VALUE UNDER ACTIVE AND PASSIVE DEBT MANAGEMENT POLICY

by

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A B S T R A C T

The relationship between debt policy and valuation has been extensively analysed in the finance literature; within a Modigliani-Miller framework, the consensus is that valuation is affected by whether debt is managed actively or passively, and that for finite projects with time varying risky cash flows, it is appropriate to use a weighted average discount rate for valuation only if it is assumed that debt is actively managed. In this paper, the relationship between debt policy and valuation is re-examined. In particular, it is shown that, under one of the most plausible forms of passive debt policy, valuation using a simple weighted average discount rate is in fact possible.

1. INTRODUCTION

The relationship between debt policy and valuation within a Modigliani-Miller framework has been extensively analysed in the finance literature (see for example Modigliani and Miller [1958], [1963]; Miles and Ezzell [1980], [1983]; Lewellen and Emery [1986]), and more recently, this has been extended to incorporate personal tax effects (for example, Ashton [1989], Clubb and Doran [1992], Taggart [1991]) and implications for beta degearing formulae (for example, Appleyard and Strong [1989], Clubb and Doran [1991]). The original motivation for the Miles-Ezzell analysis was to explore the circumstances under which it is appropriate to value a finite uneven risky cash flow using a constant weighted average of the firm's after-tax costs of debt and equity. The Miles-Ezzell analysis demonstrated that valuation is affected by whether debt is managed actively or passively, and appeared to show that, for finite projects with time varying cash flows, it is appropriate to use a weighted average discount rate for valuation only if it is assumed that debt is actively managed (see especially Ezzell and Miles [1983, p25]).

In fact, the Miles-Ezzell analysis only establishes sufficient conditions for the above result. In this paper, the relationship between debt policy and valuation is re-examined; it is shown that active debt management is by no means a necessary condition; in particular, it is shown that under one of the more plausible forms of passive debt policy, correct valuation using a simple weighted average discount rate is also in fact possible. Furthermore, the appropriate discount rate in this case is precisely the same as that appropriate when debt is actively managed.

Under passive debt management policy (PDMP), the outstanding debt associated with a project (or firm) has a time profile determined *ab initio* and thereafter adhered to. This is the kind of policy implicit in the models of Modigliani and Miller ([1958], [1963]). Active debt management policy (ADMP) by contrast allows the firm to take account of new information; financial transactions can be made at each point in time in order to maintain the actual leverage ratio at any particular desired level (Miles and Ezzell ([1980], [1983]) and Lewellen and Emery [1986]).

For any given (positive) target leverage ratio (assumed constant over time), it has been shown that value under ADMP is less than under PDMP (Lewellen and Emery [1986]). The standard explanation for the difference is that the debt interest tax shields are riskier under ADMP because of an extra layer of managerial action required to actively manage the debt (Ruback [1986]) and hence are more heavily discounted (see Miles and Ezzell [1980, 1983]). In this paper, it is shown that this explanation is only part of the story; the fundamental 'value driver' turns out to be the leverage concept used in PDMP.

Whilst under ADMP, leverage is an unproblematic and well defined concept, under PDMP, this is not the case; there are several candidate leverage concepts which might plausibly be used. Different leverage concepts impound different amounts of debt and hence add different amounts of value via the debt tax shields. When a value based PDMP leverage concept is selected, the weighted average discount rate can again be used to value a project with finite and variable cash flows and that, furthermore, the value under ADMP and PDMP are the same; there is no value discrepancy.[1]

The above discussion has implications for the appropriate choice of beta gearing/degearing formulae since it suggests that formulae originally derived for ADMP (set out in Appleyard and Strong [1989]) can also be used when a particular type of PDMP is assumed. This in turn has some policy implications; for example, estimates of the cost of capital play a central role in regulatory economics, and the choice of beta gearing/degearing formulae can materially affect such estimates. The above observations suggest that the ADMP formulae are appropriate in a greater range of circumstances than previously envisaged.

In section 2, values under ADMP and PDMP are compared in a 2-period model. Section 3 extends this to deal with value for the T-period case whilst section 4 discusses implications for Modigliani-Miller debt policies when there are risky cash flow perpetuities. Section 5 then discusses implications for work on valuation, beta degearing/regearing calculations and project appraisal.

