INTERTEMPORAL PRICE CAP REGULATION UNDER UNCERTAINTY

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ABSTRACT

This paper examines the intertemporal price cap regulation of a firm that has market power. Under uncertainty, the unconstrained firm 'waits longer' before investing or adding to capacity and as a corollary, enjoys higher prices over time than would be observed in an equivalent competitive industry. In the certainty case, the imposition of an inter-temporal price cap can be used to realise the competitive market solution; by contrast, under uncertainty, it cannot. Even if the price cap is optimally chosen, under uncertainty, the monopoly firm will generally (a) under-invest and (b) impose quantity rationing on its customers.

1. Introduction

Price cap regulation has been extensively studied over recent years in both atemporal and inter-temporal contexts. In the atemporal context, the focus has often been on how to deal with multiple and new products, on efficiency and incentive issues, or on how to regulate complex tariffs (e.g. Hillman and Braeutigan, 1989; Laffont and Tirole, 1990a,b; Armstrong, Cowan and Vickers, 1995), whilst in the intertemporal context, the construction of price adjustment processes and their potential manipulation by regulated firms has been examined (e.g. Hagerman, 1990; Braeutigan and Panzar, 1993). This paper by contrast focuses on the impact of inter-temporal price cap regulation on the firm's choice of investment in capacity when such investment is largely irreversible and when evolution of key variables such as product demand, technology etc. are governed by stochastic processes.

The impetus for the present work lies in the literature on access pricing[1] which has recently begun to recognise firstly that the access pricing problem is an inter-temporal problem, in that prices are for short run access to long lived (typically network) capacity (see Salinger, 1998; Sidak and Spulber, 1997), and secondly that uncertainty and option value could be important in this context (Hausman, 1996; 1997; 1999).[2] Following from this, it has been suggested that firms that control bottleneck facilities or capacity, when required to provide access, should be allowed to set a price no higher than could be expected to hold if the capacity was provided by a competitive industry - where this price should take account of the impact of uncertainty (Hausman, 1997; Laffont and Tirole, 2000). This suggestion, whilst perhaps intuitively plausible, has not been subject to formal analysis - and this motivates the present paper, in which the performance of an explicit price cap constraint is examined in some detail.

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[1] 'Bottleneck facilities' arise in many industries, particularly network industries, such as telecoms, railtrack, water, electricity and gas. Firms that control such facilities have typically been required by regulators to provide open access to these facilities. Of course, access is offered at a price – and so, given the inherent monopoly power associated with access provision, the question arises as to what constitutes a fair or efficient access price (see for example Armstrong, Doyle and Vickers, 1996; Armstrong, 1998; Baumol and Sidak, 1994; Laffont, Rey and Tirole, 1998a,b).

[2] There is now a fair body of work on the option value that arises out of the firm being able to defer the date at which irreversible capacity investment is made (see e.g. Dixit, 1989; Lucas and Prescott, 1971; McDonald and Siegel, 1986; Pindyck ,1988). Dixit and Pindyck (1994) is probably the seminal text in this area.

Whether under certainty or uncertainty, capping prices at the competitive level is the best that can be done. Indeed, under certainty in this model, such a competitive price cap will induce the firm to emulate the competitive solution. However, under uncertainty, this is no longer the case. When subject to a price cap, whether or not the price cap is set at the competitive level, the monopoly firm will have an incentive to under-invest in capacity. As its selling price becomes price cap constrained, the monopoly firm defers adding to capacity for the same reason as in the unconstrained monopoly case - because of the downside risk that demand will fall away. Deferring investment and rationing demand can make sense simply because, if demand does fall in the future, with less installed capacity future prices are less depressed. Of course, if the level of demand increases sufficiently, the risk of such downside movements becomes less - so when there is enough weight of demand, the price cap constrained firm will eventually be induced to add to capacity.

It has been observed in earlier work (Dixit and Pindyck, 1994) that the entry trigger price that stimulates investment is often the same for monopoly and competitive industries - in the case where the monopolist is contemplating the undertaking of a single fixed size investment. However, in the model developed in this paper, the monopoly firm is able to choose *both* the level *and* timing for its investments. As a consequence, its choices will diverge from those manifest in a competitive industry – with the extent of the divergence increasing with the extent of its market power (the less elastic the industry demand curve). For most industries, including network industries such as telecoms, electricity, gas etc., the assumption that the firm can control the level of investment as well as its timing is fairly realistic; the initial level of capacity is a choice variable, and capacity can be subsequently and incrementally upgraded and expanded. In such circumstances, monopolists tend to restrict the level of investment in capacity so as to enjoy higher prices over time. As will be seen, imposing price caps, cannot eliminate this general effect.

Section 2 outlines the basic model and identifies the intertemporal 'trigger price' which would induce the firm to add to capacity, and compares the solution with that for the competitive industry case and also with solutions under certainty. Section 3 then examines how an inter-temporal price cap would modify the behaviour of the firm whilst section 4 gives numerical examples and sensitivity analysis. Finally, section 5 draws together the principal conclusions.

2. Solutions under certainty and uncertainty in the absence of price caps

Capacity is assumed long lived but subject to physical depreciation, with technical progress reducing the unit cost of capacity provision. Industry demand is assumed to have constant elasticity, with the 'strength' of demand uncertain. The assumption of constant elasticity demand is useful in two ways: Firstly, as a convenient parameterisation facilitating the exploration of alternative assumptions regarding this elasticity. Secondly, as it facilitates the derivation of closed form solutions which are easy to interpret and debate.[3] In the case of access to bottleneck facilities, the firm, having installed capacity, is required to offer access at a price to downstream users. In the case where the firm produces some other product via a production function, it is assumed that output is strictly proportional to installed capacity. Thus in either case, the firm effectively gets a short term price for each unit of capacity (either as a price for providing access to the capacity, or for the sale of output from the capacity).

Table 1 here

Table 1 gives a glossary of notation for ease of reference. Space considerations also preclude full derivations; the core structure of the models and the key results are presented in the sections below, with derivations given in the appendix.

Let Q_t denote installed capacity at time t, whilst Q_t^d is the demand for capacity. The demand and inverse demand functions for capacity at time t are given by

$$Q_t^d = A_t p_t^\gamma, \quad \text{and} \quad p_t = A_t^{-\eta} \left(Q_t^d \right)^\eta \qquad \text{where } \eta \equiv 1/\gamma < 0 \qquad (1)$$

and p_t is the instantaneous price gained from the sale of output/access, per unit of capacity, whilst $\gamma < -1$ is the elasticity of demand.[4] For simplicity, uncertainty enters

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[3] The stimulus for the present work originated in the access pricing debate in UK Telecoms regarding whether (and how much) allowance should be made for uncertainty when assessing reasonable levels for access prices. The results obtained in this paper facilitate the computation of the allowance that should be made for uncertainty in the assessment of such prices.

[4] Demand is assumed elastic for the usual reason that, if demand was inelastic, profit $\rightarrow +\infty$ as $Q_t \rightarrow 0$.

solely through the level of demand variable, A_t and this process is assumed to be a geometric Brownian motion (*GBM*);

$$dA_t / A_t = \alpha dt + \sigma d\varpi_t. \tag{2}$$

Here α is the trend rate of growth in demand (which could be positive or negative) and σ is the associated volatility. With elastic demand, it is easy to show that it pays the firm to fully utilise its installed capacity Q_t at all times. Thus price p_t is always set such that $p_t = A_t^{-\eta}(Q_t)^{\eta}$; the evolution of price p_t over time is thus determined by the evolution of demand along with capacity investment choices over time.[5]

Technical progress is assumed to reduce the unit cost of capacity, denoted K_t at a constant rate δ (so that $K_t = K_0 e^{-\delta t}$, $\dot{K}_t = -\delta K_t$). It is straightforward to extend the model to incorporate technical progress as a stochastic process, and to allow correlations between this and the demand process. However, such extensions merely add notational clutter without altering the basic properties of the model, and so are not pursued here.[6] Capacity once installed is assumed to physically depreciate at a constant rate θ.[7] As there are no variable costs associated with its use, capacity is an irreversible investment; at all times, installed capacity will be fully utilised since output can always be sold at a non-negative price.

Competition: Results under Certainty

Before analysing the uncertainty case, it is useful, as a benchmark, to present some results for the certainty case. The competitive equilibrium market price under certainty, relative to unit capacity cost, has been calculated in earlier work as $p_t / K_t = (r + \theta + \delta)$ (see Salinger, 1998; Sidak and Spulber, 1997; Laffont and Tirole, 2000, p. 151). That is, the sum of the interest rate, the rate of depreciation and the rate of technical progress. This makes sense; in the absence of depreciation and technical progress, the price of selling

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[5] In the absence of the price cap, price is set such that demand is always equal to installed capacity; in section 3, when the price cap binds, it is shown that there is quantity rationing.

[6] This more general case is covered in some detail in an original working paper focusing on access pricing (Dobbs, 2000; available as a .pdf file at http://www.staff.ncl.ac.uk/i.m.dobbs/).

