CIV2002: Statistics for Civil Engineers
Exercises 2: solutions

QUESTION 1

Step 1: Hypotheses

H_0: $\mu = 50$
H_1: $\mu \neq 50$

Step 2: Which test?

This is a one-sample test. And since the population standard deviation is unknown, we will perform a one-sample t-test.

Step 3: Test statistic

To summarise, we have

$\bar{x} = 45.3$, $s = 5.3551$, and $n = 10$, and so we get

$$t_9 = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{45.3 - 50}{5.3551 / \sqrt{10}} = -2.775$$

(ignoring sign)

Step 4: *p*-value

Using tables, we get (on ν = n−1 = 9 degrees of freedom):

***p*-value**	**10%**	**5%**	**1%**
Critical value	1.833	2.262	3.250

Ignoring the negative sign, we find that our p-value lies between 1% and 5%.

Step 5: Conclusion

- We have moderate evidence against the null hypothesis
- Thus, we should reject the null hypothesis in favour of the alternative
- There is evidence to suggest that the average lead content in this area of the city is *not* equal to 50μg/litre

Assumptions

- Independence
- Data are drawn from a Normal distribution

QUESTION 2

Step 1: Hypotheses

H_0: $\mu_{\text{GOTHIC}} = \mu_{\text{EIXAMPLE}}$
H_1: $\mu_{\text{GOTHIC}} \neq \mu_{\text{EIXAMPLE}}$

Step 2: Which test?

This is a two-sample test. Since we do not know the population standard deviations, we will perform a two-sample t-test.

Step 3: Test statistic

To summarise, we have

$\bar{x} = 45.3$, $s = 5.3551$, and $n = 10$ for the gothic quarter, and

$\bar{x} = 41.0$, $s = 6.4142$, and $n = 15$ for the gothic quarter, and

Thus, we get

$$s = \sqrt{\frac{(10-1)\times 5.3551^2 + (15-1)\times 6.4142^2}{10+15-2}} = 6.022,$$

and so

$$t_{23} = \frac{45.3 - 41}{6.022 \times \sqrt{\frac{1}{10} + \frac{1}{15}}} = 1.749$$

Step 4: *p*-value

Using tables, we get (on $\nu = 23$ degrees of freedom):

***p*-value**	**10%**	**5%**	**1%**
Critical value	1.714	2.069	2.807

Ignoring the negative sign, we find that our *p*-value lies between 5% and 10%.

Step 5: Conclusions

- We have only slight evidence against the null hypothesis
- Thus, we should retain the null hypothesis
- There is insufficient evidence to suggest a difference in the average lead content of water between the two areas of the city.

Assumptions

- Independence
- Both samples are drawn from Normal distributions
- Equality of population variances – this seems OK, since both sample standard deviations are fairly close

QUESTION 3

Since, in both case, the population standard deviation is unknown, then the formula for a confidence interval for the population mean lead content is:

$$\bar{x} \pm t \times \frac{s}{\sqrt{n}}$$

For the first sample, we have

$\bar{x} = 45.3$, $s = 5.3551$, $n = 10$ and $t_9 = 2.262$, giving

45.3 ± 3.831, i.e. **(41.469, 49.131)**

For the second sample, we have

$\bar{x} = 41.0$, $s = 6.4142$, $n = 15$ and $t_{14} = 2.145$, giving

41.0 ± 3.552, i.e. **(37.448, 44.552)**

Since both these confidence intervals overlap, there is insufficient evidence to suggest a difference between the population mean lead content between the two areas. This conclusion supports the outcome of the hypothesis test performed in question 2.

QUESTION 4

Since the population standard deviation is unknown, the formula for a confidence interval for the population mean collision cost is:

$$\bar{x} \pm t \times \frac{s}{\sqrt{n}}$$

We have

$\bar{x} = 542, s = 70$, and $n = 80$.

