values for α^{crit} give some idea of how robust the results are. To illustrate a simple form of sensitivity analysis, consider the effect of varying the price level; panels (2c) through to (2h) illustrate the impact, under the zero and 5%SSNIP tests, of varying the price

level (from a price cost margin of 40%, to 30% to 20%) whilst holding the values of other parameters constant.

Whilst the SSNIP test does not necessarily identify unique market boundaries (for example, four distinct ‘markets’ are identified in panel (2c)), clearly the information generated by performing the above calculations, and the type of results reported in Table 2, would be clearly useful in any given regulatory enquiry. Notice that quantitative magnitudes for Δ_K^0 , Δ_K^α and α^{crit} might also be of interest as they also give a measure of the extent of market power in the region of benchmark prices.⁸

Chains of Substitution

When examining the concept of market delineation, the notion of chains of substitution is often referred to. That is, two products may be unrelated (have low or zero cross price elasticities, and yet still form part of the same market under the HMT. The idea is that they may be connected through linkages with other products (via significant cross price elasticities with these other products). This type of effect can easily arise in geographic markets; if products feature high cross price elasticities across contiguous regions but low or zero ones across non-contiguous regions, it is still possible for all these products to be regarded as being in the same market under the HMT. To date, the chains of substitution idea has generally been assessed at an

⁸ Only fairly simple forms of sensitivity analysis are discussed here. However, fairly clearly, it should be possible to extend the analysis to examine the sensitivity of market boundaries to variation in key parameters. One fairly natural possibility would be to embed the market definition algorithm into a simulation model. For example, suppose the interest is in a particular subset L . Given the covariance matrix associated with the elasticity estimates, and perhaps with distributional assumptions made for marginal costs, one could envisage taking drawings from these distributions, computing Δ_L^α - and then repeating this process for a number of runs. The proportion of the runs for which $\Delta_L^\alpha > 0$ could thus be used to back up statements such as “ L is the relevant market at 95% level of significance.”

intuitive level. The problem with this is that it is difficult to assess the ‘rate of decay’ of the influence of such cross effects. In situations where the information is available, the SSNIP test algorithm naturally will take account of such effects. To illustrate this, consider the stylised example presented in Table 3.

Table 3 here

Table 3 illustrates the case where there are zero cross price elasticities except for ‘nearest neighbours’. The results are given in panels (3b) through (3h), with the tables studying the impact of varying the price cost margin from 90% down to 20% and for a zero and a 5%SSNIP test. The higher the price cost margins, naturally, the more likely it will prove to be the case that products are seen to be in the same market through the chain of substitution effect. Thus under the 5%SSNIP test, at a 90% mark-up, all products are deemed to be in the same market; that is $K=\{1,2,3,4,5\}$. This indicates that chains of substitution can be important when deciding market boundaries, although it is necessary to consider not only the magnitude of cross price elasticities, but also market shares and price cost margins. In this illustrative example, as the price cost margins are reduced (holding other parameter values constant), the extent to which chains of substitutability matter decreases; for example, under the ZERO%SSNIP test, when price cost margins are 20%, all products are viewed as being separate markets in their own right.

5. CONCLUDING COMMENTS

The principal object of this paper has been to examine how data on elasticities, prices, costs, and sales might be used to delineate market boundaries according to the ‘hypothetical monopoly’ or SSNIP test. A clear cut procedure has been established

for undertaking this form of analysis for an α %SSNIP test, where α can be set at any desired value.

In the one or two product cases, it is often not too hard to provide rough ball park figures for the elasticities and price cost margins, so the test can be operationalised. Things are less straightforward where many (>2) products are involved. However, in any regulatory enquiry into market power, there is a *prima facie* case for trying to establish the relevant parameter values. That means there is a *prima facie* case for a serious assessment of the feasibility and potential value of a formal econometric study of demand in such enquiries. When it is feasible, the debate will then move on to the adequacies and inadequacies of different models and modelling choices, to the robustness or otherwise of the key parameters (elasticities, price cost margins). Such assessments (including assessments of robustness) can then be built into the study of market power and market boundaries using the algorithm established in this paper.

