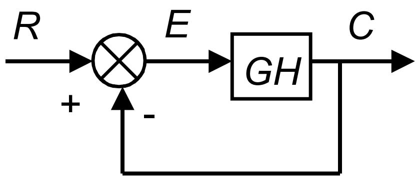
Nyquist Stability Simplified

$$\frac{C_{(s)}}{R_{(s)}} = T_{(s)} = \frac{G_{(s)}}{1 + G_{(s)}H_{(s)}}$$

General expression: $F_{(s)} = 1 + G_{(s)}H_{(s)} = 0$

Roots of $F_{(s)}$ are poles of $T_{(s)}$ and must therefore lie in left half plane for stability

Unity Feedback:



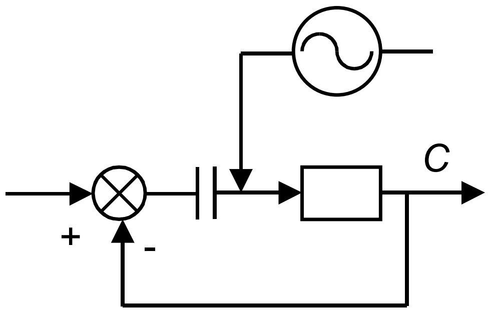
Defines Performance:

Closed Loop $\frac{C_{(s)}}{R_{(s)}} = \frac{G\ H}{1 + G\ H}$

Open Loop $\frac{C_{(s)}}{E_{(s)}} = G\ H$ & $E_{(s)} = R_{(s)} - C_{(s)}$

Input / Error $\frac{R_{(s)}}{E_{(s)}} = \frac{R_{(s)}}{R_{(s)} - C_{(s)}}$

Any of above three equations defines behaviour
If open loop transfer function known, closed loop can be predicted
Measurement is important
break loop and remove normal input:



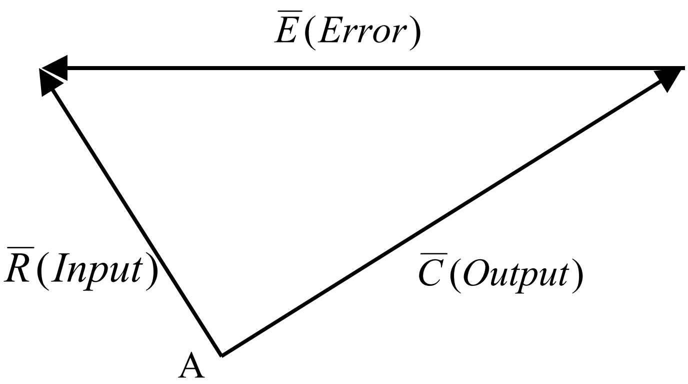
$E_{(s)} = R_{(s)} - C_{(s)}$; only 2 need be known

When $s \to j\omega$ these signals form a closed phasor set

Linear system? – absolute magnitude not important, so relative magnitude or relative phase

Can choose one signal in absolute magnitude and phase – other signals relative to that choice

Phasor Diagram:



Make $\left|E\right| = 1$ and zero phase, $\therefore$ draw horizontally

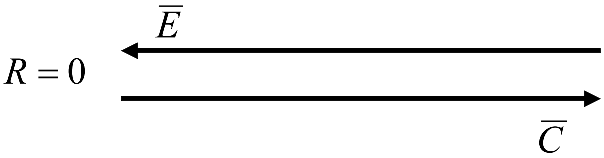
Location of point A specifies response (at this frequency)

$\frac{\overline{C}}{\overline{E}} = \overline{G}\,\overline{H}$, $\therefore$ plot $GH_{(j\omega)}$

Locus of point A is locus of open loop frequency response - specifies response for all frequencies covered

$\overline{R}(Input)$

Tip of $\overline{E}$ above lies at $(-1, j0)$, special case:



$\overline{E} = -\overline{C}$, $R = 0$ i.e. unstable output for no input

Nearness of A to $(-1, j0)$ indicates closeness to instability

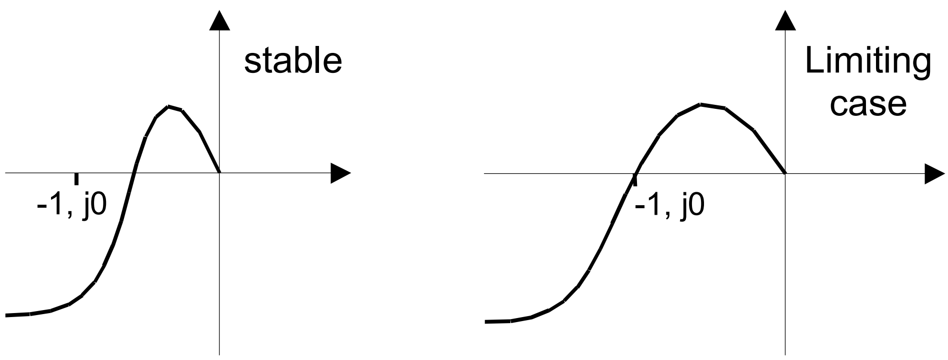
Nyquist Criteria

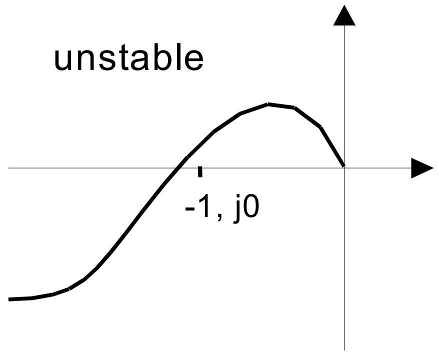
If open loop stable - simplified criteria

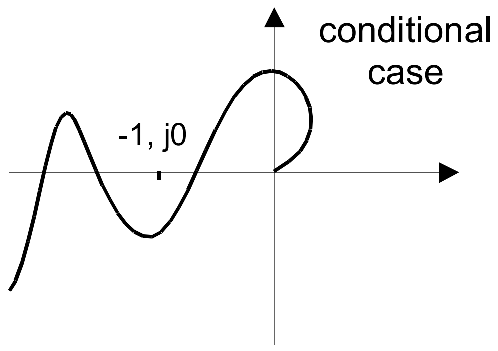
Locus of the open loop frequency response must pass to the right of $(-1, j0)$[Not enclosing] as the frequency goes from $0 \rightarrow \infty$

Polar plot of $GH_{(j\omega)}$ is Nyquist Diagram

Examples



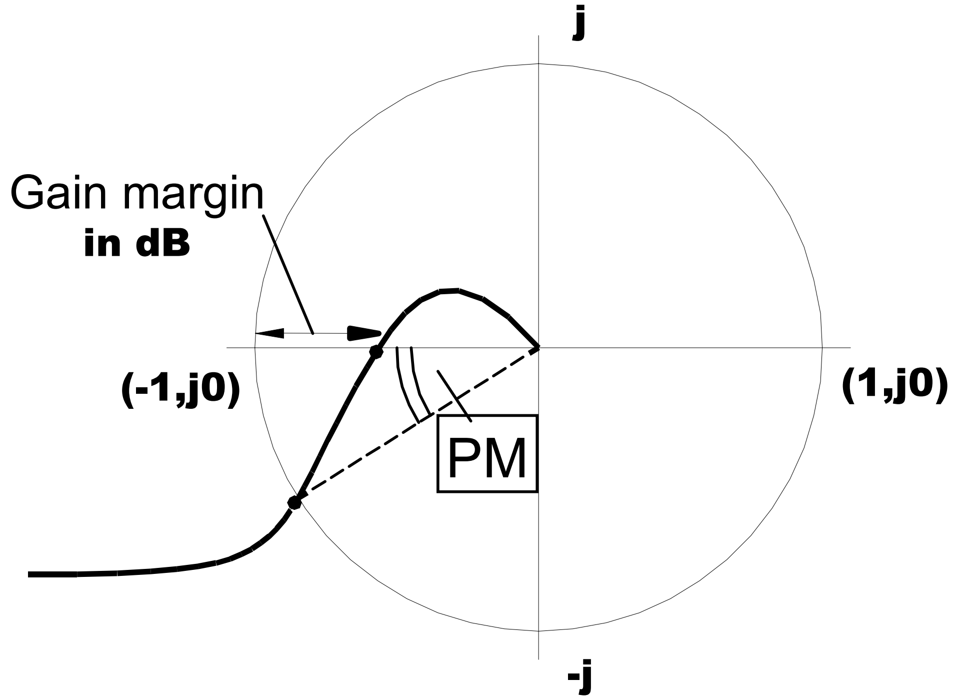




Measure of nearness to instability.

Gain = 1, Phase?

Phase = -180°, Gain?