[7] Equivalently, in terms of the ensuing mathematical analysis, one could assume that each individual unit of capacity was subject to a stochastic 'death process' in which the probability of the plant ceasing to be operational is a constant per unit time; see Merton (1976).

access to, or output from, capacity is simply rK (interest rate × unit capacity cost); the present value of this revenue stream then just covers the initial unit capacity cost. Physical depreciation and technical progress simply push up the effective interest rate from r to $r+\theta+\delta$. In the present model, the certainty price is indeed given by this formula, so long as demand is not collapsing at too fast a rate:

> **Result 1.** Under certainty,
> (a) if $\delta > \eta(\alpha+\theta)$, there is continuous investment over time and $p_t / K_t = \theta + r + \delta$ for all $t \geq 0$.
> (b) if $\delta \leq \eta(\alpha+\theta)$, there is an initial pulse of investment at time zero, and no subsequent investment. The initial price is $p_0 / K_0 = (\theta + r + \eta(\alpha+\theta))$ and then $p_t = p_0 e^{-\eta(\alpha+\theta)t}$ for $t > 0$.

> **Proof:** Omitted

The intuition for result 1 is straightforward. At time 0, there is an instantaneous pulse of investment in capacity. It can then be shown that there is either continuous further investment – or none at all. If there is continuous investment, the hire price falls at the same rate δ as for unit capacity cost K_t, such that the hire price is any time t given as $p_t = (\theta + r + \delta) K_t$. The present value of such future hires is, of course, just equal to the initial capital outlay.[8] By contrast, if the trend in demand is sufficiently negative, whilst physical depreciation is sufficiently slow, then there is simply an initial pulse and no subsequent investment. This occurs if $(\alpha+\theta)/\gamma > \delta$ or equivalently, if $\alpha < \gamma\delta - \theta$. Since the elasticity of demand $\gamma < 0$ and depreciation $\theta \geq 0$, this can only occur if demand is falling sufficiently fast (α sufficiently negative). When this occurs, the demand effect depresses price at a faster rate than δ, and hence the initial trigger entry price has to be higher to compensate for the ensuing faster decline in the price profile (to motivate the initial investment, competitive firms must expect future prices to be such that

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[8] That is, for any starting time τ, given that $K_t = K_\tau e^{-\delta(t-\tau)}$, that $p_t = (\theta + r + \delta) K_t$, and that at time t the unit capacity has depreciated to $e^{\theta(\tau-t)}$, it follows that

$$K_\tau = \int_\tau^\infty p_t e^{(\theta+r)(\tau-t)} dt = p_\tau \int_\tau^\infty e^{(\theta+r+\delta)(\tau-t)} dt = p_\tau / (\theta + r + \delta).$$

the investment is at least a zero *NPV* transaction).[9] Demand has to fall at a fairly high trend rate for this to happen, and so the case is of little practical importance. Accordingly, in the rest of this paper, it is assumed that $\delta \geq \eta(\alpha + \theta)$ and hence that

$$\xi_c \equiv \theta + r + \delta \tag{3}$$

is indeed the certainty relative price.[10]

Results under Uncertainty

After the initial investment in capacity at time zero, under uncertainty, investment is characterised by periods of continuous investment and periods on which the firm chooses not to invest. Clearly, periods of falling demand will tend to be associated with non-investment whilst expansion of demand will tend to stimulate further additions to capacity. In what follows, it suffices to focus on a time interval on which there is no investment, followed by a consideration of the conditions on the boundary at which investment commences. The evolution of capacity on a non-investment time interval is described by the process $dQ_t = -\theta Q_t dt$, whilst the price process is driven by (2) through (1); applying Itô's lemma, and defining

$$\mu_p \equiv -\eta\left(\alpha + \theta - \tfrac{1}{2}(\eta+1)\sigma^2\right), \tag{4}$$

the price process is also *GBM* and can be written as

$$dp_t = \mu_p p_t dt - \eta\sigma p_t d\varpi_t. \tag{5}$$

Notice that, from (4), demand volatility affects the trend rate in the price process. The firm is assumed to maximise expected present value; at some time τ during an interval of non-investment, this is

$$V\left(p_\tau, K_\tau, Q_\tau\right) = E_\tau\left\{\int_\tau^{\tilde{i}} p_t Q_t e^{-r(t-\tau)} dt + V\left(p_{\tilde{i}}, K_{\tilde{i}}, Q_{\tilde{i}}\right)e^{-r(\tilde{i}-\tau)}\right\}. \tag{6}$$

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[9] In this case, price is driven by the inverse demand function. Since $Q_t = Q_0 e^{-\theta t}$, $A_t = A_0 e^{\alpha t}$ and $p_t = A_t^{-\eta} q_t^\eta = p_0 e^{-\eta(\alpha+\theta)t}$, then

$$K_0 = \int_0^\infty p_t e^{-(\theta+r)t} dt = p_0 \int_0^\infty e^{-(\theta+r+\eta(\alpha+\theta))t} dt = p_0 / \left(\theta + r + \eta(\alpha+\theta)\right).$$

[10] The original working paper covered the general case, but restricting attention to the normal case where $\delta > \eta(\alpha+\theta)$ reduces complexity without substantively changing any of the results.

Here r denotes an appropriate discount rate[11] and E_τ denotes the expectations operator, where expectations are formed at time τ. The time $\tilde{t}$ denotes the end of the period of non-investment, a point in time at which new investment adds further to capacity.

The value function V is homogenous in prices and is also linear in Q_τ, and so can be written as

$$V(p_t, K_t, Q_t) \equiv \psi(x_t) K_t Q_t \qquad \text{where } x_t \equiv p_t / K_t \qquad (7)$$

denotes the relative price. It is also useful to define the 'per unit capacity' value function as

$$v(p_t, K_t) = V(p_t, K_t, Q_t) / Q_t = \psi(x_t) K_t. \qquad (8)$$

Using this, (6) can be simplified to give

$$\psi(x_\tau) K_\tau = E_\tau \left\{ \int_\tau^{\tilde{t}} p_t e^{-(r+\theta)(t-\tau)} dt + \psi(x_{\tilde{t}}) K_{\tilde{t}} e^{-(r+\theta)(\tilde{t}-\tau)} \right\}. \qquad (9)$$

(using the fact that $Q_t = Q_\tau e^{-\theta(t-\tau)}$). Equivalently, in terms of the per unit capacity value function, this becomes

$$v(p_\tau, K_\tau) = E_\tau \left\{ \int_\tau^{\tilde{t}} p_t e^{-r(t-\tau)} dt + v(p_{\tilde{t}}, K_{\tilde{t}}) e^{-r(\tilde{t}-\tau)} \right\}. \qquad (10)$$

The optimisation of (6) or (10) is fairly routine, although the process is somewhat involved. Essentially, the process involves solving an ordinary differential equation for $\psi(x)$; the solution can be shown to take the form (see appendix)

$$\psi(x) = B_0 x + B_1 x^{\lambda_1} + B_2 x^{\lambda_2}, \qquad (11)$$

where

$$B_0 = \frac{1}{\theta + r - \mu_p} = \frac{1}{(\theta + r) + \eta\left(\alpha + \theta - \frac{1}{2}(\eta + 1)\sigma^2\right)}, \qquad (12)$$

$$\lambda_1 = \left(-R_1 + R_2\right)/\eta^2 \sigma^2, \qquad (13)$$

$$\lambda_2 = \left(-R_1 - R_2\right)/\eta^2 \sigma^2, \qquad (14)$$

(λ_1, λ_2 are roots to a fundamental quadratic equation) and where

$$R_1 \equiv \left(\mu_p + \delta - \frac{1}{2}\eta^2\sigma^2\right), \qquad (15)$$

$$R_2 \equiv \left(\left(\mu_p + \delta - \tfrac{1}{2}\eta^2\sigma^2 \right)^2 + 2\eta^2\sigma^2 \left(\theta + r + \delta \right) \right)^{1/2}. \tag{16}$$

Notice that $2\eta^2\sigma^2 \left(\theta + r + \delta \right) > 0$ if $\sigma^2 > 0$, so the roots are real and of opposite sign when uncertainty is present. The arbitrary constants B_1, B_2 are determined by boundary conditions. Given $\lambda_2 < 0$, as relative price $x \to 0$, if per unit value, $v(p_t, K_t)$, is to be finite, it must be that $B_2 = 0$ - see Dixit (1993) on this type of boundary condition. By contrast, if relative price increases sufficiently, then a value is reached at which new investment is triggered. The constant B_1 is determined by an analysis of smooth pasting conditions at this boundary – and the value is different depending on whether the industry is competitive or a monopoly. For the competitive market case, the relative price at which investment is triggered under certainty is denoted ξ_c; under uncertainty, it is denoted ξ_u and in the monopoly case under uncertainty, ξ_M. The results can be summarised as follows:

> **Result 2.** In the presence of uncertainty ($\sigma > 0$), new investment is triggered when the relative price rises to the level
> (i) If the industry is perfectly competitive:
> $$\xi_u = \left(\frac{\lambda_1}{\lambda_1 - 1} \right)\left(\theta + r - \mu_p \right) = \left(\frac{\lambda_2 - 1}{\lambda_2} \right)\left(\theta + r + \delta \right) = \left(\frac{\lambda_2 - 1}{\lambda_2} \right)\xi_c .$$
> (ii) If a monopoly firm supplies the industry:
> $$\xi_M = \left(\frac{\gamma}{1+\gamma} \right)\left(\frac{\lambda_2 - 1}{\lambda_2} \right)\left(\theta + r + \delta \right) = \left(\frac{\gamma}{1+\gamma} \right)\left(\frac{\lambda_2 - 1}{\lambda_2} \right)\xi_c = \left(\frac{\gamma}{1+\gamma} \right)\xi_u .$$
>
> **Proof:** see appendix