For a 90%, 95% and 99% confidence interval, t = 1.645, 1.960 and 2.576 (respectively). Thus, we get:

90% CI: **(£529.13, £554.87)**
95% CI: **(£526.66, £557.34)**
99% CI: **(£521.84, £562.16)**

If the population standard deviation was *known* to be £70, i.e. σ = 70, then, in practice, we would get exactly the same confidence intervals as those given above. The confidence intervals calculated above are only *approximate*; recall that, as $n \to \infty$, Student's t distribution → the standard Normal distribution, and so, when n is quite large (as in this example), there is very little difference between critical values from the t distribution and corresponding values from the standard Normal. So much so, in fact, that for large sample sizes, (typically $n > 30$), we simply use critical values from the standard Normal distribution whether σ is know or not – this corresponds to using the ∞ row in table 1 of the formulae booklet.

QUESTION 5

Step 1: Hypotheses

H_0: $\mu_A = \mu_R$
H_1: $\mu_A \neq \mu_R$

Step 2: What test?

This is a two-sample test, and since both population standard deviations are known, we use a two-sample Z-test.

Step 3: Test Statistic

We have

$$Z = \frac{\bar{x}_A - \bar{x}_R}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_R^2}{n_R}}} = \frac{26.4 - 23.5}{\sqrt{\frac{4.28^2}{12} + \frac{3.30^2}{15}}} = 1.932$$

Step 4: *p*-value

From tables, we get:

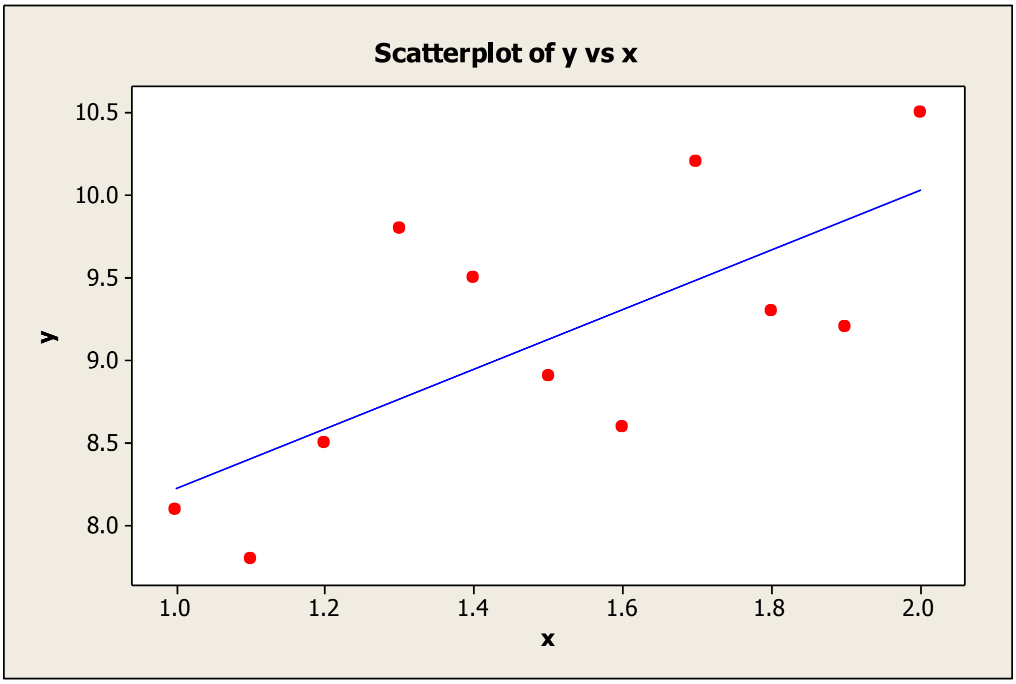
***p*-value**	**10%**	**5%**	**1%**
Critical value	1.645	1.960	2.576

Thus, our p-value lies between 5% and 10%.

Step 5: Conclusions

- We have only slight evidence against the null hypothesis
- Thus, we retain the null hypothesis (but should consider further investigation)
- There is insufficient evidence to suggest a difference in the average pulse rate increase between American and Russian astronauts.

QUESTION 6



(a) Comment: there is strong positive linear relationship between x and y.