As remarked in the introduction, this paper has been concerned with operationalising the hypothetical monopoly test. The uses to which it should be put have not been discussed in any detail. However, given the predominance in competitive and regulatory enquiries of dynamic, nascent and innovative industries, some comment on how the HMT might be used is perhaps in order. The assessment of market power (and market boundaries) via the HMT is essentially a short run static assessment (although one can try to study and forecast the evolution of the key variables - elasticities, margins etc. - over time). Whilst one might wish to put a fair degree of weight on this assessment where the markets under scrutiny are themselves judged to be fairly 'static', clearly in dynamic innovatory markets, which often feature high

fixed/sunk costs and low variable costs, there are good reasons for questioning the uncritical use of such market power tests.⁹ In such markets, assessments of market power are still informative regarding the current situation in the market – and such information is useful as an input into any regulatory assessment – but any assessment for such markets must take account of the details of the case. As Lind et al [2002, p. 56] conclude, rather than using the HMT to identify a situation of potential abuse (and hence as requiring some form of regulation), in dynamic markets, it may be preferable to reverse this line of reasoning: That is, “to look at the alleged abuse of the competition .. and ask how market definition... can help us to understand the issue at hand.”¹⁰ However, either way, there is still a role for the HMT to play.

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⁹ Dynamic and innovative markets feature dynamic pricing strategies in which prices can rationally diverge significantly from unit variable cost. Thus skimming pricing can involve initial high prices; more significantly, and widespread in many markets, penetration pricing involves very low initial prices, often below marginal cost. Such policies impact on prices, margins, and sales volumes over time, and hence complicate any assessment of market power. It can also be argued that any attempt to squeeze profit in such industries will damage the incentive to invent, which is the lifeblood for dynamic competition.

¹⁰ Lind et al [2002] provides an excellent assessment of how competition policy might best be applied in dynamic innovative markets. The pitfalls, and the usefulness, of SSNIP tests in such cases is discussed at pp. 54-56.

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APPENDIX

(i) Concavity of $\Delta\Pi^K(z)$

$\Delta\Pi^K(z)$ is a concave function if $\Pi^K(\mathbf{p})$ is concave in prices (as assumed in propositions 1, 2). To see this, let $\Pi_p^K(\mathbf{p})$ denote the (row) vector of partial derivatives and $\Pi_{pp}^K(\mathbf{p})$, the Hessian matrix. Then $\Delta\Pi^K(z)$ is defined as

$$\Delta\Pi^K(z) = \Pi^K(\bar{\mathbf{p}}) - \Pi^K(\mathbf{p}) \quad (1)$$

where

$$\bar{\mathbf{p}} = (1+z)\mathbf{p} \text{ so that } d\bar{\mathbf{p}}/dz = \mathbf{p} \quad (2)$$

If $\Pi^K(\mathbf{p})$ is concave, then

$$\mathbf{h}'\Pi_{pp}^K(\mathbf{p})\mathbf{h} \leq 0 \text{ for any } \mathbf{p} > \mathbf{0} \text{ and any } \mathbf{h} \neq \mathbf{0} \quad (3)$$

Differentiating (1) with respect to z gives

$$d\Delta\Pi^K(z)/dz = d\Pi^K(\bar{\mathbf{p}})/dz = \Pi_p^K(\bar{\mathbf{p}})\mathbf{p} \quad (4)$$

and so

$$d^2\Delta\Pi^K(z)/dz^2 = d^2\Pi^K(\bar{\mathbf{p}})/dz^2 = \mathbf{p}'\Pi_{pp}^K(\bar{\mathbf{p}})\mathbf{p} \leq 0, \quad (5)$$

the latter inequality following from (3). This establishes that $\Delta\Pi^K(z)$ is a concave function. *Inter alia*, also note, from (4) and (6), by setting $z=0$, that

$$d\Delta\Pi^K(0)/dz = \Delta_K^0 = \Pi_p^K(\mathbf{p})\mathbf{p} \quad (6)$$

(ii) Proof for Proposition 2.