The result that $\xi_u = \left(\frac{\lambda_1}{\lambda_1 - 1} \right)\left(\theta + r - \mu_p \right)$ has been seen several times in earlier work in which a price process has been assumed to be *GBM* in competitive markets, and also in the case where a monopoly firm is considering a fixed size, all-or-nothing type of investment (see e.g. Dixit and Pindyck, 993; Hausman, 1997). The term $\lambda_1 / \left(\lambda_1 - 1 \right)$ is often termed an option value multiplier as it multiplies what is taken to be the price under certainty. However, in this model, uncertainty also affects the investment relative trigger price not simply through the standard option multiplier $\lambda_1 / (\lambda_1 - 1)$ but through its effect on μ_p which is also affected by volatility σ, via (4). This is logical since, given a

downward sloping industry demand function, demand uncertainty will naturally tend to impact on the trend rate in price, μ_p. Relative to models which simply assume the trend in price is a fixed datum (for example, Dixit and Pindyck, 1994, c. 6,7; Hausman, 1997; 1999), this alters the comparison of certainty and uncertainty solutions and tends to *reduce* the overall impact of uncertainty. This is reflected in the option multiplier on the certainty price, which is $(\lambda_2 - 1)/\lambda_2$ - although this 'option multiplier' is not one which has been noted in previous work. As explained above, for plausible parameter values, the multiplier $(\lambda_2 - 1)/\lambda_2$ takes a smaller value than $\lambda_1/(\lambda_1 - 1)$.

Dixit, Pindyck, and Sodal (1999), in dealing with an 'all-or-nothing' fixed size monopoly investment, interpreted the option multiplier $\lambda_1/(\lambda_1 - 1)$ as an elasticity mark-up. Here, it can be seen in Result 2(ii) that when the firm controls not only the initial timing but also the initial scale, and has the ability to subsequently add to this investment, then there is an additional 'demand elasticity' based mark-up $\gamma/(\gamma + 1)$. As in the single period single product case, the monopoly firm has an incentive to reduce its investment in capacity in order to enjoy higher prices than would be possible under competition. Also, note that removing market power by letting $\gamma \to -\infty$, the monopoly solution in Result 2(ii) converges on that for a competitive industry in result 2(i), as one would expect.

Relative prices are prices relative to unit capacity cost, and of course, capacity cost is falling at the rate $K_t = K_0 e^{-\delta t}$. Thus, denoting the absolute level for the competitive entry price at which new investment enters the market as $p_e^u(t)$, and for the monopoly case as $p_e^M(t)$, then

$$p_e^u(t) = \xi_u K_t \tag{17}$$

and

$$p_e^M(t) = \xi_M K_t = \left(\frac{\gamma}{1 + \gamma}\right) p_e^u(t). \tag{18}$$

That is, the investment trigger price at which a monopolist adds to capacity is given as the competitive investment entry trigger price (under uncertainty) multiplied by the standard

monopoly mark-up.[12] Hence, since $\xi_M > \xi_u$, the monopolist only adds to capacity when price reaches a higher value than would be the case under competition; prices are at all times higher under monopoly than under competition, whilst, concomitantly, installed capacity is less.

From (1), the firm at time 0, installs capacity Q_0^M so that its initial selling price is $p_e^M(0) = \xi_M K_0$ where K_0 is initial unit capacity cost. That is, it chooses Q_0^M, such that $Q_0^M = A_0(p_e^M(0))^\gamma = A_0(\xi_M K_0)^\gamma$. By contrast, a competitive industry would install $Q_0^u = A_0(\xi_u K_0)^\gamma$, so a measure of the extent of monopoly under-investment is given by the ratio $Q_0^M / Q_0^u = (\xi_M / \xi_u)^\gamma = \left(\frac{\gamma}{1+\gamma}\right)^\gamma$. Following the initial investment, demand evolves via (2), capacity depreciates at the rate θ and price evolves according to (5). At each point in time, the firm must decide whether to wait or whether to add to its current level of capacity. If and when price p_t reaches the level given in (18), the monopoly firm starts adding to capacity; the level of capacity at any point in time at which the firm is undertaking positive investment can then be calculated as $Q_t^M = A_t(p_e^M(t))^\gamma = A_t(\xi_M K_t)^\gamma$, whilst, for a competitive industry $Q_t^c = A_t(\xi_c K_t)^\gamma$. Thus at any point in time when both the monopolist and the competitive industries are adding to capacity, their levels of capacity can easily be compared, since $Q_t^M / Q_t^c = \left(\frac{\gamma}{1+\gamma}\right)^\gamma$ (<1).

3. Monopoly subject to Price Cap

In this section, the price the firm chooses to set at time t, denoted p_t^s, is restricted by a price cap constraint of the form discussed in section 1, namely that

$$p_t^s \le \overline{p}_t \qquad \text{where } \overline{p}_t = \overline{\xi} K_t \qquad\qquad (19)$$

and $\overline{\xi}$ is a constant chosen by the regulator. If the regulator sets $\overline{\xi} = \xi_u$, then the maximum price the firm is allowed to set is indeed the competitive price at which further investment would be stimulated. Notice that, in this formulation, in the absence of

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[12] In the single period Monopoly pricing problem under certainty, profit maximisation requires setting a price $p_M = \left(\frac{\gamma}{1+\gamma}\right) MC$, where MC denotes marginal cost - which would also correspond to the competitive price p_u in a competitive market. That is, $p_M = \left(\frac{\gamma}{1+\gamma}\right) p_u$.

technical progress reducing the unit cost of capacity, K_t is a constant, and the price cap is constant over time. Where technical progress reduces the cost of capacity provision, this translates into a tightening of the absolute price cap. However, in terms of relative price, the cap is constant over time.

Let p_t now stand for the **market clearing price**, the price which would equate the level of demand to the currently available capacity, such that (1) gives the relationship between this market clearing price and installed capacity. Of course, the firm's **actual** choice of price, p_t^s, must satisfy the price cap (19) and so is given as

$$p_t^s = Min[p_t, \overline{p}_t] = Min[p_t, \overline{\xi} K_t].\qquad(20)$$

There are now two possible non-investment regimes. When demand falls sufficiently relative to installed capacity, price will be below the price cap – and the firm will wait (regime 1). This is a situation in which $p_t / K_t < \overline{\xi}$. As demand increases relative to capacity (and recall that installed capacity is constantly depreciating), the firm may allow the market clearing relative price to rise above the level imposed by the intertemporal price cap $\overline{p}_t$. At such a point, the firm is price constrained, as it has to set a price p_t^s such that $p_t^s / K_t = \overline{\xi}$. Under uncertainty, it can be shown that the value maximising choice of the firm is indeed to choose not to add immediately to capacity – but to wait for a further increase in demand. On such intervals, the firm imposes quantity rationing on customers (regime 2). Thus, from (1), demand at the price $\overline{p}_t$ is $Q_t^d = A_t \overline{p}_t^{\gamma}$ whilst installed capacity is related to the market clearing price p_t by $Q_t = A_t p_t^{\gamma}$. Given $\gamma < 0$, when $p_t > \overline{p}_t$, clearly $Q_t^d > Q_t$ and there is excess demand. Finally, if demand increases sufficiently relative to installed capacity, the firm is induced to add to capacity (regime 3).

The market clearing relative price at which new investment is triggered is denoted ξ. Whilst being required to set the price $p_t^s \leq \overline{\xi} K_t$, for its given level of capacity, the market clearing price p_t is the price the firm would like to set (if it was not constrained by the price cap). Whenever this market clearing price $p_t \to \xi K_t$, the firm starts to add to capacity. In the presence of uncertainty ($\sigma > 0$), the formal analysis parallels that for

the unconstrained case although in the price-cap case there are two regime boundaries at which smooth pasting conditions apply. A sketch of the solution procedure is given in the appendix, and full step by step derivations can again be found in the original working paper,[13] available at the website http://www.staff.ncl.ac.uk/i.m.dobbs/. Focusing on ξ, the solution is:

> **Result 3.** The price cap monopoly relative entry market clearing price can be written as
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> $$\xi = \left\{ \left(\overline{\xi} - \xi_c \right) \xi_M \overline{\xi}^{\,\lambda_2 - 1} \Big/ \left(\xi_M - \xi_c \right) \right\}^{1/\lambda_2} .$$
>
> **Proof:** in Dobbs (2001).

Result 3 gives the *relative* market clearing price ξ at which new investment is triggered (price relative to unit capacity cost), expressed as a function of the (relative) price cap, $\overline{\xi}$. The implications for capacity investment are discussed later – but it is perhaps worth re-emphasising the connection between prices and quantities. Thus, note that, at any point in time *t* where demand is sufficient to induce new investment under the price cap, the market clearing price is $p_t = \xi K_t$ and the level of capacity invested is

$Q_t = A_t p_t^{\gamma} = A_t \left(\xi K_t \right)^{\gamma}$ (where $\gamma < -1$). It follows that the higher the value for ξ, the less the installed capacity *ceteris paribus*. In particular, the larger the value for ξ, the smaller the time zero initial level of investment will be.