2. COMPARISONS IN A 2-PERIOD MODEL

The project generates risky net (ie. net of corporate tax) cash flows denoted $X=\{X_1,..,X_T\}$ where X_t denotes the risky net cash flow in period t. In this section T=2; section 3 deals with the more general case. A valuation framework is assumed in which assets exist in the Capital Market which allow each of the uncertain cash flows X_t to be priced at time zero and, in this 2-period case, state conditionally at time τ ($0<\tau<t$). The value or price at time zero of an uncertain net cash flow X_t is denoted P_t.[2]

ASSUMPTIONS:

A1: The cost of debt, r, is exogenous, constant over time and risk free.

A2: Debt is rolled over on a one period maturity.

A3: The tax environment is that of Modigliani-Miller; there is a constant corporate tax rate t_c, and the tax shield on debt is always available.[3]

A4: There is a fixed probability distribution f(X) associated with the cash flow $X=(X_1,..,X_T)$.

A5: All net cash flows are paid out to shareholders.

A6: An expected risky cash flow X_t can be valued at time τ ($0 \le \tau < t$) by discounting at a constant risky rate R (R>r).

Assumptions A1-A7 set out a standard framework for discussing debt management policy as established by Modigliani-Miller, Miles and Ezzell, and Lewellen and Emery. Although this section is only concerned with a 2-period example, it is useful to establish some general notation for use in all sections, as follows. The expected value of X_t with expectations formed at time τ ($0 \le \tau < t$) is denoted $E_\tau(X_t)$. *A6* implies that the value at time τ of the risky cash flow X_t is

$$PV_\tau(X_t) = E_\tau(X_t)/(1+R)^{t-\tau} \tag{1}$$

where R denotes the constant risky discount rate.[4] Note that equation (1) implies that, if α is any given constant, then $PV_\tau(\alpha X_t) = \alpha PV_\tau(X_t)$. Let V_{ijk} (where $0 \le i \le j \le k < T$) denote the value at time j, given information up to time i, of the uncertain net cash flows from period k+1 onward. That is, V_{ijk} is the price an agent would be willing at time i to commit to paying at time j in order to receive the uncertain cash flow stream $\{X_{k+1}, X_{k+2}, ..\}$. Clearly, V_{000} denotes the present (time zero) value of the whole future cash flow $\{X_t\}$, V_{00t} represents the present (time zero) value of the cash flow occurring after time t whilst V_{0tt} represents the forward price an individual would be willing to commit now to paying at time t for the cash flow ensuing from that time (if $i<j$, then V_{ijk} is a forward price). The arbitrage-free relationship between these two prices is of course that

$$V_{0tt} = V_{00t}(1+r)^t. \tag{2}$$

V_{ttt} by contrast is the price at time t of the remaining cash flow. Note that viewed from time zero, V_{ttt} is a random variable; it will in general depend on realizations $X_1, X_2, .., X_t$. To see this, consider the 2-period case:

$$V_{111} = E_1(X_2)/(1+R) \tag{3}$$

where

$$E_1(X_2) = \int_{x_2 \in \chi} x_2 g(x_2 | X_1) dx_2 \tag{4}$$

and

$$g(X_2 | X_1) = f(X_1, X_2) / \int_{x_2 \in \chi} f(X_1, x_2) dx_2 \tag{5}$$

where $g(X_2|X_1)$ is the conditional density function and χ denotes the range for X_2. In this case, clearly in general V_{111} depends on the value taken by X_1.

Given the above definitions, the unlevered value of the cash flow is

$$V_u \equiv V_{000} = \sum_{t=1}^{2} E_0(X_t) / (1+R)^t \tag{6}$$

The rest of this section deals with how debt adds to this unlevered value under ADMP and PDMP. First, the Miles-Ezzell value rule is established for the 2-period case, as a benchmark for the ensuing analysis of the PDMP case.

Value under ADMP

The ADMP target involves choosing D_t at time t so as to maintain debt to value constant (denoted L) over time. Value here is the value current at time t, V_{ttt}. That is,

$$D_t = L.V_{ttt} \tag{7}$$

The ADMP solution can be established recursively:

$$V_{111}(X_1) = PV_1(X_2) + \frac{t_c r D_1}{1+r} \tag{8}$$

We write $V_{111} = V_{111}(X_1)$ to emphasize that it is conditional on X_1. The debt D_1 generates a riskless tax shield $t_c rD_1$ in (8) which is accordingly discounted at the riskless rate. Using (7) gives

$$V_{111}(X_1) = PV_1(X_2)/\psi, \tag{9}$$

where

$$\psi = 1 - \frac{t_c r L}{1+r}. \tag{10}$$

Moving back to time zero, the value associated with an ADMP, denoted V_a, is given by

$$V_a \equiv V_{000} = \frac{E_0(X_1)}{1+r} + PV_0\{PV_1(X_2)/\psi\} + \frac{t_c r D_0}{1+r} \tag{11}$$