The profit functions are concave in prices; this implies $\Delta\Pi_L(z)$ is concave in z (for all $z > 0$, $L \subseteq K$, $L \neq \emptyset$). As in proposition 1, $\Delta_K^0 > 0$ guarantees there exists an interval $(0, \delta)$ for some $\delta > 0$ such that $\Delta\Pi_K(z) > 0$ for $z \in (0, \delta)$, which is a sufficient condition for there to be market power with respect to subset K .

For any non-null set L , if $\Delta_L^0 \leq 0$, then since $\Delta\Pi_L(z)$ is concave, it follows that $\Delta\Pi_L(z) \leq 0$ for all $z \geq 0$. More formally, this is established by taking an exact Taylor series expansion: thus

$$\Delta\Pi^L(z) = \Delta\Pi^L(0) + \left[d\Delta\Pi^L(0)/dz \right] z + \frac{1}{2} \left[d^2\Delta\Pi^L(\psi z)/dz^2 \right] z^2 \quad (7)$$

where $0 < \psi < 1$. Now $\Delta\Pi^L(0) = 0$, $d\Delta\Pi^L(0)/dz = \Delta_L^0 \leq 0$, and by concavity, $d^2\Delta\Pi^L(\psi z)/dz^2 \leq 0$ for all $z \geq 0$. Hence if $\Delta_L^0 \leq 0$ then $\Delta\Pi^L(z) \leq 0$ for all $z \geq 0$.

Thus $\Delta_K^0 > 0$ is not only sufficient, but also is a necessary condition for market power with respect to subset K - since by concavity, if $\Delta_K^0 \leq 0$ then $\Delta\Pi^K(z) \leq 0$ for all $z \geq 0$. Finally, if there exists a subset $L \subset K$ for which $\Delta_L^0 > 0$, then this subset has market power, so K is not a smallest subset - so $\Delta_L^0 > 0$ is a necessary condition for it to be a smallest subset, and if $\Delta_L^0 \leq 0$ for all subsets $L \subset K$, then this is a sufficient condition for K to be a smallest subset. This completes the proof of proposition 2.

(iii) Proof for Proposition 3.

First note that

$$\partial\Delta\Pi^K / \partial p_i = q_i + \sum_{k \in K} (p_k - c_k) (\partial q_k / \partial p_i) \quad (8)$$

$$\begin{aligned} \partial^2\Delta\Pi^K / \partial p_j \partial p_i &= (\partial q_i / \partial p_j) + (\partial q_j / \partial p_i) + \sum_{k \in K} (p_k - c_k) (\partial^2 q_k / \partial p_j \partial p_i) \\ &= (\partial q_i / \partial p_j) + (\partial q_j / \partial p_i) \end{aligned} \quad (9)$$

and

$$\partial^3\Delta\Pi^K / \partial p_m \partial p_j \partial p_i = (\partial^2 q_i / \partial p_m \partial p_j) + (\partial^2 q_j / \partial p_m \partial p_i) = 0, \quad (10)$$

the latter because demand is assumed linear in the relevant region in proposition 3.

The profit function in (1) can be written as an exact Taylor series expansion of the form

$$\Delta\Pi^K(z) = \Delta\Pi^K(0) + \left[d\Delta\Pi^K(0)/dz \right] z + \frac{1}{2} \left[d^2\Delta\Pi^K(0)/dz^2 \right] z^2 + \frac{1}{6} \left[d^3\Delta\Pi^K(\psi z)/dz^3 \right] z^3 \quad (11)$$

where $\Delta\Pi^K(0) = 0$, $d\Delta\Pi^K(0)/dz = \Delta_K^\alpha$ from (6), and from (5),

$$d^2\Delta\Pi^K(0)/dz^2 = \mathbf{p}' \cdot \Pi_{pp}^K(\mathbf{p}) \cdot \mathbf{p} = 2\psi_K \quad (12)$$

where

$$\begin{aligned}
2\psi_K &\equiv \sum_{i \in K} \sum_{j \in K} \left(p_i p_j \left(\frac{\partial^2 \Pi^K}{\partial p_i \partial p_j} \right) \right) \\
&= \sum_{i \in K} \sum_{j \in K} \left(p_i p_j \left(\frac{\partial q_i}{\partial p_j} + \frac{\partial q_j}{\partial p_i} \right) \right) \\
&= 2 \sum_{i \in K} \left(p_i q_i \sum_{j \in K} \eta_{ij} \right)
\end{aligned} \tag{13}$$