Table 1 about here

Fig. 1 illustrates how ξ behaves as a function of $\overline{\xi}$, the tightness of the constraint in (19) .[14] The key point to note is that $\xi > \overline{\xi}$ when the price cap $\overline{\xi}$ is set in the range (ξ_c, ξ_M). That is, under uncertainty, the market clearing relative price at which new investment is triggered lies above the price cap and, as a corollary, there is quantity rationing (the extent of this rationing is studied later).

Fig. 1 here

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[13] The results obtained in this paper depend on smooth pasting conditions; these are akin to first order necessary conditions for value maximisation; as in the static optimisation case, formulae derived from such conditions might in principle identify local maxima, minima or inflection points. It is reasonably straightforward to verify the results obtained in this paper are associated with value maximisation through the use of numerical simulation (a Fortran programme for this is available at http://www.staff.ncl.ac.uk/i.m.dobbs/).

[14] The numerical values originate from the benchmark parameter values given in table 2 below; these are discussed in more detail in section 4.

An analysis of the formula for ξ in result 3 establishes the following structural characteristics:

> **Result 4**: Properties of the function $\xi(\overline{\overline{\xi}})$:
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> (i) $\xi(\overline{\overline{\xi}})$ attains its global minimum on the interval (ξ_c, ξ_M) at $\overline{\overline{\xi}} = \xi_u$ with $\xi(\overline{\overline{\xi}})$ strictly decreasing on (ξ_c, ξ_u) and strictly increasing on (ξ_u, ξ_M): that is, $\xi_u = Argmin_{\xi_c \leq \overline{\overline{\xi}} \leq \xi_M} \xi(\overline{\overline{\xi}})$.
>
> (ii) $Lim_{\overline{\overline{\xi}} \downarrow \xi_c} \xi(\overline{\overline{\xi}}) = +\infty$
>
> (iii) $Lim_{\overline{\overline{\xi}} \uparrow \xi_M} \xi(\overline{\overline{\xi}}) = \xi_M$
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> (iv) $\xi(\xi_u) > \xi_u$ **if** $\sigma > 0$
>
> (v) $Lim_{\sigma \to 0} \xi(\xi_u) = \xi_c$

> **Proof:** See appendix.

Result 4 establishes the general shape of the function $\xi(\overline{\overline{\xi}})$ as that depicted in Fig. 1. Result 4 (i), clearly illustrated in Fig. 1, indicates that, if the regulator's aim is to get prices under monopoly as close as possible to what they would be under competition, ξ_u, then setting the maximum allowed relative price $\overline{\overline{\xi}} = \xi_u$, the competitive price, is the best that can be done. Result 4(ii) can be explained as follows. Under certainty, the firm gets zero net present value from installing a unit of capacity only if it is *guaranteed* able to sell the product (or access to its capacity) at the certainty relative price ξ_c for ever. By contrast, in the uncertainty case, if the certainty relative price was set as the maximum price under the price cap (i.e. $\overline{\overline{\xi}} = \xi_c$), then there is positive probability that demand will shift sufficiently adversely for the relative price to drop below this level. Thus, the firm would always see the investment as having negative expected *NPV* – and hence would not invest at all. As the price cap is tightened down to the level ξ_c, the firm sets a higher and higher market clearing price before it is willing to add capacity. That is, at time zero, it installs less and less capacity, the closer the relative price cap $\overline{\overline{\xi}}$ is to ξ_c. Result 4 (ii) shows that, in the limit, as $\overline{\overline{\xi}} \to \xi_c$, so $\xi \to +\infty$ and the firm will not install any capacity at all; this is the left asymptote in Fig. 1.

Result 4 (iii) is the intuitively obvious fact that as the price cap ceases to bind (ever), the price constrained trigger price converges on that of the unconstrained monopolist. Result

4 (iv) establishes that, if there is uncertainty, setting the price cap at the competitive level ($\bar{\bar{\xi}} = \xi_u$) does not realise the competitive solution, a point clearly illustrated in Fig. 1. Given uncertainty ($\sigma > 0$), $\xi(\xi_u) > \xi_u$; recall also that $\xi > \xi_u$ implies under-investment and periods on which the firm will impose quantity rationing. Result 4 (v) finally states that, as $\sigma \to 0$, then $\xi(\xi_u) \to \xi_c$. That is, under certainty, the competitive relative trigger price is ξ_c; setting $\bar{\bar{\xi}} = \xi_c$ in this case implies that $\xi(\bar{\bar{\xi}}) = \xi(\xi_c) = \xi_c$. That is, as $\sigma \to 0$, the regime 2/3 (investment/no investment) boundary converges on the regime 1/2 boundary and the firm chooses to emulate the competitive solution.

The above discussion was couched in terms of relative prices; it is straightforward to translate this into absolute prices, and to then translate this into implications regarding quantities - the extent of under-investment and the extent of quantity rationing. First, define the associated *absolute* level for the market clearing price at which the price capped monopolist (*PCM*) would choose to start to add incremental capacity as

$$p_e^{PCM}(t) = \xi K_t. \tag{21}$$

where ξ is defined in Result 3. Recall that this *market clearing price* $p_e^{PCM}(t)$ at which investment is triggered is not that which is observed in the market place (because the price cap is binding, the observed price is $\bar{p}_t = \bar{\bar{\xi}} K_t$); the monopolist holds back and only commences investment in capacity when a time arrives where demand reaches a state such that, '*if only*' the monopolist was allowed to set price freely, it would be able to sell all its currently installed capacity at the price $p_e^{PCM}(t)$.

Just as the difference in relative price (ξ compared with $\bar{\bar{\xi}}$) gives an index of the extent to which the firm is under-investing, so too does the difference between the market clearing price in (21) and the price cap $\bar{p}_t$. However, the quantitative level of under-investment in capacity is also affected by demand elasticity; it can be directly calculated, using demand function (1). Thus, at time zero, the firm installs an initial level of capacity Q_0^{PCM} so that the market clearing price at that time is given by (21); thus

$Q_0^{PCM} = A_0 \left(p_e^{PCM}(0) \right)^\gamma = A_0 \left(\xi K_0 \right)^\gamma$. This can be compared to the level of investment under unconstrained monopoly, $Q_0^M = A_0 \left(\xi_M K_0 \right)^\gamma$ and competition, where under

uncertainty it is $Q_0^u = A_0 \left(\xi_u K_0 \right)^\gamma$ whilst under certainty it can be written as

$Q_0^c = A_0 \left(\xi_c K_0 \right)^\gamma$. Comparisons can thus be made using ratios (these eliminate the

influence of A_0, K_0). For example, the competitive level of investment under uncertainty

relative to that under certainty is given as

$$Q_0^u / Q_0^c = \left[A_0 \left(\xi_u K_0 \right)^\gamma \right] \Big/ \left[A_0 \left(\xi_c K_0 \right)^\gamma \right] = \left(\xi_u / \xi_c \right)^\gamma . \tag{22}$$

This is studied numerically in section 4 Table 2, which examines the impact of

uncertainty on the competitive industry, whilst the level of investment of the price capped

monopolist, relative to the competitive case, $Q_0^{PCM} / Q_0^u = \left(\xi / \xi_u \right)^\gamma$, is examined in Table 4.

Fig. 2 illustrates the general structure of this investment behaviour.[15]

Fig. 2 here

Fig. 2 illustrates the effect on initial investment; thus, in the presence of uncertainty,

investment by the monopoly firm is less than under competition, and investment for the

monopoly firm subject to the price cap is also always less than that under competition.

Investment by the price capped firm goes to zero as the price cap is tightened toward the

certainty relative price level ξ_c, converges on that of the unconstrained monopoly firm

as the price cap is relaxed toward the monopoly price (i.e. as $\bar{\xi} \to \xi_M$) and attains its

maximum level when $\bar{\xi} = \xi_u$, the competitive relative price level. However, this level

remains below that for the competitive industry case.

Whenever the price cap binds (whether the firm invests or not), in the presence of

uncertainty, the price capped firm sheds demand through quantity rationing. This follows

from (1); when the price cap binds, quantity demanded is $Q_t^d = A_t \bar{p}_t^\gamma = A_t \left(\bar{\xi} K_t \right)^\gamma$ whilst at

a point at which capacity investment is occurring, which includes time zero, the level of

capacity is given by the demand function (1) at the market clearing price $p_t = \xi K_t$; that is,

installed capacity is $Q_t^{PCM} = A_t p_t^\gamma = A_t \left(\xi K_t \right)^\gamma$. Thus when market clearing price exceeds

the price cap $\left(p_t > \bar{p}_t \right)$, given $\gamma < 0$, so demand exceeds installed capacity; $Q_t^d > Q_t^{PCM}$.

Thus at any time where the firm is about to add to capacity, including time zero, the

extent of quantity rationing is given as

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[15] Again, the actual numerical values are based on benchmark parameter values in Table 2 below.

$$QR = \left(Q_t^d - Q_t^{PCM}\right)/Q_t^{PCM} = \left(\overline{\xi}/\xi\right)^{\gamma} - 1 \qquad\qquad (23)$$

The behaviour of the price capped firm thus parallels that for the unconstrained monopoly firm. Firstly it installs the amount Q_0^{PCM} as described above. With this level of investment, the price cap will bind and the firm will shed demand through quantity rationing. As demand and capacity evolve over time, if the market clearing price stays below $p_e^{PCM}(t)$ as defined in (21), the firm will wait (no investment) and will also ration demand. Demand rationing may cease if demand subsequently falls sufficiently for the price cap to cease to bind. On the other hand, if demand grows sufficiently, the firm will at some point be induced to add to capacity. At such points in time, investment will bring capacity up to the level so the market clearing price is given by (21), such that capacity (relative to capacity at such a time in a competitive industry[16]) is again given by $Q_t^{PCM}/Q_t^u = \left(\xi/\xi_c\right)^{\gamma}$.