Noting that[5] $PV_0\{PV_1(X_2)/\psi\} = PV_0\{PV_1(X_2)\}/\psi = PV_0(X_2)/\psi = E_0(X_2)/\{\psi(1+R)^2\}$, and that from (7), $D_0 = LV_{000}$, equation (11) can be rearranged to obtain

$$V_a \equiv V_{000} = \sum_{t=1}^{2} \frac{E_0(X_t)}{(1+r^*)^t} \tag{12}$$

where

$$r^* = R - [t_c r L(1+R)/(1+r)] \tag{13}$$

which is of course the Miles-Ezzell [1980] ADMP result for the 2-period case. Under ADMP, the value of the after tax cash flows (including the added value from the debt tax shields) is given by discounting expected values (with expectations formed at time zero) at a constant adjusted rate r^*.

Value under PDMP

In ADMP, debt is chosen at time t so as to maintain debt at a constant proportion of value at time t; thus the leverage ratio is actually maintained constant over time. In the case of a passive debt management policy, the debt level for each future period is contractually fixed at time zero. Since the future value of the firm is a random variable, the actual leverage ratio will fluctuate through time. Thus in setting a leverage target, this can only be set in terms of valuations or expectations at time zero. In the literature on PDMP, the debt schedule set out at time zero was designed to maintain debt to expected value (with time zero expectations) constant over time. That is, $D_t / E_0(V_{ttt}) = L$, constant (see Ashton and Atkins [1978], Miles and Ezzell [1983]). It turns out that this definition does not induce a valuation rule in which any finite project's cash flows can be valued using a simple constant weighted average discount rate. However, this is by no means the only definition of 'constant leverage' available under passive debt management. One alternative target leverage would be that of expected leverage; $E_0(D_t / V_{ttt}) = L$. This would in general imply a different debt schedule and hence value to the firm (but it again does not induce a simple cost of capital valuation rule). This is not surprising because in both these definitions of leverage, a mathematical expectation forms part of the definition but is not in any sense part of a valuation operation. The only available leverage concept based on value is that which uses the forward price (definition *d2* below); this has the merit of putting the leverage definition on a consistent footing with that for ADMP (namely being value based). Using this leverage concept, it is now shown that value under PDMP coincides with that under ADMP.

Viewed from the perspective of time zero, the value of X_2 at time 1 is a random variable (it depends on the realization of X_1). The PDMP involves choosing D_0 and D_1 at time zero based on expectations at time zero so as to maintain a fixed target leverage ratio L over the two periods of the capital project. The alternative targets are as follows;

Definition *d1*: The ratio of debt to expected value is maintained constant: $D_t / E_0\{V_{ttt}\} = L$ for $t \geq 0$. Thus choose D_0, D_1 so that

$$D_0/V_{000} = D_1/E_0\{V_{111}\} = L. \tag{14}$$

Definition *d2*: The debt to value leverage ratio maintained constant, where value is defined as the forward price V_{0tt} for period t: $D_t / V_{0tt} = L$, for $t \geq 0$. Thus choose D_0, D_1 so that

$$D_0/V_{000} = D_1/V_{011} = L. \tag{15}$$

Note that, since V_{000} is not a random variable, there is no effective difference between the definitions at time zero; that is $D_0/E(V_{000}) = D_0/V_{000}$. In the two period example, different leverage targets generate different debt schedules and hence different values for the project/firm because of their differing treatment of debt at time 1. Definition *d1* is of course the PDMP target adopted in earlier work (see e.g. Ashton and Atkins [1978]) and *d2*, the new forward price based definition. In the 2-period case, the forward price V_{011} is the price it is just worth committing to pay at time 1 to receive the cash flows from time 2. It is the value of that future cash flow at time 1 based on information at time zero.