and $d^3 \Delta \Pi^K(\psi z) / dz^3 = 0$ in view of (10). Thus, it follows that (11) simplifies to

$$\Delta \Pi^K(z) = \Delta_K^0 z + \psi_K z^2. \tag{14}$$

Thus $\Delta \Pi^K(\alpha/100) = \Delta_K^0(\alpha/100) + \psi_K(\alpha/100)^2 \geq 0$ as $\Delta_K^0 + \psi_K(\alpha/100) \geq 0$

Thus $\Delta_L^0 + \psi_K(\alpha/100) > 0$ for set K , and for this to be the smallest profitable set, for all subsets $L \subset K$, it must be that $\Delta_L^0 + \psi_L(\alpha/100) \leq 0$. This completes the proof for proposition 3.

(iv) An alternative form of the hypothetical monopoly test

A possible alternative definition for the hypothetical monopoly test would parallel that given in the main paper in proposition 2, but merely require that the HM could profitably increase all prices (that is, there is no longer a requirement for the increases to be proportionate). Now, the rate of increase of profit with respect to a change in the i^{th} price is given as

$$\begin{aligned}
\partial \Pi^K / \partial p_i &= q_i + \sum_{k \in K} (p_k - c_k) (\partial q_k / \partial p_i) \\
&= q_i \left(1 + \sum_{k \in K} m_k \frac{p_k q_k}{p_i q_i} \eta_{ki} \right)
\end{aligned}$$

so, defining

$$\theta_i^K = 1 + \sum_{k \in K} m_k \frac{p_k q_k}{p_i q_i} \eta_{ki},$$

as a computable statistic if prices, outputs, marginal costs, and elasticities are known at the price vector to be tested, then we have the following rather obvious implications:

Proposition 4: (i) A sufficient condition for product set K to have market power is that $\theta_i^K > 0$ for all $i \in K$, and (ii) a necessary condition for there to be no smaller subsets with market power is that there is no subset $L \subset K$ for which $\theta_i^L > 0$ for all $i \in L$.

It is possible to use the tests implied in proposition 4 in order to give some indications of what might constitute market boundaries, and a computer program of a similar type to that developed for the algorithm in the paper can be used to perform the numerical computations. However, unlike the tests developed in the paper, and even if one imposes concavity on the profit function $\Pi^K(\mathbf{p})$, the partial derivative tests outlined in proposition 4 above by no means provide necessary and sufficient conditions for identifying a relevant market. To spell this point out, notice that there may be market power associated with a product set K even though several of the θ_i^K are negative (since profit can increase if one raises the price only for a subset of the product grouping), and furthermore, even if one finds that, for a given subset K , that all subsets $L \subset K$ have at least one element $\psi_i^L \leq 0$, so that condition (ii) is satisfied, this by no means implies that there is no market power in any of these subsets (since larger price increases might be profitable even if small ones are not).

Table 1: Estimates for UK Letters business**Panel (1a): Elasticities¹¹**

	First Class	Second Class
First Class	-1.3	1.2
Second Class	0.95	-1.2

Panel (1b): Other Data

	c_i	p_i	m_i	q_i
First Class	10	20	0.50	0.5
Second Class	7	15	0.47	0.5

Panel (1c): Letters Business Product Groupings

Δ_K^0	$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Product Grouping
3.50	2.85	26.92	1
2.70	2.25	30.00	2
16.00	15.86	556.52	1 2

¹¹ These elasticity figures are drawn from the study by Cuthbertson and Dobbs [1996].

