MMM231/ENM316/CIV434 - Finite Element Method

Exercises 3 – Stresses, strains and displacements

Use the formulae on the pages indicated, i.e. pp30 and 41-4:

1. Calculating strains from direct stresses σ_x, σ_y, σ_z (p41):

   (i) Given $E = 2\times10^{11}$, $\nu = 0.3$, $\boldsymbol{\sigma} = (10^8 \quad 0 \quad 0)^T$, find the vector of strains $\boldsymbol{\varepsilon}$.

   (ii) If $E = 10^{11}$, $\nu = 0.25$, $\boldsymbol{\sigma} = (2\times10^8 \quad -10^8 \quad 5\times10^7)^T$, find the strains $\boldsymbol{\varepsilon}$.

2. Calculating stresses from strains in the case of plane stress (p30 or 42): Given $E = 2\times10^{11}$, $\nu = 0.3$, $\boldsymbol{\varepsilon} = (10^{-3} \quad -2\times10^{-3} \quad 5\times10^{-4})^T$, find the vector of stresses $\boldsymbol{\sigma}$.

3. If *x* and *y* displacements are given by $u = (1-3xy+y^2)\times10^{-2}$, $v = (2+x^2+xy+2y^2)\times10^{-2}$, find the vector of strains $\boldsymbol{\varepsilon} = (\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy})^T$, at the point (2,-3) (see p30).

4. If the vector of stresses is given by $\boldsymbol{\sigma} = (3x^2+2y+1 \quad 3y^2-2y+x \quad 2x-6xy+5)$, check that these satisfy the equilibrium equations (p44).

5. (i) Find the shape functions for the 3-node element with nodes at (2,0), (2,3), (-3,0) (exercise 1(iii) on Sheet 2).

   (ii) Write down the 2x6 matrix of shape functions **N** (p30 for this and rest of question).

   (iii) Find the matrix $\mathbf{B} = \mathbf{LN}$, i.e. the matrix of derivatives of shape functions.

   (iv) Given the vector of nodal displacements $\mathbf{u}^* = 10^{-4}\times(2 \quad 3 \quad 0 \quad -2 \quad -1 \quad 1)^T$, calculate the vector of strains, as $\boldsymbol{\varepsilon} = \mathbf{Bu}^*$.

   (v) Finally, calculate the vector of stresses as $\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}$, where D is the elasticity matrix for plane stress, using $E = 10^{11}$, $\nu = 0.4$.

6. (i) If $\sigma_x = 0.0$, $\sigma_y = 0.0$, $\tau_{xy} = 2.0$, use Mohr's circle (p43) to find the principal stresses and their orientation $\sigma_{\max}$, $\sigma_{\min}$, θ.

   (ii) If $\sigma_x = 6.0$, $\sigma_y = 0.0$, $\tau_{xy} = 4.0$, find the principal stresses and their orientation.

Solutions to Exercises 3

1. (i) $10^{-3}(0.5 \quad -0.15 \quad -0.15)^T$ (ii) $10^{-3}(2.125 \quad -1.625 \quad 0.25)^T$

2. $10^7(8.79 \quad -37.36 \quad 3.85)^T$

3. $10^{-2}(9 \quad -10 \quad -11)^T$

4. (no solution, as requirement is to satisfy equation)

5. (i) - Sheet 2, exercise 1(iii). (ii) - just put functions into correct locations.

   (iii) $\mathbf{B} = \begin{pmatrix} 1/5 & 0 & 0 & 0 & -1/5 & 0 \\ 0 & -1/3 & 0 & 1/3 & 0 & 0 \\ -1/3 & 1/5 & 1/3 & 0 & 0 & -1/5 \end{pmatrix}$, (iv) $10^{-4}(0.60 \quad -1.67 \quad -0.27)^T$

   (v) $10^7(-0.081 \quad -1.70 \quad -0.096)^T$

6. (i) $\sigma_{\max} = 2$, $\sigma_{\min} = -2$, $\theta = 45°$, (ii) $\sigma_{\max} = 8$, $\sigma_{\min} = -2$, $\theta = 26.6°$