

Tutorial Questions 3 – Introductory magnetic machines

1. Estimate the electric field strength required to make a balloon stick to a ceiling due to electrostatic force. Assume that the balloon weighs 1g and has an area of contact with the ceiling of 1 cm².
2. Two magnetic pole made of highly permeable material are separated by an air gap of length 1.0 mm. Each pole measures 10 mm by 10 mm in cross-section. A coil of 1000 turns, carrying a current of 1.0 A, drives magnetic flux across this gap.
 - (a) What is the magnitude of the flux density in the gap if all the MMF is dropped across it?
 - (b) What is the magnitude of the force attracting the poles together?
 - (c) If one pole is offset by 1.0 mm, calculate the force attracting it back.
3. Two conducting plates are separate by an air-gap of length 1.0 mm. Each plate measures 10 mm by 10 mm in cross-section. A voltage of 300 V is applied across the plates. What is the magnitude of the force attracting the plates together?
4. This question concerns the elementary rotary system shown in figure 1, with the parameters indicated with the figure.

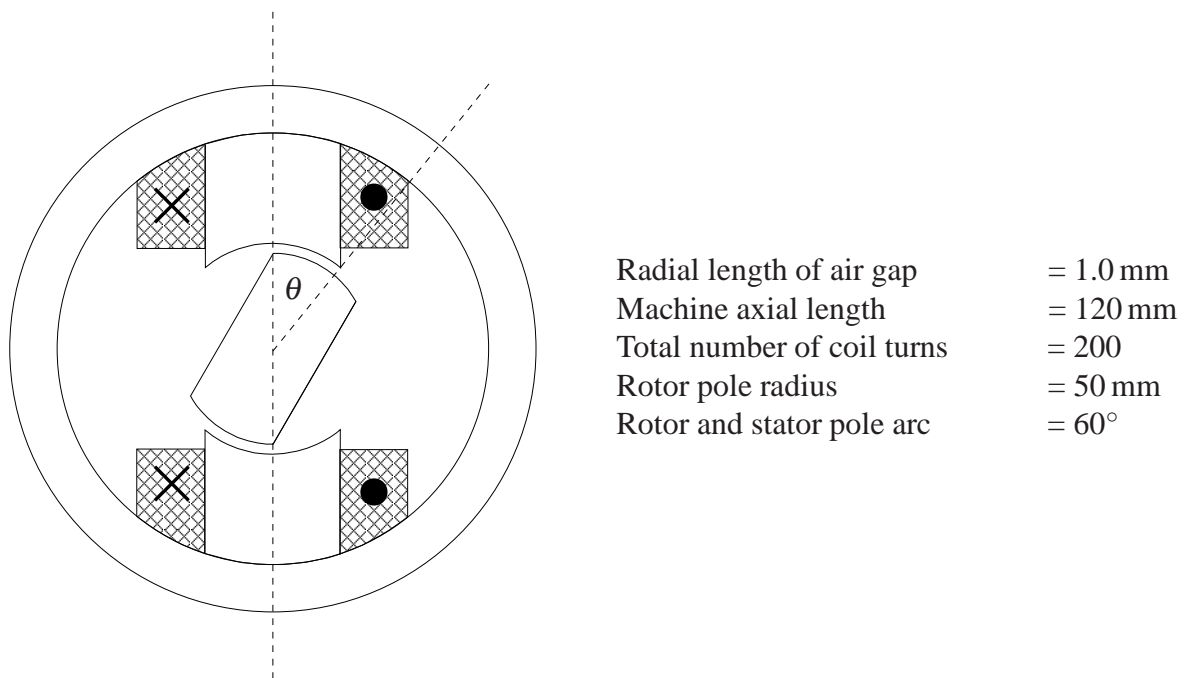


Figure 1: Rotary machine for the tutorial problems. The 200 coil turns are split into the two halves.

- (a) Assuming that the iron circuit has no significant reluctance, determine the reluctance of the magnetic circuit in figure 1, as a function of the rotor/stator overlap angle, θ .
- (b) Sketch the magnetic flux pattern for the arrangement in figure 1.

- (c) Using the angular dependence of the reluctance derived for the first part of the question, determine the variation in the coil inductance with θ .
- (d) What is the maximum torque produced when the coil carries a current of 10.0 A? You may use the equation

$$T = \frac{1}{2} i^2 \frac{dL}{d\theta},$$

remembering that in this equation θ must be in radians.