Debt and value can be readily calculated under each of the above definitions. This is done recursively; firstly D_1 is determined and then D_0. Once the debt schedule is established, value V_{000} under PDMP is given as the value of the net cash flows alone plus the value of the riskless tax shields. Since the debt tax shields are riskless, value is given by

$$V_{000} = \left(\sum_{t=1}^{2} \frac{E_0(X_t)}{(1+R)^t} \right) + \frac{t_c r L V_{000}}{1+r} + \frac{t_c r D_1}{(1+r)^2},$$

so

$$V_p \equiv V_{000} = \left\{ \left(\sum_{t=1}^{2} \frac{E_0(X_t)}{(1+R)^t} \right) + \frac{t_c r D_1}{(1+r)^2} \right\} / \psi \tag{16}$$

where V_p denotes value under PDMP and ψ is defined in (10). Clearly V_p is strictly increasing in D_1. The levels of debt implied by the alternative leverage concepts are as follows:

Debt D_1 under *d1*:

$D_1 = L\ E_0[V_{111}(X_1)] = L\ E_0[PV_1(X_2)/\psi] = LE_0\{E_1(X_2)\}/\{(1+R)\psi\}$

$$= \quad LE_0(X_2)/\{(1+R)\psi\} \tag{17}$$

Debt D_1 under *d2*:

The target leverage here is

$$D_1/V_{011} = L. \tag{18}$$

where, as discussed above, V_{011} is the forward price defined in equation (2). Viewed from time zero, the present value at time zero of the unlevered cash flow X_2 is $PV_0(X_2)$. It follows from (2) that the sum of money any individual would be willing to commit at time 0 to paying at time 1 for an unlevered risky cash flow X_2 is $(1+r)PV_0(X_2)$. Onto this must be added the value of the riskless tax shield, $t_c rD_1/(1+r)$. Using (18), V_{011} can be determined from the equation

$$V_{011} = (1+r)PV_0(X_2) + \frac{t_c rLD_1}{1+r} = (1+r)PV_0(X_2) + \frac{t_c rLV_{011}}{(1+r)}. \qquad (19)$$

It follows that

$$V_{011} = (1+r)PV_0(X_2)/\psi. \qquad (20)$$

Noting that $PV_0(X_2) = E_0(X_2)/(1+R)^2$, using (20) in (18) gives

$$D_1 = L(1+r)E_0(X_2)/\{(1+R)^2\psi\}. \qquad (21)$$

It is straightforward to compare the amount of debt outstanding under these different definitions; comparing (17) and (21), then, in an obvious notation,

$$D_1[d_2] = D_1[d_1](1+r)/(1+R), \qquad (22)$$

and, since L, R, $\psi > 0$,

$$D_1[d1] \lesseqgtr D_1[d2] \text{ as } E_0(X_2) \lesseqgtr 0 \qquad (23)$$

From (16), since $\partial V_p/\partial D_1 > 0$, it then follows that

$$V_p[d1] \lesseqgtr V_p[d2] \text{ as } E_0(X_2) \lesseqgtr 0 \qquad (24)$$

Under assumption *A3*, tax shields are always available; that is, if a net cash flow is negative, there is an associated negative tax shield (this is not particularly unrealistic - for example, if the firm as a whole is profitable, a negative cash flow associated with an individual project merely reduces the overall tax shield). Different leverage definitions impound different amounts of debt at time 1 and hence entail different values, given that debt adds value via the tax shield. Value under *d1* may in general be above or below that under *d2*. Of course, in so far as projects tend to have positive expected cash flows, there is a tendency for $V_p(d1) > V_p(d2)$. As mentioned before, *d1* is the PDMP leverage target used in previous work; it follows that $V_p[d1]$ is precisely the value that comes from the APV formula for the PDMP case derived in Ashton and Atkins [1978].

Finally, if $D_1[d2]$ from (21) is substituted into (16) and simplified, the value equation collapses to that of ADMP as in (12), (13). That is,

$$V_p[d2] = V_a \tag{25}$$

Thus, using the only value based leverage concept available for passive debt management, equation (25) establishes value equivalence of ADMP and PDMP. It then follows that the constant weighted average cost of capital which is appropriate for valuing finite lived projects when debt is actively managed is also appropriate when there is a passive debt policy. This result is explicitly demonstrated for the multi (>2) period case in the following section.

3. VALUING A T-PERIOD RISKY CASH FLOW UNDER PDMP

This section extends the analysis of passive debt management policy to the *T*-period case. Thus, the project to be evaluated lasts *T* periods (section 4 discusses the perpetuity case) and gives rise to risky net of corporate tax cash flows $X=\{X_1 ,..,X_T\}$ where X denotes the cash flow in period t. Again, let V_{ijk} (where $0 \le i \le j \le k<T$)

denote the value at time j, given information up to time i, of the uncertain net cash flows $\{X_{k+1},..,X_T\}$. Again the assumptions *A1-A5* apply, and *A6* is relaxed as follows:

A6′: Assets exist in the Capital Market which allow each of the uncertain cash flows X_t *to be priced at time zero.*

