MMM231/ENM316/CIV434 - Finite Element Method

Exercises 1

1. (i) For the matrix equations on p9 of the notes, write out fully equations 2, 3, 4, noting that $u_1 = 0,\ u_3 = 0,\ v_3 = 0$, and letting $k = 25EtA/(1-\nu^2)$. For example, equation 3 is $k(-\nu\ v_1 + u_2) = 0$.

   (ii) Add equation 2 to equation 4 to give an equation with only v_1 and u_2 in it. Combine with equation 3 and solve for v_1 and u_2. Finally use equation 4 to get v_2. For example, $v_1 = -F/[k(1-\nu^2)]$.

   (iii) If $E = 2.09\times10^{11}$, $\nu = 0.3$, calculate A and then *k*. Hence get values for v_1, u_2, v_2. Compare with results obtained using *FINEL*.

   (iv) Using solutions to (iii) (i.e. in numerical form), use equations 1, 5, 6 to find the unknown reaction forces on the r.h.s. of the equations. For example, equation 1 gives: $X_1 = k(0.65v_1 - u_2 - 0.35v_2) = 1000$.

   (v) Check that your results for the reaction forces agree with equilibrium of forces and moments for the original problem.

2. (i) If the original bracket problem is modified to look like Figure 1, sketch the expected shape when loaded.

   (ii) If the bracket is modelled by a single 3-node element as before, what deformed shape do you expect?

   (iii) If the bracket is modelled by four 3-node elements (similar to layout on p4 of notes), what shape do you expect to see?

   (iv) Number both meshes and write down the data files you would need for input to *FINEL*.

   (v) Enter and run both cases with *FINEL* and see if your predicted shapes were correct.

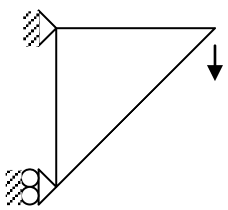


Figure 1.

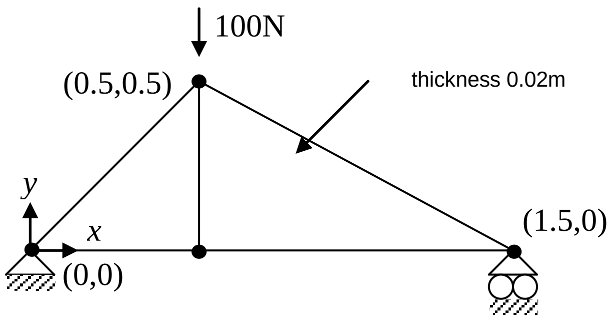


Figure 2.

3. (i) For the problem and two element mesh design shown in Figure 2, number the nodes and elements and write out the data file for *FINEL*.

   (ii) Use *FINEL* to find the vertical displacement of the highest point and the horizontal displacement at the right-hand support.

   (iii) Also look at the output file to find the reaction forces at the supports, and check these satisfy equilibrium of forces and moments for this problem.

Solutions to Exercises 1

1. (i)
$$k[(1+p)v_1 - \nu u_2 - pv_2] = 0$$
$$k[-\nu v_1 + u_2] = 0$$
$$k[-pv_1 + pv_2] = -F$$

(ii)
$$v_1 = -F/[k(1-\nu^2)]$$
$$u_2 = -\nu F/[k(1-\nu^2)]$$
$$v_2 = -F(3+2\nu)/k(1-\nu^2)]$$

(iii) $A = 0.02m^2$, $k = 1.148\times10^9\,Nm^{-1}$, hence

$v_1 = -9.57\times10^{-7}m$, $u_2 = -2.87\times10^{-7}m$, $v_2 = -3.44\times10^{-6}m$,

(iv) $X_1 = 1000N$, $X_3 = -1000N$, $Y_3 = 1000N$.

2. (iv) Data file is exactly as that on p8 of the notes, except that node 2 has coordinates (0.2,0.2).

(v) There are various ways of numbering the mesh, but make sure you have included a constraint in the *x*-direction at the bottom corner, and constraints in both directions at the top left corner. The force should act downwards, i.e. if the right-hand tip is node 6, the force should be: 6 2 -1000.

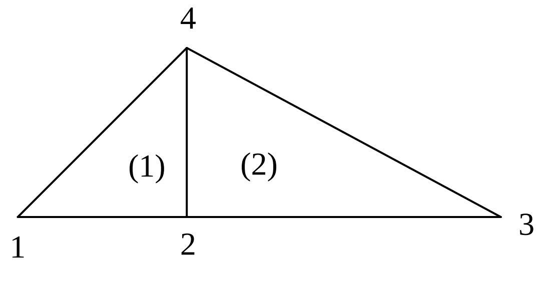
3. (i) One way of numbering the mesh is shown below. In this case, elements and constraints data will look like this:

```
elements
1 1 2 4
2 2 3 4

constraints
1 1 0.0
1 2 0.0
3 2 0.0
```

(ii) $v_4 = -9.89\times10^{-8}m$, $u_3 = 1.44\times10^{-8}m$

(iii) $X_1 = 0$, $Y_1 = 66.67N$, $Y_3 = 33.33N$



JC Appleby, February 2003