Table 2: A numerical example (Lite Beers)**Panel (2a):** Elasticities¹²

Product	1	2	3	4	5
1	-3.763	0.464	0.397	0.254	0.201
2	0.569	-4.598	0.407	0.452	0.482
3	1.233	0.956	-6.097	0.841	0.565
4	0.509	0.737	0.587	-5.039	0.577
5	0.683	1.213	0.611	0.893	-5.841

Other data, not varied in panels (1a)-(1g) below: $c_i, q_i = 1.0, i = 1, \dots, 5$ **Panel (2b):** Beer Market Example - with prices $p_1, \dots, p_5 = 1.4$

Δ_K^0	$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Product Grouping
-0.11	-0.37	n/a	1
-0.44	-0.76	n/a	2
-1.04	-1.47	n/a	3
-0.62	-0.97	n/a	4
-0.94	-1.35	n/a	5
-0.13	-0.64	n/a	1 2
-0.49	-1.07	n/a	1 3
-0.42	-0.98	n/a	1 4
-0.69	-1.30	n/a	1 5
-0.93	-1.59	n/a	2 3
-0.58	-1.17	n/a	2 4
-0.70	-1.31	n/a	2 5
-1.08	-1.76	n/a	3 4
-1.50	-2.26	n/a	3 5
-0.96	-1.62	n/a	4 5
0.03	-0.70	0.2	1 2 3
0.03	-0.70	0.2	1 2 4
-0.04	-0.78	n/a	1 2 5
-0.23	-1.01	n/a	1 3 4
-0.60	-1.45	n/a	1 3 5
-0.41	-1.22	n/a	1 4 5
-0.50	-1.32	n/a	2 3 4
-0.72	-1.58	n/a	2 3 5
-0.25	-1.03	n/a	2 4 5
-0.96	-1.86	n/a	3 4 5
0.76	-0.08	4.5	1 2 3 4
0.59	-0.28	3.4	1 2 3 5
0.72	-0.14	4.2	1 2 4 5
0.24	-0.69	1.3	1 3 4 5
0.30	-0.63	1.6	2 3 4 5
1.92	1.03	10.8	1 2 3 4 5

¹² These elasticity figures are actually drawn from the study by Hausman et al. [1999] for low alcohol beers.

Panel (2c): Beer Market Example - Prices $p_1, \dots, p_5 = 1.4$ Zero% SSNIP Test

Δ_K^0	α^{crit}	Market				
0.03	0.2	1	2	3		
0.03	0.2	1	2	4		
0.24	1.3	1	3	4	5	
0.30	1.6	2	3	4	5	

Panel (2d): Beer Market Example - Prices $p_1, \dots, p_5 = 1.4$ 5% SSNIP Test

$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market				
1.03	10.8	1	2	3	4	5

Panel (2e): Beer Market - Prices $p_1, \dots, p_5 = 1.3$ Zero% SSNIP Test

Δ_K^0	α^{crit} (%)	Market				
0.17	3.5	1				
0.07	0.6	2	4			
0.21	1.3	2	3	5		
0.03	0.2	3	4	5		

Panel (2f): Beer Market – Prices $p_1, \dots, p_5 = 1.3$ 5% SSNIP Test

$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market				
0.09	5.7	1	2	3		
0.10	5.7	1	2	4		
0.03	5.3	1	2	5		
0.31	6.8	1	3	4	5	
0.36	7.1	2	3	4	5	

Panel (2g): Beer Market - Prices $p_1, \dots, p_5 = 1.2$ Zero % SSNIP Test

Δ_K^0	α^{crit} (%)	Market				
0.45	9.9					1
0.28	5.1					2
0.19	3.2					4
0.03	0.5					5

Panel (2h): Beer Market – Prices $p_1, \dots, p_5 = 1.2$ 5% SSNIP Test

$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market				
0.222	9.9	1				
0.005	5.1	2				
0.245	6.6	3	4	5		

Table 3: A numerical example illustrating ‘chains of substitution’**Panel (3a)** Elasticities

Product	1	2	3	4	5
1	-5.0	2.0	0	0	0
2	2.0	-5.0	2.0	0	0
3	0	2.00	-5.0	2.0	0
4	0	0	2.0	-5.0	2.0
5	0	0	0	2.0	-5.0

Other data, not varied in panels (1a)-(1g) below: $c_i, q_i = 1.0, i = 1, \dots, 5$