Given this assumption, the implicit rate for discounting $E_0(X_t)$ to time zero may be time varying. Thus the associated discount rate can be defined as ${}_0R_t$ in the equation

$$P_t = E_0(X_t)/(1+{}_0R_t)^t \tag{26}$$

The special case of a constant discount rate case (${}_0R_t=R$, constant) occurs if prices and expected cash flows can be related by the formula $P_t = E_0(X_t)/(1+R)$, although this is <u>not</u> assumed in the ensuing analysis. To begin with, consider the cash flow in the absence of debt. The value of X viewed from time zero is

$$V_{000} = \sum_{t=1}^{T} P_t. \tag{27}$$

The present (time zero) value of the net cash flow $\{X_\tau,..,X_T\}$ is

$$V_{00\tau} = \sum_{t=\tau+1}^{T} P_t. \tag{28}$$

The value of the cash flow $\{X_\tau,..,X_T\}$ at time τ but viewed from the perspective of time zero (ie. with information at time zero) is the forward price. This is defined using (2) as

$$V_{0\tau\tau} = V_{00\tau}(1+r)^\tau . \tag{29}$$

The forward price $V_{0\tau\tau}$ denotes the value of the project at time τ based on information at time zero. It is the value worth committing at time zero to paying at time τ to receive the net cash flow $\{X_{\tau+1},..,X_T\}$. Note that under *A6′*, the only way of valuing the net cash flow $\{X_{\tau+1},..X_T\}$ at time τ based on information at time zero is through the forward price - that is, by valuing it at time 0 and then compounding this forward at the risk free rate.

Under a passive debt management policy, a debt schedule $D = \{D_0,..,D_T\}$ (with $D_T = 0$) is decided on at time zero. The aim, by assumption, is to choose a debt schedule so as to maintain the leverage ratio (definition *d2*) constant at an arbitrarily fixed number L.

Since the Debt schedule is fixed at the outset, the associated tax shields are riskless and hence are discounted at the riskless rate r. Hence the value at time zero of the levered cash flow $\{X_{\tau+1},..X_T\}$ is

$$V_{00\tau} = \sum_{\tau+1}^{T} P_t + \sum_{\tau}^{T-1} t_c r D_t \,/\, (1+r)^{t+1}. \qquad (30)$$

The value of this cash flow at time τ, viewed from the information perspective of time zero is $V_{0\tau\tau}$ as defined by (29). A debt schedule D is chosen at time zero in order to make the expected leverage ratio a constant. That is, D is chosen to satisfy

$$D_t / V_{0tt} = L, \qquad t = 0,1,..,T\text{-}1. \qquad (31)$$

From (29), (30), and (31), the following expression for D_τ can be obtained;

$$\begin{aligned} D_\tau \quad &= L(1+r)^{\tau}.V_{00\tau} \\ &= L(1+r)^{\tau}\left(\sum_{\tau+1}^{T} P_t + \sum_{\tau}^{T-1} \{t_c r D_t \,/\, (1+r)^{t+1}\}\right), \qquad (32) \\ &(\mathit{for}\ \ \tau < T). \end{aligned}$$

This equation holds for all τ<T, so it holds for $D_{\tau\text{-}1}$. Multiplying the equation (32) for $D_{\tau\text{-}1}$ by (1+r) and subtracting this from the equation for D_τ gives the following result:

$$(1+r)D_{\tau-1} - D_\tau = LP_\tau(1+r)^{\tau} + Lt_c r D_{\tau-1}. \qquad (33)$$

or

$$D_{\tau-1} = \theta D_\tau + \theta P_\tau L(1+r)^\tau \qquad (0 < \tau < T) \qquad (34)$$

where

$$\theta \equiv 1/[1+r(1-t_c L)]. \qquad (35)$$

There is no debt at the last period of course; D_T =0. Furthermore, (34) implies the explicit solution for D_{T-1};

$$D_{T-1} = \theta L(1+r)^T P_T. \qquad (36)$$

Solving the difference equation (34) (using (36)) gives the explicit solution for value under a PDMP;

$$D_\tau = \sum_{t=\tau+1}^{t=T} L\theta^{t-\tau}(1+r)^t P_t. \qquad (0 \le \tau < T-1) \qquad (37)$$

Setting τ=0, the initial choice of debt is

$$D_0 = \sum_{t=1}^{T} L\theta^t (1+r)^t P_t. \qquad (38)$$

The initial value of the project can now be computed from (31); setting t=0 in that equation gives

$$V_{000} = D_0 / L = \sum_{t=1}^{T} \theta^t (1+r)^t P_t. \qquad (39)$$

Substituting for θ using (35), this gives

$$V_{000} = \sum_{t=1}^{T} P_t / \psi^t, \qquad (40)$$

where ψ = 1 - [$rt_c L$/(1+r)] as in (10).