Phase Margin $= 180^\circ +$ phase at which the gain is equal to unity

Gain Margin is the reciprocal of $\left|GH_{(j\omega)}\right|$ at frequency when phase reaches -180°

$$GM = 20\log_{10}\left|\frac{1}{GH_{(j\omega)}}\right| = -20\log_{10}\left|GH_{(j\omega)}\right|$$

Aims

Gain Margin ~ $8dB(3-12dB)$

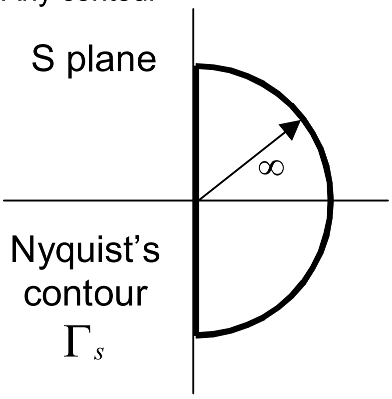
Phase Margin ~ $45^\circ(30^\circ - 60^\circ)$

Complete form of Nyquist Criteria

Open loop unstable
i.e. $GH_{(s)}$ has positive real part roots (roots in right half plane)

Cauchy's Encirclement Theorem

$s \to F_{(s)}$
Any contour



Let
$N =$ number of clockwise encirclements
$P =$ number of poles enclosed
(usually poles of *GH(s)* in right hand plane)
$Z =$ number of roots or zeros of function of the complex variables
(usually zeros of $F_{(s)} = 0$ in right hand plane)

then $N = Z - P$

(NB if P is finite this means an open loop unstable system)

Conformal transformations - direction of passage is retained

SISO (Single Input Single Output)
$F_{(s)} = 1 + GH_{(s)}$

Therefore can examine $GH_{(s)} = -1 \to$ more convenient

Plot of $GH_{(j\omega)}$ is always mirror image for $(-j\omega)$ frequencies of the "real" frequency response i.e. $(+j\omega)$
Real system has zero response at infinite frequency, therefore zero point at infinite radius

Nyquist Complete Criteria

Requires that for system to be stable, number of counter clockwise encirclements of $(-1, j0)$ by $GH_{(j\omega)}$ as ω goes from $-\infty \rightarrow +\infty$ must equal number of poles of $GH_{(s)}$ with positive real parts

If $P = 0$ reduces to simplified criteria since passing to right of $(-1, j0)$ as ω goes from $0 \rightarrow +\infty$ means that is not enclosed for full contour

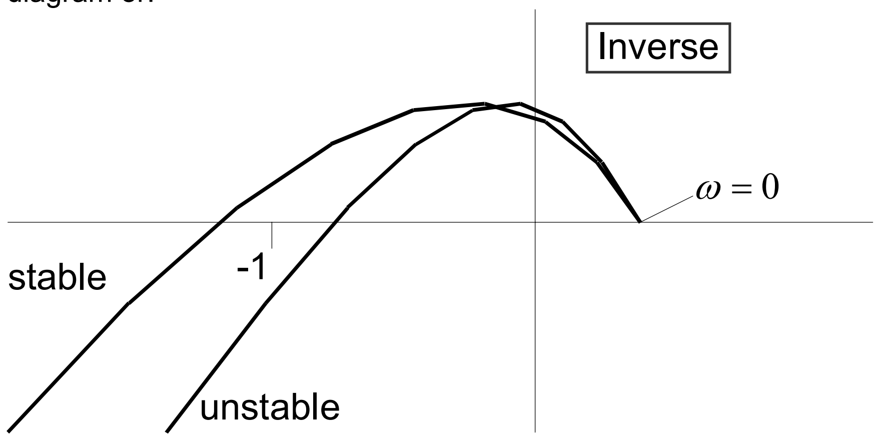
If P is finite closed loop system is still stable if N is negative such that $N + P \leq 0$, i.e. counter clockwise encirclements balance unstable open loop poles

Inverse Nyquist

General Case $\dfrac{C}{R} = \dfrac{G}{1+GH}$

Inverse $\dfrac{R}{C} = \dfrac{1+GH}{G} = \dfrac{1}{G} + H$

Inverse form of Nyquist – useful if H is a compensator, i.e. plot $\dfrac{1}{G}$ on a polar diagram or:



M and δ Circles

Nyquist plot “contains” the whole closed loop response
Performance specification – frequency domain

Bandwidth – range of ω where response has tolerance, typically $\pm 3dB$ for control $0 \rightarrow \omega_c$

ω_c is cut off frequency at which response is -3dB

Cut-off rate, rate of fall of response beyond ω_c

Resonance peak, M_r peak “magnification”; $M_r = \left(\frac{C_{(j\omega)}}{R_{(j\omega)}} \right)_{\max}$.

Often suitable M_r from $1.1 \rightarrow 1.5$, and is measure of how well damped or “lively” the system is

ω_r is resonant frequency