Panel (3b): Chains Study - Prices $p_1, \dots, p_5 = 1.9$ **Zero% SSNIP Test**

Δ_K^0	α^{crit} (%)	Market			
		1	2	3	4
0.4	2.6				
0.4	2.6	2	3	4	5

Panel (3c): Chains Study – Prices $p_1, \dots, p_5 = 1.9$ **5% SSNIP Test**

$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market				
		1	2	3	4	5
0.84	8.2					

Panel (3d): Chains Study - Prices $p_1, \dots, p_5 = 1.4$ **Zero% SSNIP Test**

Δ_K^0	α^{crit} (%)	Market	
		1	2
0.4	4.8	1	2
0.4	4.8	2	3
0.4	4.8	3	4
0.4	4.8	4	5

Panel (3e): Chains Study – Prices $p_1, \dots, p_5 = 1.4$ **5% SSNIP Test**

$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market		
		1	2	3
0.91	14.3	1	2	3
0.91	14.3	2	3	4
0.91	14.3	3	4	5

Panel (3f): Chains Study - Prices $p_1, \dots, p_5 = 1.3$ Zero and 5% SSNIP Tests

Δ_K^0	$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market	
0.8	0.41	10.26	1	2
0.8	0.41	10.26	2	3
0.8	0.41	10.26	3	4
0.8	0.41	10.26	4	5

Panel (3g): Chains Study - Prices $p_1, \dots, p_5 = 1.2$ Zero% SSNIP Test

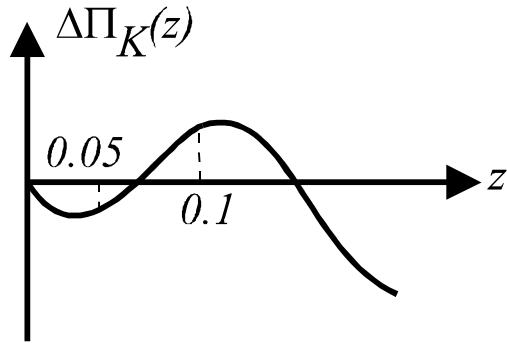
Δ_K^0	α^{crit} (%)	Market	
0.2	3.3	1	
0.2	3.3	2	
0.2	3.3	3	
0.2	3.3	4	
0.2	3.3	5	

Panel (3h): Chains Study - Prices $p_1, \dots, p_5 = 1.2$ 5% SSNIP Test

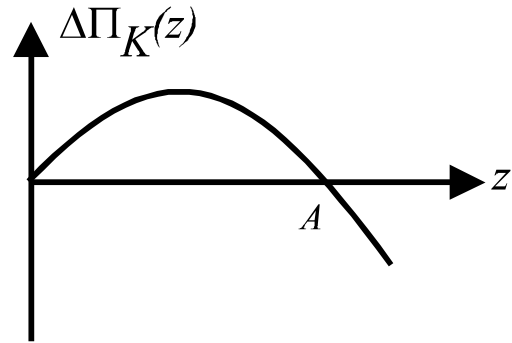
$\Delta_K^{\alpha=5\%}$	α^{crit} (%)	Market	
0.84	16.7	1	2
0.84	16.7	2	3
0.84	16.7	3	4
0.84	16.7	4	5

Figure 1: Illustrating possible behaviours of the function $\Delta\Pi_K(z)$

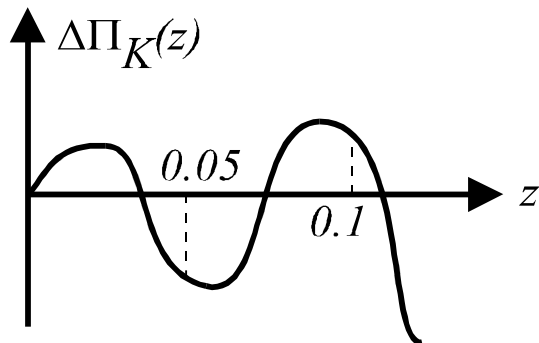
Panel (a): $\Delta_K < 0$



Panel (c): $\Delta_K > 0$



Panel (b): $\Delta_K > 0$



Panel (d): $\Delta_K < 0$