As argued in section 2, this valuation should be the same as that obtained for the case of active debt management and this is indeed the case. Miles and Ezzell [1980] showed that, for the ADMP case where it is appropriate to value risky net cash flows by discounting their expected values at a constant discount rate denoted R, the following valuation formula obtains:

$$V_{ADMP} = \sum_{t=1}^{T} E_0(X_t) / (1 + r^*)^t \qquad (41)$$

where

$$r^* = R - rt_c L(1 + R) / (1 + r) \qquad (42)$$

(cf. equations (12), (13) in section 2). As discussed earlier, assuming a constant discount rate imposes the following structure on the market values of unlevered cash flows, namely that

$$P_t = E_0(X_t)/(1+R)^t , \quad t=1,..,T$$
(43)

Thus, assuming R is constant (as is conventional in this literature) is equivalent to assuming prices P_t satisfy (43). It is straightforward to show that if (43) is substituted into the formula for P_t in (40), the general PDMP valuation equation (40) collapses to the same formula as V_{ADMP} given above in equations (41) and (42). Thus we have established that (contrary to previous claims e.g. Lewellen and Emery [1986]) the standard weighted average cost of capital as developed by Miles and Ezzell for investment appraisal under ADMP can be equally appropriate for valuing finite lived projects with passively managed debt. The relevant PDMP involves maintaining value based leverage (definition *d2*) constant.

4. THE PERPETUITY CASE

Letting $T \rightarrow \infty$, the above valuation formula (40) continues to hold. The Modigliani-Miller model is an example of PDMP which assumes a constant absolute level of debt outstanding over time. Naturally, this has implications for the leverage profile as viewed from time zero. In this section, the M-M+Taxes value formula is compared with that associated with a constant expected (time zero) leverage ratio using the forward price, value based, leverage concept *d2* above.

For simplicity, we consider the case where the project or firm generates a level risky perpetuity. Such a perpetuity features expected net cash flow $E_0(X_t) = \overline{X}$, a constant for all t>0 (where expectations are taken at time zero). By assumption, R continues to denote the (assumed intertemporally constant) discount rate appropriate for valuing risky cash flows. The value of an unlevered firm (V_U) is

$$V_U = \overline{X} / R \tag{44}$$

and of the levered firm (V_L) , from (41), is

$$V_L = \overline{X} / r^* \tag{45}$$

where r^* is defined by (42). Using (42), (44) and the fact that $V_L \equiv V_{000} = D_0/L$ in (45), it is possible to obtain the valuation equation

$$V_L = V_U + \left(\frac{r}{1+r}\right) . \left(\frac{1+R}{R}\right) t_c D_0 \tag{46}$$

which may be contrasted with the M-M valuation equation that

$$V_L = V_U + t_c D_0. \tag{47}$$

Comparing these, the value based leverage concept *d2* impounds a decreasing level of debt over time (according to (34)), with, in consequence, lower added value from the tax shields; that is, since $R>r$,

$$\left(\frac{r}{1+r}\right)\left(\frac{1+R}{R}\right) < 1. \tag{48}$$

The two valuation equations (46), (47) reflect two different types of passive debt management policy, and, indeed two different time profiles of debt. Modigliani and Miller [1963] discuss the case of a firm which chooses at time zero a PDMP in which it commits itself to maintaining a constant absolute level of debt in its capital structure. That is, having the initial level D_0, this is maintained over time. The M-M valuation equation (47) is appropriate for this case. Equation (46), by contrast, is the appropriate valuation equation when the problem involves a firm which chooses to commit itself to a PDMP which maintains the leverage ratio, definition *d2*, at a fixed constant over time. The constant leverage ratio in fact entails a gradually diminishing absolute level of debt outstanding.

Thus, the appropriate firm valuation rule is seen to depend on whether intertemporal debt invariance or intertemporal capital structure invariance is assumed.

It is possible to solve for D_0 in equation (46), the perpetuity case using (38). This gives

$$D_0 = \frac{L\overline{X}(1+r)}{R(1+r) - rLt_c(1+r)} \tag{49}$$

(the initial value of the firm being $V_L = LD_0$). Thus under definition *d2*, debt monotonically decreases over time as dictated by equation (33) starting at the level D_0 given in (49). The outstanding debt is decreased over time since the futures price V_{0tt}, decreases (so as to maintain the constant leverage ratio *d2*).