The essential reason why the firm chooses to 'under-invest' (relative to the competitive benchmark) when constrained by a price cap is that, given demand uncertainty, it takes account of possible future adverse market movements. The firm cannot get a higher price than the price capped price – but if it restrains investment in periods when the price cap is binding, although it loses the extra revenue this would generate, it also takes a smaller 'hit' on prices in the scenario when future demand falls.

4. Sensitivity Analysis

There is a perennial debate between regulators and the firms they regulate regarding the impact of uncertainty on investment incentives – and on how regulation can adversely affect such investment - see Lind, Muysert and Walker (2002) for an extensive review. The model presented in this paper suggests that any form of price regulation (for example, access price regulation) should take account of option value effects arising out of uncertainty in underlying processes such as technological change or demand, and that such effects may be quantifiably significant. How significant clearly depends on the estimates given for the key parameters involved. It is straightforward to set out the core

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[16] The comparison applies at time zero. More generally, the competitive industry investment trigger price is different from that for the monopolist – but in times of expanding demand it is possible that both would be adding to capacity at the same time – on time intervals where both are investing, the capacity comparison discussed here again applies.

equations in a spreadsheet, so as to explore how variations in parameter values translate into impacts on the prices set in competitive markets, under monopoly and price capped monopoly along with the associated levels of investment in capacity. This section carries out an illustrative sensitivity analysis based on benchmark parameter values in table 2.

Table 2 here

Given values for these parameters, and a value for capacity cost at the time entry takes place (standardised here as $K_0 = £100$), it is straightforward to first compute values for ξ_c, ξ_u, ξ_M, the relative entry trigger prices. These hold for all $t \geq 0$; however, multiplying these by the initial benchmark Fig. $K_0 = 100$ gives the initial entry trigger prices as $p_e^c(0)$, $p_e^u(0)$, $p_e^M(0)$. Given any specification for the price cap $\bar{\bar{\xi}}$, the value for $\xi(\bar{\bar{\xi}})$, the relative market clearing price at which the price constrained monopolist would choose to enter, can be obtained from Result 3, and hence also the value for the market clearing price $p_e^{PCM}(0)$.

The benchmark (risky) discount rate is taken as 5%, although a range up to 30% is reported; the value for θ is 5% with a range from 0-50% considered (infinite life down to 2 years expected life[17]); the value for elasticity is –2, with a range from -1 to -10. The trend in demand, α, is 5% with a range from –30 to +30%. The rate of technical progress is set at $\delta = 5\%$ with a range from 0-25%. Dixit and Pindyck (1994) use 20% as an estimate for volatility for price processes (based on the volatility of the S&P index). However, the volatility of demand processes may tend to be greater. One rather crude way of examining this is to look at the volatility of sales revenue , σ_R. For the UK Telecom sector, for example, this averages around the 20% mark. One of the complicating factors in translating this into an estimate for demand volatility is that demand elasticity makes a difference, as one would intuitively expect. It is possible to show that the volatility of the revenue process implied in this model, on intervals where there is no investment, is related to the volatility of the underlying demand process by the formula $\sigma = -\gamma \sigma_R$; Thus setting $\sigma_R = 0.2$ and $\gamma = -2$ as in Table 1, then this would give

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[17] Depreciation can be thought of as a probabilistic death rate for the unit of capacity (as in Merton, 1976), or as physical depreciation in the available capacity over time. In the former case, the expected life of plant is $\int_0^\infty e^{-\theta t} dt = 1/\theta$; in the latter, this is the average availability over time.

$\sigma = -\gamma\sigma_R =0.4$ or 40%. This is the value used for σ in Table 1, with a range from 0-80% considered in the sensitivity analysis. Finally, an arbitrary value of $K_0 = £100$ is used for the initial unit capacity cost.

Table 3 here

Table 3 uses the parameter values of Table 2, and reports the impact of unilaterally varying each parameter in turn on the relative price, $p_e^u(0)/p_e^c(0) = \xi_u/\xi_c$, and initial levels of capacity investment, $Q_0^u/Q_0^c = \left(\xi_u/\xi_c\right)^\gamma$ for the case where the industry is fully competitive. As the table shows, increasing the interest rate r, depreciation θ, technical progress δ, or the rate of growth of demand α tends to reduce the impact of uncertainty on the relative investment trigger prices and so reduces the impact on investment level. Increasing the elasticity of demand (to more elastic) has a similar effect, as one would expect. Increasing the level of volatility, σ, naturally increases the price differential; notice that the quantitative impact is relatively small up to around 20% volatility but then increases dramatically in the final column of panels (a) and (b) of Table 3.[18]

Table 4 here

Table 4 gives the response of the price capped monopolist to variations in the tightness of the price cap (the numerical values in Fig.s 1 and 2 come from this table – Fig. 1 illustrates the relative market clearing price effect, and Fig. 2, the impact on the relative level of investment relative to the competitive uncertainty case). At the Table 2 benchmark values, the imperfect nature of the firm's response is clear. Thus the certainty competitive entry price is £15, under uncertainty, it is £18.24 but even with the best choice for the price cap, setting $\bar{\xi} = \xi_u = 0.1824$, the initial market clearing price at which the price capped firm enters is £23.64 and, if the firm was commencing investment at time zero, it would install only just over 59.6% of the capacity the competitive industry would supply at this time. This illustrates the general argument presented in section 3 above that the price cap cannot be used to realise the competitive outcome in the presence of uncertainty.

Table 5 here

Table 5 explores the consequences of varying each parameter from the Table 2 values, whilst maintaining an optimal price cap (keeping $\bar{\xi} = \xi_u$ as the value of ξ_u varies with

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[18] Of course, this form of sensitivity analysis involves moving each parameter value unilaterally. However, it only requires a simple spreadsheet in order to explore alternative 'what if' questions.

variations in parameter values). This gives some idea of the conditions under which the optimally set price cap is most effective. The first panel in Table 4 reports the relative entry price $p_e^{PCM}(0)/p_e^{u}(0) = \xi(\xi_u)/\xi_u$. In the benchmark case this takes the value 1.296 (market clearing price at which entry occurs is 29.6% higher than that for the competitive case) with initial installed capacity at 59.6% of the competitive level. The price cap of course improves the situation over that of unconstrained monopoly, when capacity is only $\left(\frac{\gamma}{1+\gamma}\right)^{\gamma} = 25\%$ of the competitive level (at benchmark $\gamma = -2$). However, Table 5 clearly shows that the price cap does not get particularly close to inducing the firm to mimic the competitive industry solution. The only cases where the price cap works well are (a) where there is little monopoly power (with elasticity $\gamma = -10$ in column 2) or (b) if there is little volatility ($\sigma = 0.01$ in the final column). Finally, Result 4 (vi) indicated that the price cap works well under certainty and the final column of table 4 bears this out.

5. Concluding Comments

Whilst uncertainty has an impact on the price at which firms choose to invest in capacity, firms with monopoly power who are able to control the scale of their investments will under-invest and will wait too long before adding to such investment. As a consequence, prices to final customers are always higher than in competitive markets. The extent of this effect depends on the values chosen for various parameters, although naturally enough, the most important is that of demand elasticity; as in the single period monopoly problem, unless demand is really quite elastic, the level of under-investment can be quite substantial. Following this basic insight, the response of the firm with monopoly power to the imposition of a simple form of inter-temporal price cap was examined. The price cap took the form of limiting the maximum price the firm is allowed to charge over time. As a special case, this constraint could be used to impose a maximum price equal to that which would arise in a competitive market. Under *certainty*, it was shown that this form of intertemporal price cap could be used to encourage the monopoly firm to emulate the competitive solution. However, under *uncertainty*, it was shown that, whilst an intertemporal price cap may be beneficial, it cannot be used to realise the competitive solution.

The essential problem with the price cap is that, when it is set at a level below the unconstrained monopoly entry price, the firm does not start investing immediately the price cap is hit. It delays investment, and sheds demand through quantity rationing, until a point is reached where demand is sufficiently strong to motivate it to invest. The rationale for not immediately expanding capacity when the price cap begins to bite is that the firm takes account of the future possibility that demand may fall to a point where the price cap no longer binds. In having less installed capacity at that time, the firm enjoys higher prices thereafter (keeps prices closer to the maximum allowable).

Although in this model the best choice of price cap is indeed the competitive price, it remains the case that price capped firms will have a general incentive to both under-invest and to impose quantity rationing. This would be manifest in service industries by the firm allowing the quality of service to degrade. For example, particularly in periods where there is a significant upswing in demand such that the price cap binds, the firm has a clear incentive to drag its feet on investment, an incentive to find excuses for why it cannot keep up with such upswings in demand.[19] Whilst the model omits some potentially important factors (such as brand loyalty and reputation), it suggests that careful consideration should be given to these potential 'side effects' in any proposed application of price cap regulation.