(b) n=11,

$$S_{xy} = \sum xy - n\bar{x}\bar{y}$$
$$= 152.59 - 11 \times \frac{16.5}{11} \times \frac{100.4}{11} = 1.99,$$

And

$$S_{xx} = \sum x^2 - n\bar{x}^2$$
$$= 25.85 - 11 \times \left(\frac{16.5}{11}\right)^2 = 1.1.$$

Thus,

$$\hat{\beta} = \frac{S_{xy}}{S_{xx}} = 1.809$$
$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 6.414$$

The fitted regression model is

$$\mathrm{y} = 6.414 + 1.809\mathrm{x} + \varepsilon$$

The fitted line can be found in the diagram in part (a).

(c) (i) From the normal PP plot, the normality assumption seems reasonable although the histogram seems not support it. This may be caused by the small sample size.
(ii) The assumption of constant variance seems reasonable.

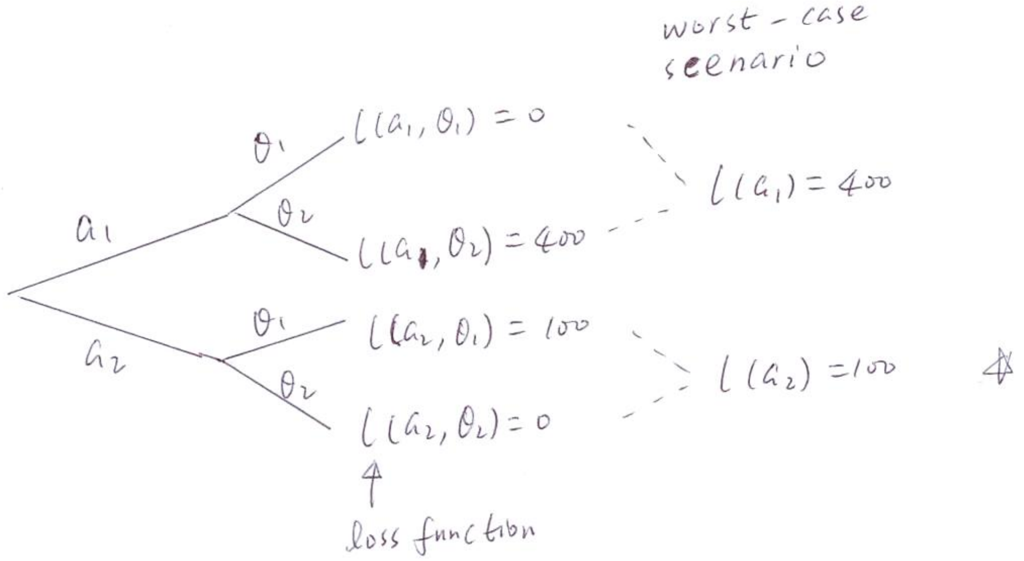
(d) When x=1.5, the prediction is

$$\hat{\mathrm{y}} = 6.414 + 1.809 \times 1.15 = 8.494.$$

The predicted amount of converted sugar is 8.494 at a temperature of 1.15.

QUESTION 6

a) The decision tree is given as follows



b) From the above decision tree, it is easy to know that, in the worst-case scenario, the loss is 400 for action a1 and 100 for action a2. So we should take action a2 to minimize the loss in the worst-case scenario using the minimax criterion.

c) The expected values of the loss functions are

$$l(a_1) = E\big(l(a_1, \theta)\big) = l(a_1, \theta_1) * \frac{5}{6} + l(a_1, \theta_2) * \frac{1}{6} = 0 * \frac{5}{6} + 400 * \frac{1}{6} = 66.67$$

And

$$l(a_2) = E\big(l(a_2, \theta)\big) = l(a_2, \theta_1) * \frac{5}{6} + l(a_2, \theta_2) * \frac{1}{6} = 100 * \frac{5}{6} + 0 * \frac{1}{6} = 83.33$$

So the expected-value decision is to take action a1 to minimize the expected-values of loss functions.