The effect on the expected leverage ratio *d2* of an M-M PDMP where there is a positive constant level of debt maintained is as follows. Given that $D_t = D > 0$, constant, equations (28), (29) become

$$V_{00\tau} = \left(\frac{1}{1+R}\right)^{\tau}\left(\frac{\overline{X}}{R}\right) + \left(\frac{1}{1+r}\right)^{\tau} t_c D, \tag{50}$$

$$V_{0\tau\tau} = \left(\frac{1+r}{1+R}\right)^{\tau}\left(\frac{\overline{X}}{R}\right) + t_c D. \tag{51}$$

With a fixed debt level, the expected leverage ratio under *d2* varies with τ;

$$L_{\tau} = D / V_{0\tau\tau} \tag{52}$$

where V_{0tt} is given by (51). Since $R > r$ and $0 < t_c < 1$,

$$\mathrm{Lim}_{\tau\to\infty} L_{\tau} = 1/t_c > 1 \tag{53}$$

Thus under the Modigliani-Miller PDMP, if a constant absolute level of debt is maintained, the time zero expected leverage ratio under definition *d2* monotonically increases, and for a corporate tax rate t_c $(0 < t_c < 1)$, it turns out that for sufficiently large τ, the leverage ratio (D/V) exceeds unity. The price or value anyone would be willing to commit at time zero to paying at time τ for the future cash flow $\{X_t\}_{\tau+1}^{\infty}$ is (for sufficiently large τ) considerably less than the debt outstanding at that time!

Within the context of the model, both the constant debt and the constant leverage case are equally plausible. This is so because the M-M framework rules out the kind of considerations which would motivate choosing one type of policy rather than the other (debt capacity, risk of financial distress etc.). However, the perpetuity model discussed here is often used as the basis for cost of capital computations (e.g. OFTEL [1992], OFWAT [1991], OFGAS [1992]) because it is reasonably straightforward to operationalise. If it is so used, it follows that the assumptions over debt management policy will affect the estimates of a firm's beta (and hence of its cost of capital within a CAPM framework).

V. IMPLICATIONS FOR APPLIED WORK AND POLICY

The primary motivation for this paper was to show that the weighted average cost of capital could be used to value finite variable risky cash flows not only when debt was actively managed (a result previously obtained by Miles and Ezzell [1980]), but also under a passive debt management policy. Value under PDMP was shown to depend upon the choice of leverage concept; with the same constant target level over time, different leverage concepts imply different debt profiles and hence different firm values. Value under ADMP and PDMP are not in general equal when the PDMP leverage concept is that of debt to expected value. However, when the PDMP leverage concept is placed on a value basis, a basis consistent with that used in ADMP, there is no value difference, and the weighted average discount rate correctly values the cash flow under both policies.[7]

Empirically, this raises a question as to which leverage concept is adopted in practice by those who pursue PDMP leverage targets. If firms tend to adopt a particular PDMP target leverage ratio based on a particular leverage concept, this will affect the choice of calculation appropriate here. If it is debt to expected value, then the PDMP valuation equation developed by Ashton and Atkins [1978] from the general APV approach of Myers [1974] is appropriate. However, if the value based leverage concept is used, then the appropriate valuation formula for PDMP is that established by Miles and Ezzell [1980] for ADMP. The value based leverage concept is not only theoretically more appealing but also would appear to be a more plausible representation of the PDMP in practice. In the classic passive debt strategy, debt is issued at time zero and repaid in installments over the life of the project/firm. The terms of this debt contract are effectively a series of forward contracts. Since they are being related to a value base, it is logical to view this base in terms of forward values.

The cost of capital is of central importance in the practical regulation of privatised industries such as telecommunications, electricity, gas and water since it is one of the factors which influence a regulator's choice of an appropriate 'price-cap'. Estimation of a company's cost of capital requires not only a specification of the tax environment (usually assumed to be some version of M-M) but also the specification of its debt management policy. To illustrate the potential difference in the discount rates based on the old PDMP rule and the 'new' PDMP (=ADMP) rule, consider these rates for the perpetuity case. For the old PDMP rule, the discount rate is $r^* = R(1\text{-}t_cL)$ (see e.g. Brealey and Myers [1991, p462]), whilst the rate under the 'new' PDMP rule is given by (13). The difference, in a UK context, could amount to around one percentage point or so in the estimate of the discount rate.[6] This may not seem a lot, but it does imply a shift in the choice of price cap (and indeed, at least theoretically, the choice of price cap can, in some cases, be quite sensitive to the estimate of the cost of capital). Furthermore, even small effects of this type regarding debt management policy can have important distributional effects - as shareholders gain *vis a vis* consumers from higher estimates in the cost of capital. In the UK, the last few years have seen considerable debate between regulators and the companies they regulate regarding this assessment of the cost of capital, with the two parties' assessments often diverging by several percentage points; the choice of formulae for estimating the cost of capital is clearly an important issue here (see for example WSA [1991], OFWAT [1991,1994], OFTEL [1992a, 1992b], British Telecom [1992], OFGAS [1992]).