Quantity rationing of existing customers tends to carry adverse reputation effects for the firm, and this consideration may help mitigate the extent of rationing. However, there is another form of rationing that is not only less easy to monitor, but also has little or no impact on the firm's reputation. This is rationing by exclusion, where the exclusion typically has a geographic dimension. For example, at current prices, many households would choose gas for domestic heating purposes, but find there is no network supply in the local area. The same is true for cable TV and various Telecom services. Local demand is excluded because networks rarely have 100% coverage. If price cap regulation takes no account of the significant levels of uncertainty which are often present in innovative industries, the consequence is likely to be that that price caps will be set too tight - and this could have a significant adverse impact on the rate at which networks are developed – and on the overall extent of coverage of such networks.

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[19] This kind of problem appears to be currently manifesting itself in UK Telecoms (although peak/off-peak tariff rebalancing is one of many other issues involved in this case).

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APPENDIX

This appendix gives an outline of the analysis involved in obtaining results 2-4. For an exposition of smooth pasting/boundary conditions, see Dixit (1993) and Dumas (1991). Detailed step by step derivations are given in the appendices to the original working papers (Dobbs, 2000; 2001). To reduce notational clutter, time subscripts and function arguments are dropped in what follows (where this results in no loss of intelligibility).

A1 Competition/ Monopoly without a Price Cap

The arbitrage equation (Dixit, 1993, p. 15), from (10) is that

$$(r+\theta)vdt = pdt + E(dv).$$ (A.1)

Applying Itô's lemma and simplifying, this yields the following fundamental differential equation:

$$\tfrac{1}{2}\eta^2\sigma^2 x^2\psi'' + \left(\mu_p+\delta\right)x\psi' - (\delta+\theta+r)\psi + x = 0.$$ (A.2)

The general solution to (A.2) can be shown to have the form

$$\psi(x) = B_0 x + B_1 x^{\lambda_1} + B_2 x^{\lambda_2},$$ (A.3)

where $B_0, \lambda_1, \lambda_2, R_1, R_2$ are given by (12)-(16). The arbitrary constants B_1, B_2 are determined by boundary conditions. Given $\lambda_2 < 0$ from (14), as $x \to 0$, if value is to be finite, it must be that $B_2 = 0$. The other constant B_1 is determined by an analysis of smooth pasting conditions at the boundary (at which new investment is triggered). This is now done in turn for the competitive and monopoly cases.

Competitive Industry under Uncertainty

In a competitive industry, new investment occurs when expected value for a unit of capacity rises to equal the unit cost of investment. This value matching condition occurs at a time $\tilde{t}$ at which the price $p_{\tilde{t}}$ reaches the level $p_{\tilde{t}} = \xi_u K_{\tilde{t}}$ (equivalently $x_{\tilde{t}} = \xi_u$) where ξ_u is the competitive uncertainty relative price that triggers new investment:

$$v(p_{\tilde{t}}, K_{\tilde{t}}) = \psi(p_{\tilde{t}}/K_{\tilde{t}})K_{\tilde{t}} = K_{\tilde{t}} \Rightarrow \psi(p_{\tilde{t}}/K_{\tilde{t}}) = 1 \qquad \Rightarrow \psi(\xi_u)=1 \quad \text{(A.4)}$$

Smooth pasting additionally requires the following 'first order condition' to hold;

$$\tfrac{d}{dp}\left[v(p,K)-K\right] = \tfrac{d}{dp}\left(\psi(p/K)K\right) = \psi'(p/K) = 0 \Rightarrow \psi'(\xi_u) = 0.$$ (A.5)

The conditions (A.4) and (A.5) can be used to determine B_1, ξ_u. After some manipulation the result for ξ_u is that

$$\xi_u = \left(\frac{\lambda_1}{\lambda_1-1}\right)(\theta+r-\mu_p).$$ (A.6)

which is the first part of result 2(i) in the paper. After further routine algebra, it is possible to show that equation (A.6) can also be re-expressed as

$$\xi_u = \left(\frac{1-\lambda_2}{\lambda_2}\right)(\theta+r+\delta) = \left(\frac{1-\lambda_2}{\lambda_2}\right)\xi_c$$ (A.7)

as reported in result 2(i).

Monopoly under Uncertainty

In the monopoly case, investment commences at a time $\tilde{t}$ at which price $p_{\tilde{t}}$ reaches the level $p_{\tilde{t}} = \xi_M K_{\tilde{t}}$, where ξ_M is the relative price at which new capacity is added. Since ξ_M is a free choice by the firm, smooth pasting involves first and second derivative conditions (see Dumas, 1991). The first derivative condition is that, with respect to the control variable, the rate of change of value should just equal the rate of change of cost;

$$\partial V(p_{\tilde{t}}, K_{\tilde{t}}, Q_{\tilde{t}})/\partial p_{\tilde{t}} = \partial (K_{\tilde{t}} Q_{\tilde{t}})/\partial p_{\tilde{t}} \tag{A.8}$$

where $Q_{\tilde{t}} = A_{\tilde{t}} p_{\tilde{t}}^{\gamma}$. The second derivative condition is

$$\partial^2 V(p_{\tilde{t}}, K_{\tilde{t}}, Q_{\tilde{t}})/\partial p_{\tilde{t}}^2 = \partial^2 (K_{\tilde{t}} Q_{\tilde{t}})/\partial p_{\tilde{t}}^2 \tag{A.9}$$

These conditions imply:

$$\gamma[\psi(\xi_M) - 1] + \xi_M \psi'(\xi_M) = 0, \tag{A.10}$$

$$\gamma(\gamma - 1)[\psi(\xi_M) - 1] + (\gamma - 1)\xi_M \psi'(\xi_M)$$
$$+ \left\{ (1 + \gamma)\xi_M \psi'(\xi_M) + \xi_M^2 \psi''(\xi_M) \right\} = 0 \quad . \tag{A.11}$$

These serve to define the unknowns B_1, ξ_M. After some routine algebra, the solution for ξ_M can be simplified to give

$$\xi_M = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\lambda_2 - 1}{\lambda_2}\right)(\theta + r + \delta), \tag{A.12}$$

as in Result 2(ii).

A2 Price capped Monopoly

Let ψ denote the solution when there is zero investment and no price constraint, whilst ψ_2 denotes the solution when the price constraint applies but there is no investment. There are now three regimes; unconstrained with no investment, price constrained with no investment, and price constrained with investment. The solution in the first two regimes is first discussed, followed by an analysis of the smooth pasting conditions at the interfaces between the regimes.

Regime 1: Unconstrained price, no investment.

The solution here is naturally identical to that already established for the unconstrained monopoly firm – that is, the solution is given by (11) where B_0 is given by (12). As before, note that, as $x \to 0$, if $v(p, K)$ is to be finite, it must be that $B_2 = 0$. The constant B_1 in this new problem is determined via an analysis of boundary conditions (see below).

Regime 2: Price constrained, no investment

In this region, the price cap binds and $p_t = \bar{\xi} K_t$; the arbitrage equation becomes

$$(r + \theta)v dt = \bar{\xi} K dt + E(dv) \tag{A.13}$$

The analysis parallels that for (A.2); it yields (compare with (A.2)):

$$\tfrac{1}{2}\eta^2\sigma^2 x^2 \psi'' + (\mu_p + \delta) x \psi' - (\delta + \theta + r)\psi + \bar{\xi} = 0, \tag{A.14}$$

as the fundamental differential equation. The solution in regime 2 is denoted ψ_2 and this is given as

$$\psi_2(x) = \left(\bar{\xi}/(\delta + \theta + r)\right) + C_1 x^{\lambda_1} + C_2 x^{\lambda_2}. \tag{A.15}$$

where the arbitrary constants C_1, C_2 are determined by a consideration of boundary conditions.

Analysis of transition boundary conditions:

Let $\tilde{t}_1$ denote a hitting time at which there is a transition between the regimes 1 and 2 whilst $\tilde{t}_2$ denotes a hitting time at which there is a transition between the regimes 2 and 3 (at which new investment commences).

Regime 1/2 boundary:

At this boundary, by definition, the price cap binds, so $x_{\tilde{t}_1} = \bar{\xi}$. As far as the firm is concerned, $\bar{\xi}$ is exogenous; as a consequence, smooth pasting involves matching value and first derivatives for the solutions as they meet at the boundary (Dumas, 1991). Since $v(x_t) = \psi(x_t)K_t$, this requires

$$\psi\left(\bar{\xi}\right) = \psi_2\left(\bar{\xi}\right), \tag{A.16}$$

$$\psi'\left(\bar{\xi}\right) = \psi_2'\left(\bar{\xi}\right) \tag{A.17}$$

where, from the definitions of ψ, ψ_2, these are calculated as $\psi(\bar{\xi}) = B_0\bar{\xi} + B_1\bar{\xi}^{\lambda_1}$, $\psi'(\bar{\xi}) = B_0 + \lambda_1 B_1\bar{\xi}^{\lambda_1-1}$, $\psi_2(\bar{\xi}) = \left(\bar{\xi}/(\theta+r+\delta)\right) + C_1\bar{\xi}^{\lambda_1} + C_2\bar{\xi}^{\lambda_2}$ and $\psi_2'(\bar{\xi}) = \lambda_1 C_1\bar{\xi}^{\lambda_1-1} + \lambda_2 C_2\bar{\xi}^{\lambda_2-1}$.