5. This question concerns the elementary rotary system shown in figure 2, with the parameters indicated.

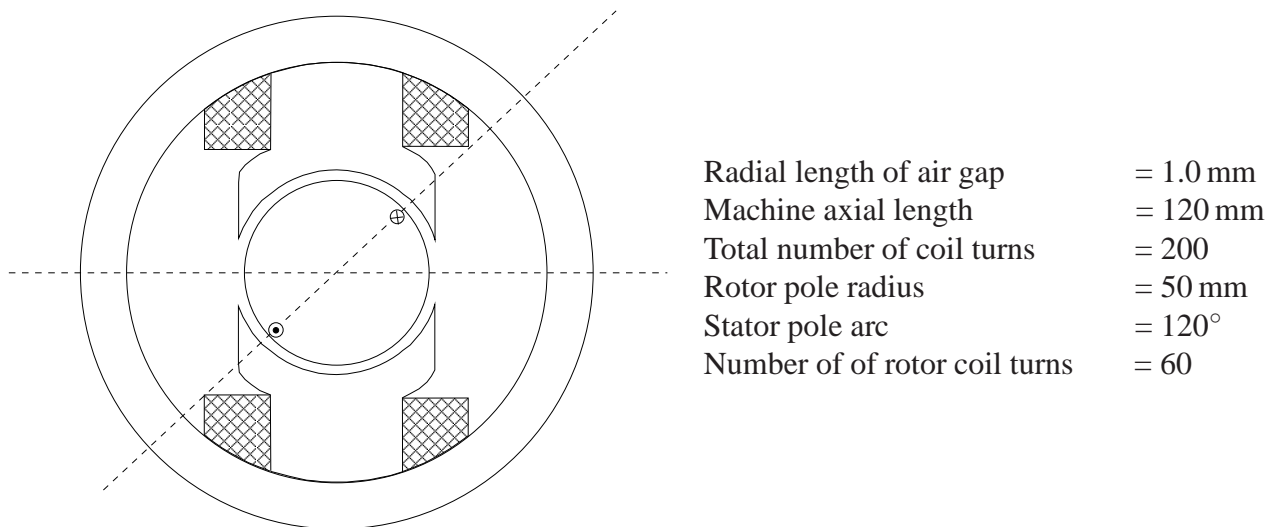


Figure 2: Rotary machine for the tutorial problems. The 200 coil turns are split into the two halves.

- (a) If the stator coil in figure 2 has a current of 10.0 A, what is the radial flux density in the air-gap?
- (b) If the rotor coil of figure 2 carries a current of 6.0 A, what is the torque produced? [Hint: this is a $F = BiL$ problem.]

Tutorial Questions 3 – solutions

1. We use $F = mg$ for the downward force due to gravity (m mass, g acceleration due to gravity), and $F = \epsilon E^2 A / 2$ for the upward force of electrostatic attraction (E electric field strength, A area, ϵ permittivity). At the point at which the electrostatic force is *just enough* to hold the balloon, we have $mg = \epsilon E^2 A / 2$, which rearranges to

$$E = \sqrt{\frac{2mg}{\epsilon A}} = \sqrt{\frac{2 \times 0.001 \times 9.81}{8.854 \times 10^{-12} \times 10^{-4}}} \approx 4.7 \times 10^6 \text{ V/m.}$$

2. (a) The flux density is related to the MMF by

$$H = \frac{B}{\mu} = \frac{\text{MMF}}{l},$$

so

$$B = \frac{\mu Ni}{l} = \frac{4\pi \times 10^{-7} \times 1000 \times 1}{0.001} = 1.26 \text{ T.}$$

- (b) See notes.

- (c) The force of attraction is given in the notes to be

$$F = \frac{\mu H^2 A}{2},$$

(H field strength, A area and μ permeability) and we can substitute the magnetic field, which is Ni/l (N number of coil turns, i coil current and l magnetic path length), to give

$$F = \frac{\mu N^2 i^2 A}{2l^2} = \frac{4\pi \times 10^{-7} \times 1000^2 \times 1^2 \times 0.01^2}{2 \times 0.001^2} = 62.8 \text{ N.}$$

Alternatively, the inductance can give the force by

$$F = \frac{1}{2} i^2 \frac{dL}{dl},$$

where L is the inductance of the coil, given by

$$L = \frac{N^2}{S} = \frac{N^2}{l/\mu A},$$

resulting in the same formula.

- (d) When the poles are displaced laterally by 1 mm, we can use the inductance to give the force:

$$L = \frac{N^2}{S} = \frac{N^2}{l/\mu xy},$$

where now x and y are the lengths of overlapping poles in these directions. Then, assuming displacement in the x direction, we have

$$F = \frac{1}{2} i^2 \frac{dL}{dx} = \frac{\mu N^2 i^2 y}{2l} = \frac{4\pi \times 10^{-7} \times 1000^2 \times 1^2 \times 0.01}{2 \times 0.001} = 6.28 \text{ N.}$$

3. The force in this case is given by

$$F = \frac{\epsilon E^2 A}{2} = \frac{\epsilon V^2 A}{2l^2} = \frac{8.854 \times 10^{-12} \times 300^2 \times 0.01^2}{2 \times 0.001^2} = 4.0 \times 10^{-5} \text{ N.}$$

4. (a) Since the reluctance of the iron core is negligible, all the significant reluctance lies in the air gaps. The total path length is twice the air-gap, *i.e.* 2 mm. The path area is the axial length multiplied by the rotor radius, multiplied by the angle of overlap between rotor and stator,

$$A = L_a \times r \times \theta,$$

where we're using L_a to represent the axial length. Using the expression in the notes, the reluctance is therefore given by

$$S = \frac{l}{\mu A} = \frac{l}{\mu L r \theta} = \frac{0.002}{4\pi \times 10^{-7} \times 0.12 \times 0.05 \times \theta} = \frac{2.65 \times 10^5}{\theta}.$$

- (b) The inductance is given by

$$L = \frac{N^2}{S},$$

as given in the notes. Thus,

$$L = \frac{200^2}{S} = 0.151\theta.$$

This is a maximum then θ is maximum, which as specified is at 60° or $\pi/3$ radians. Then $L = 0.158 \text{ H}$.

- (c) The torque is the turning force, as given in the question, so $T = \frac{1}{2} i^2 \frac{dL}{d\theta} = 0.5 \times 10^2 \times 0.151 = 7.54 \text{ N.m}$.
5. (a) Again, the only place where the MMF is dropped in this system is in the gaps, since the reluctance of the iron cores are relatively small. Hence

$$B = \mu H = \mu \frac{Ni}{2l} = 4\pi \times 10^{-7} \frac{200 \times 10.0}{2 \times 0.001} = 1.26 \text{ T.}$$

- (b) If the rotor coil has N turns on each side, then each contributes to the torque produced, so

$$T = 2NBiLr = 2 \times 60 \times 1.26 \times 6.0 \times 0.12 \times 0.05 = 5.43 \text{ N.m.}$$