A further implication of our analysis is that the degearing formulae developed for the active case by Appleyard and Strong [1989] also holds when firms follow a passive debt management policy in which the aim is to maintain leverage constant. This has implications for estimation of firm betas and the estimation of the cost of capital via the 'pure play' CAPM approach, as follows. In the M-M model, the gearing of levered equity in a debt-equity scenario as developed by Hamada [1972] is (in an obvious notation)

$$\beta_e = \beta_u + (1 - t_c)(\beta_u - \beta_d)D / E, \qquad (54)$$

while for the Miles Ezzell ADMP case,

$$\beta_e = \beta_u + (1 - \frac{t_c r_d}{1 + r_d})(\beta_u - \beta_d) D / E \tag{55}$$

(see Appleyard and Strong [1989]). The difference in these beta estimates is particularly clear when r_d is small, when the ADMP degearing formula (55) can be approximated by the formula

$$\beta_e \approx \beta_u + (\beta_u - \beta_d) D / E. \tag{56}$$

Under the assumption that any firm operating a PDMP sets a constant debt to value target of the type discussed in this paper, the above beta gearing-degearing formulae ((55) or (56)) for active debt policy applies equally to a M-M type passive policy for finite lived projects. When the weighted cost of capital is estimated using the CAPM approach, the procedure involves first the estimation of β_e, and then the estimation of β_u via (54) or (55). Thus different estimates of β_u arise from different beta degearing formulae, and hence different estimates of the firm's cost of capital.[8]

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F O O T N O T E S

[1]In practice of course, outside the Modigliani-Miller environment assumed in this literature, there are good reasons why value under ADMP might be greater (rather than less) than that under PDMP. This is so because PDMP fixes the debt profile *ab initio*. With the equity market value fluctuating over time, so too does the leverage ratio; the potential for financial distress is thus greater than in the case where the debt is adjusted from period to period. However, in the M-M framework, financial distress and bankruptcy are ruled out by assumption.

[2]This notation (P_t) is used primarily in sections III and IV.

[3]There is thus no risk of tax exhaustion or financial distress.

[4]State conditional values are required if the ADMP and PDMP leverage concept used in earlier work are to be operational.

[5]$PV_0\{PV_1(X_2)\} = PV_0(X_2)$ or there would be an arbitrage opportunity.

[6]Of course, introducing the possibility of tax exhaustion and risky debt would lead to other sources of divergence in value as between PDMP and ADMP. These effects would tend to make ADMP relatively more attractive. For example, active debt management policy can take advantage of the latest information on future expected cash flow in order to maximise the gain from carry forward and carry back provisions for tax losses etc.

[7]Thus, suppose R=10%, r=5% and t_c=0.3. The following table illustrates the effect of variation in the debt ratio L.

<u>Discount Rates (DR) (%)</u>

Debt Ratio, L:	0.0	0.3	0.6	0.9
DR under 'New' PDMP/ADMP:	10.0	9.53	9.06	8.59
DR under 'Old' PDMP:	10.0	9.10	8.20	7.30

Difference:	0.0	0.43	0.86	1.29

Thus for the UK at present, for typical debt ratios and typical rates of interest, we may be looking at differentials of around 0.5-1.5%.

[8]An additional difficulty faced by external (but not internal) observers of the firm is that valuation formulae under PDMP hold theoretically only at $t=0$ (this is just as much true for level perpetuity Modigliani-Miller valuation models as for those discussed here). Whilst it might be possible to identify the point in time when a firm was deemed to institute a PDMP, this would be clearly difficult to operationalise in practice. No doubt it is for this reason that the empirical literature which makes use of these alternative valuation and gearing formulae implicitly or explicitly appears to take the pragmatic view that alternative models are validated primarily by their predictive performance (as for example with Fuller and Kerr's [1981] assessment of whether gearing adjustments improve the quality of estimates of the cost of capital).