Regime 2/3 boundary:

Here ξ denotes the relative trigger market clearing price at which the firm would choose to start to invest when the firm is subject to a price cap. Since the choice of ξ is free, the smooth pasting conditions at $\tilde{t}_2$ require the first and second derivatives of the value function in regime 2 to satisfy equivalent conditions to those specified above in the unconstrained monopoly case. That is,

$$\gamma\left[\psi_2\left(\xi\right)-1\right] + \xi\psi_2'\left(\xi\right) = 0 \tag{A.18}$$

$$\gamma(\gamma-1)\left[\psi_2\left(\xi\right)-1\right] + (\gamma-1)\xi\psi_2'\left(\xi\right) + \left[(1+\gamma)\xi\psi_2'\left(\xi\right) + \xi^2\psi_2''\left(\xi\right)\right] = 0 \tag{A.19}$$

where the derivatives are calculated as in the analysis at the regime 1/ 2 boundary (but evaluated at ξ).

Analysis of smooth pasting conditions:

After some routine algebra, it is possible to solve the equations (A.16)-(A.19) to determine the arbitrary constants B_1, C_1, C_2 and the value of ξ (as a function of $\bar{\xi}$ and the other parameters in the problem). The solution for ξ is given in Result 3 (the full 'step by step' derivation being given in the appendix to the working paper (Dobbs, 2001).

A3 Proof for Result 4.

From the formula for $\xi(\bar{\xi})$ in Result 3 was

$$\xi = \left\{\left(\bar{\xi}-\xi_c\right)\xi_M\bar{\xi}^{\lambda_2-1}\Big/\left(\xi_M-\xi_c\right)\right\}^{1/\lambda_2} \tag{A.20}$$

Differentiating with respect to $\bar{\xi}$ gives

$$\frac{d\xi}{d\bar{\xi}} = \left(1/\lambda_2\right)\left\{\left(\frac{\bar{\xi}-\xi_c}{\xi_M-\xi_c}\right)\xi_M\bar{\xi}^{\lambda_2-1}\right\}^{(1/\lambda_2)-1}\left(\frac{\xi_M}{\xi_M-\xi_c}\right)\frac{d}{d\bar{\xi}}\left(\left(\bar{\xi}-\xi_c\right)\bar{\xi}^{\lambda_2-1}\right) \tag{A.21}$$

where, using the definition for ξ_u ,

$$\frac{d}{d\bar{\xi}}\left(\left(\bar{\xi}-\xi_c\right)\bar{\xi}^{\lambda_2-1}\right)=\frac{d}{d\bar{\xi}}\left(\bar{\xi}^{\lambda_2}-\xi_c\bar{\xi}^{\lambda_2-1}\right)=\left(\lambda_2\bar{\xi}^{\lambda_2-1}-\left(\lambda_2-1\right)\xi_c\bar{\xi}^{\lambda_2-2}\right)$$

$$=\lambda_2\bar{\xi}^{\lambda_2-2}\left(\bar{\xi}-\xi_u\right) \tag{A.22}$$

so

$$\frac{d\xi}{d\bar{\xi}}=\left\{\left(\frac{\bar{\xi}-\xi_c}{\xi_M-\xi_c}\right)\xi_M\bar{\xi}^{\lambda_2-1}\right\}^{(1/\lambda_2)-1}\underbrace{\left(\frac{\xi_M}{\xi_M-\xi_c}\right)\xi^{\lambda_2-2}}_{(+)}\underbrace{\left(\bar{\xi}-\xi_u\right)}_{}$$

where the underbraced term $\left\{\left(\frac{\bar{\xi}-\xi_c}{\xi_M-\xi_c}\right)\xi_M\bar{\xi}^{\lambda_2-1}\right\}^{(1/\lambda_2)-1}$ is $(+)$. $\tag{A.23}$

hence

$$\frac{d\xi}{d\bar{\xi}}\gtreqless 0 \quad \text{as} \quad \bar{\xi}\gtreqless\xi_u. \tag{A.24}$$

This completes the proof for result 4(i). As $\bar{\xi}\downarrow\xi_c$, the term in brackets $\{\ \}\to 0$ in equation (A.20); since $1/\lambda_2<0$, it follows that $\xi\to+\infty$, which is result 4(ii). Letting $\bar{\xi}\to\xi_M$ in (A.20), clearly $\xi(\bar{\xi})\to\xi_M$, which is result 4(iii). Setting $\bar{\xi}=\xi_u$, from (A.20),

$$\xi^{\lambda_2}=\left\{\left(\xi_u-\xi_c\right)\xi_M\xi_u^{\lambda_2-1}/\left(\xi_M-\xi_c\right)\right\} \tag{A.25}$$

Now, $\xi(\xi_u)\gtreqless\xi_u$ as $\xi(\xi_u)^{\lambda_2}\lesseqgtr\xi_u^{\lambda_2}$ (since $\lambda_2<0$). Using (A.25) this implies $\xi(\xi_u)\gtreqless\xi_u$ as

$$\left(\xi_u-\xi_c\right)\xi_M\xi_u^{\lambda_2-1}/\left(\xi_M-\xi_c\right)\lesseqgtr\xi_u^{\lambda_2}$$

$$\Rightarrow\left(\xi_u-\xi_c\right)\xi_M\lesseqgtr\xi_u\left(\xi_M-\xi_c\right)\Rightarrow\xi_M-\xi_u\gtreqless 0$$

In fact $\xi_M-\xi_u>0$ and hence $\xi(\xi_u)>\xi_u$, which is Result 4 (iv).

To establish Result 4 (v), first substitute in (A.25) using result 2, for $\xi_u=(\lambda_2-1)\xi_c/\lambda_2$ and $\xi_M=(\lambda_2-1)\xi_c/\lambda_2(1+\eta)$ to get $\xi(\xi_u)=\left(1+\eta\lambda_2\right)^{-1/\lambda_2}\xi_u$. From the definition for λ_2, note that $Lim_{\sigma\to 0}\lambda_2=-\infty$, and so $Lim_{\sigma\to 0}\left(\frac{\lambda_2-1}{\lambda_2}\right)=1$. Hence from result 3, $Lim_{\sigma\to 0}\ \xi_u=\xi_c$. Also $Lim_{\sigma\to 0}\left(1+\eta\lambda_2\right)^{-1/\lambda_2}=Lim_{\lambda_2\to-\infty}\left(1+\eta\lambda_2\right)^{-1/\lambda_2}=1$. Hence

$$Lim_{\sigma\to 0}\ \xi\left(\xi_u\right)=Lim_{\sigma\to 0}\left(1+\eta\lambda_2\right)^{-1/\lambda_2}Lim_{\sigma\to 0}\ \xi_u=\xi_c \tag{A.26}$$

which is Result 4 (v).

Table 1: Glossary of Notation

Basic Parameters		Prices for output/access to capacity at which incremental investment is triggered:	
Discount rate	r	Competitive, certainty	$p_e^c(t)$
Rate of physical depreciation,	θ	Competitive, uncertainty	$p_e^u(t)$
Rate at which technical progress reduces the price of capacity,	δ	Monopoly, uncertainty	$p_e^M(t)$
Demand growth rate,	α	Price capped monopoly demand price	$p_e^{PCM}(t)$
Instantaneous standard deviation for the demand function	σ	Competitive certainty relative price	ξ_c
Elasticity of demand	γ	Competitive uncertainty relative price	ξ_u
Inverse elasticity of demand	$\eta = 1/\gamma$	Monopoly uncertainty relative price	ξ_M
Variables		Price capped monopoly demand relative price	ξ
Unit price of capacity,	K_t	**Capacity measures**	
Price of output/access to capacity	p_t	Demand for capacity at price p_t	Q_t^d
Relative price	$x_t = p_t / K_t$	Capacity under certainty	Q_t^c
Trend rate, price process	μ_p	Capacity under uncertainty	Q_t^u
The maximum price under the price cap	$\bar{p}_t$	Monopoly capacity under uncertainty	Q_t^M
The maximum relative price under the price cap	$\bar{\xi}(\equiv \bar{p}_t / K_t)$	Price capped monopoly capacity under uncertainty	Q_t^{PCM}

Table 2: Benchmark Values

Parameter	Benchmark Value
Discount rate, r	5%
Rate of physical depreciation, θ	5%
Rate at which technical progress reduces the price of capacity, δ	5%
Demand growth rate, α	5%
Instantaneous standard deviation σ	40%
Elasticity γ	-2
Benchmark unit price of capacity, K_0	£100

Table 3: Competitive industry comparisons (Effect of a unilateral variation in parameter values from those given in Table 2)

(a) Ratio of initial entry price under uncertainty to entry price under certainty

α	$\dfrac{p_e^u(0)}{p_e^c(0)}$	γ	$\dfrac{p_e^u(0)}{p_e^c(0)}$	θ	$\dfrac{p_e^u(0)}{p_e^c(0)}$	δ	$\dfrac{p_e^u(0)}{p_e^c(0)}$	r	$\dfrac{p_e^u(0)}{p_e^c(0)}$	σ	$\dfrac{p_e^u(0)}{p_e^c(0)}$
-0.3	1.275	-10	1.015	0	1.305	0	1.400	0.03	1.224	0.01	1.000
-0.1	1.419	-5	1.052	0.01	1.282	0.01	1.345	0.05	1.216	0.05	1.003
-0.05	1.333	-3	1.119	0.05	1.216	0.05	1.216	0.08	1.206	0.1	1.013
0	1.267	-2	1.216	0.1	1.168	0.1	1.144	0.10	1.200	0.15	1.029
0.05	1.216	-1.5	1.320	0.2	1.117	0.15	1.107	0.15	1.187	0.2	1.051
0.1	1.179	-1.1	1.474	0.3	1.090	0.2	1.085	0.20	1.177	0.4	1.216
0.3	1.100	-1.01	1.527	0.5	1.062	0.25	1.070	0.30	1.161	0.8	1.957

(b) Ratio of initial capacity level under uncertainty to capacity level under certainty

α	$\dfrac{Q_0^u}{Q_0^c}$	γ	$\dfrac{Q_0^u}{Q_0^c}$	θ	$\dfrac{Q_0^u}{Q_0^c}$	δ	$\dfrac{Q_0^u}{Q_0^c}$	r	$\dfrac{Q_0^u}{Q_0^c}$	σ	$\dfrac{Q_0^u}{Q_0^c}$
-0.3	0.615	-10	0.864	0	0.587	0	0.510	0.03	0.667	0.01	1.000
-0.1	0.497	-5	0.777	0.01	0.609	0.01	0.553	0.05	0.676	0.05	0.994
-0.05	0.563	-3	0.713	0.05	0.676	0.05	0.676	0.08	0.688	0.1	0.975
0	0.623	-2	0.676	0.1	0.732	0.1	0.764	0.1	0.694	0.15	0.945
0.05	0.676	-1.5	0.660	0.2	0.801	0.15	0.816	0.15	0.709	0.2	0.905
0.1	0.720	-1.1	0.652	0.3	0.841	0.2	0.850	0.2	0.722	0.4	0.676
0.3	0.826	-1.01	0.652	0.5	0.887	0.25	0.873	0.3	0.742	0.8	0.261

Table 4: Impact of varying relative price cap $\overline{\xi}$ on $p_e^{PCM}(0)$, Q_e^{PCM}/Q_e^u and QR

Relative Price Cap $\overline{\xi}$	$\xi(\overline{\xi})$	$p_e^{PCM}(0)$	Q_e^{PCM}/Q_e^u	%Quantity Rationing, QR
0.1501 (=ξ_c)	0.650	65.0	0.078	1782.90%
0.1550	0.291	29.1	0.394	251.38%
0.1800	0.237	23.7	0.595	72.66%
0.1824 (=ξ_u)	0.236	23.6	0.596	67.87%
0.2000	0.241	24.1	0.574	44.88%
0.2500	0.272	27.2	0.451	18.22%
0.3000	0.311	31.1	0.345	7.34%
0.3649 (=ξ_M)	0.365	36.5	0.250	0.00%

Parameter values as in Table 1. Based on these values,

$p_e^c(0) = 15.00$, $p_e^u(0) = 18.24$, $p_e^M(0) = 36.49$

Table 5: Comparison of price capped monopoly with competition under uncertainty (when price cap optimally set in all cases i.e. setting $\overline{\xi} = \xi_u$)

(a) Price cap monopoly initial market clearing price compared to competitive entry price, $p_e^{PCM}(0)/p_e^u(0)$

α	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$	γ	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$	θ	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$	δ	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$	r	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$	σ	$\dfrac{p_e^{PCM}(0)}{p_e^u(0)}$
-0.3	1.490	-10	1.031	0	1.345	0	1.383	0.03	1.301	0.01	1.016
-0.1	1.390	-5	1.085	0.01	1.333	0.01	1.362	0.05	1.296	0.05	1.048
-0.05	1.357	-3	1.172	0.05	1.296	0.05	1.296	0.08	1.289	0.1	1.087
0	1.325	-2	1.296	0.1	1.261	0.1	1.241	0.1	1.285	0.15	1.129
0.05	1.296	-1.5	1.434	0.2	1.215	0.15	1.204	0.15	1.276	0.2	1.216
0.1	1.269	-1.1	1.662	0.3	1.185	0.2	1.178	0.2	1.268	0.4	1.296
0.3	1.196	-1.01	1.746	0.5	1.147	0.25	1.159	0.3	1.255	0.8	1.495

(b) Price cap monopoly capacity level relative to the competitive case†, Q_0^{PCM}/Q_0^u.

α	$\dfrac{Q_0^{PCM}}{Q_0^u}$	γ	$\dfrac{Q_0^{PCM}}{Q_0^u}$	θ	$\dfrac{Q_0^{PCM}}{Q_0^u}$	δ	$\dfrac{Q_0^{PCM}}{Q_0^u}$	r	$\dfrac{Q_0^{PCM}}{Q_0^u}$	σ	$\dfrac{Q_0^{PCM}}{Q_0^u}$
-0.3	0.450	-10	0.739	0	0.553	0	0.523	0.03	0.591	0.01	0.969
-0.1	0.518	-5	0.664	0.01	0.563	0.01	0.539	0.05	0.596	0.05	0.911
-0.05	0.543	-3	0.621	0.05	0.596	0.05	0.596	0.08	0.602	0.1	0.847
0	0.569	-2	0.596	0.1	0.629	0.1	0.650	0.1	0.606	0.15	0.784
0.05	0.596	-1.5	0.583	0.2	0.677	0.15	0.690	0.15	0.615	0.2	0.676
0.1	0.621	-1.1	0.572	0.3	0.712	0.2	0.721	0.2	0.622	0.4	0.596
0.3	0.699	-1.01	0.570	0.5	0.761	0.25	0.745	0.3	0.635	0.8	0.447

†Note that the extent of quantity rationing can be measured as $\left(1-(Q_0^{PCM}/Q_0^u)\right)$, given that $\overline{\xi} = \xi_u$.
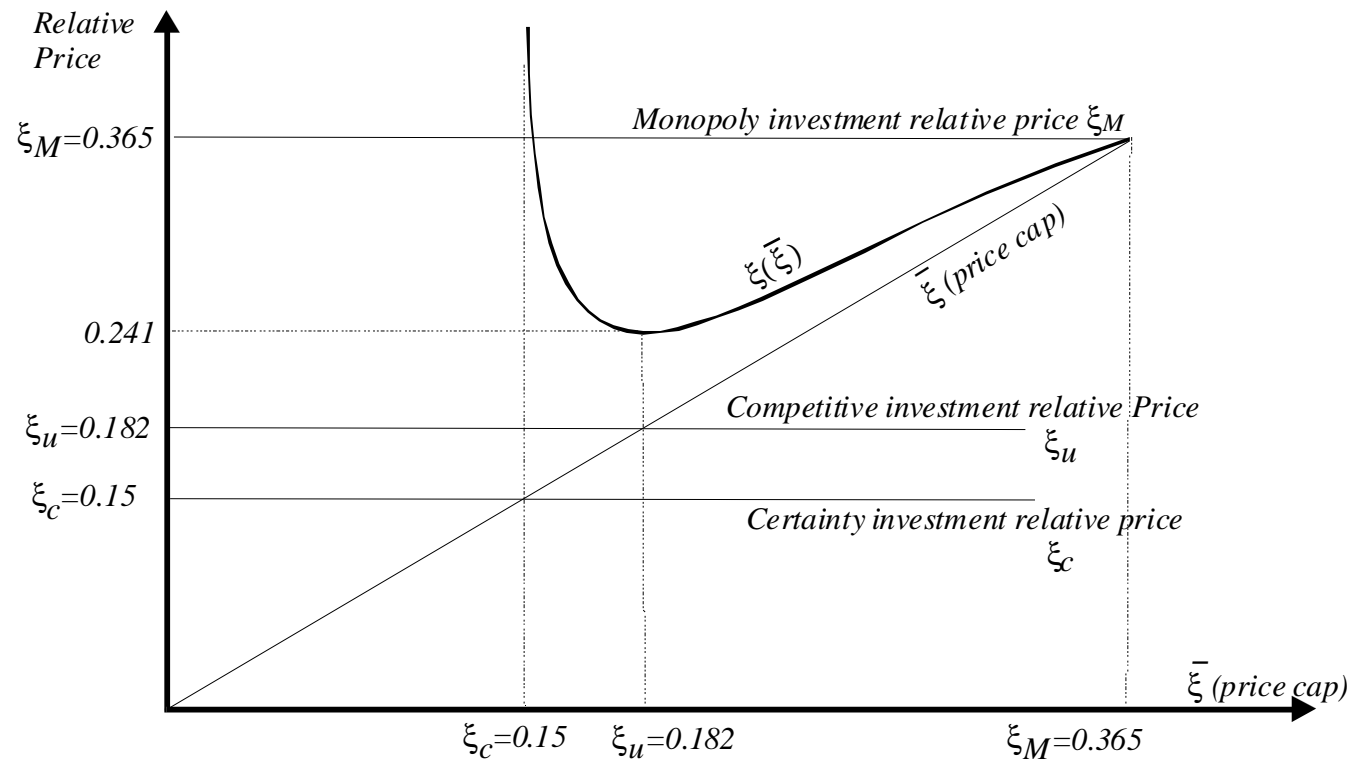
Fig. 1   Market clearing relative price ξ

Fig. 2   Price capped monopoly installed capacity