EPITOMIZ: <u>E</u>LASTO-<u>P</u>LAST<u>I</u>C GEOME<u>T</u>RICALLY N<u>O</u>NLINEAR STRUCTURAL OPTI<u>MIZ</u>ATION

BACKGROUND

Shape optimization is a fundamental objective of designers of components, devices or systems. It aims to determine the dimensional variables such that they maximise the ability of the part to perform the intended function under the prescribed operating conditions. An important aim of engineering activities is to improve and to optimize technical designs, structural components and structural assemblies. The task of structural optimization is to support the engineer in searching for the best possible design alternatives of specific structures. The "best possible" or "optimal" structure is the structure which corresponds most closely to the designer's desired concept whilst at the same time meeting functional, manufacturing and application criteria, e.g., all multidisciplinary requirements. In comparison to the "trial & error" method generally used in the engineering environment and based on an intuitive heuristic, the determination of optimal solutions by applying mathematical optimization procedures is more reliable and efficient if correctly applied.

In the field of mechanical design, the first known shape optimization study was by Galileo (1638), who presented a logical scheme for selecting the form of a bent beam for uniform strength. Subsequently, the development of differential calculus provided a mathematically elegant general tool for the evaluation of maxima and minima of differentiable relationships or functionals. Mathematical optimization procedures have been developed from the classical works of Bernoulli, Euler & Lagrange (Cajori (1919)). In 1847 Cauchy made a significant contribution to optimization theory in proposing the gradient search algorithm. It was at the beginning of the century (1910) when the shape optimization field was presented as a branch of optimal control theory, leading (from 1960s) to the combination of numerical approximation techniques (i.e. finite element method) with mathematical programming methods in the development of structural optimization algorithms.

The origin of plasticity, as a branch of the mechanics of continua, may be attributed to Trseca (1864-1872), who published a series of papers on the extrusion of metals. St. Venant developed the fundamental theory in 1870. Generalized equations in three dimensions were given by Levy later in the same year, derived independently by von Mises in a landmark paper in 1913. Between 1940 and 1960 intensive research was undertaken in the field, culminating in the development of the classical theory of metal plasticity. Notable contributors included Melan, Prager, Drucker & Koiter.

Research within the field of plasticity applied to problems in structural mechanics is well established. Most recently, research in this area has concentrated on advanced problems including finite element formulations / algorithms (i.e. Yoshimura et al (1994)), composites / composite construction (i.e. Swaddiwudhipong (1996)), contact (i.e. Ko et al (1996)) and combined material and geometric nonlinearities. Meek & Loganathan (1990) presented results of a study into the geometric and material nonlinear behaviour of space frame structures comprising thin walled circular and square hollow sections. Based on a simple beam-column element formulation, nonlinear governing equations were derived to represent large displacement elasto-plastic responses and solved using a combined arc length / Newton-Raphson method. Most relevant to the research proposed in this case for support, for the realistic prediction of the non-linear behaviour of a structure, the results of the study identified the necessary incorporation of the effect of both geometric and material non-linearities.

Recent advances in thin shell finite element formulations and applications have been reviewed in a paper by Yang et al (1990). The paper highlights the research trend into the development of shell finite elements for the material and geometric nonlinear analysis of flat plates, axisymmetric and curved shells, with particular emphasis on discrete Kirchoff and degenerated theories. Most research activities in this area have concentrated on improving formulations based upon the classic starting point of an assumed displacement field for analysis. The avoidance of shear and membrane locking in thin shells and the appropriate introduction of drilling degrees of freedom have been the primary topics of interest. In recognition of the maturity of research into material & geometric nonlinearities and shell element formulations, a number of commercially available finite element codes support a significant range of analysis algorithms in these areas (i.e., ABAQUS, MARC, LUSAS, ANSYS, etc.).

Predominantly, the geometrically nonlinear optimization of beams, trusses, plates and (to a lesser extent) shells have been investigated. Three different measures of nonlinear structural stiffness were proposed by Kamat & Mesquita (1985). When optimizing these measures, it was shown that identical designs were obtained in the linear case, but significantly different solutions were generated when the response was nonlinear. Procedures for design sensitivity analysis of nonlinear structural response using an incremental solution scheme have been described by Wu & Arora (1987), while Cardoso & Arora (1988) presented a unified variational theory of design sensitivity analysis of linear and nonlinear structures for shape, geometry and material selection problems. Both physical and geometric nonlinearities were treated by Mroz et al (1985). A uniform formulation of sensitivity analysis for beams and plates was given in terms of generalised strains and stresses. Optimal design problems with stress and deflection constraints were defined with optimality conditions derived using the concept of a linear adjoint structure.

Most recent papers in the field of material and geometric nonlinear optimization describe a variety of research outcomes. A paper by Bletzinger & Ramm (1993) demonstrated the use of optimization techniques for the form-finding of shells. Stress distributions in shell structures have been shown to be extremely parameter sensitive, especially where the parameter mid-surface is shape. A method was established on the synthesis of design modelling, structural analysis and mathematical optimization techniques. The design of structures with non-smooth mechanical behaviour has been investigated by Karlbring & Ronnqvist (1995). The work applied the method of moving asymptotes to this class of problem. Typical problems solved included the deformation and force distribution in trusses subjected to frictionless contact. Barthold & Stein (1996) described a shape design sensitivity analysis for hyperelastic material behaviour. A rigorous analysis based on convected curvilinear coordinates was used to decompose all continuum mechanical functions into independent geometry and displacement mappings. A clear definition of the dependency of physical quantities on geometry and displacement was provided by the proposed decomposition permitting the use of standard linearization techniques for sensitivity calculations. A study into the optimal design of elastic-plastic beams was described in a paper by Lepik (1995). The location of an additional support was selected as the design variable, with the compliance of the structure as the objective function to be minimised.

While much research has been published on elasto-plastic optimization, with most notable contributions from Hinton, Choi, Bendsoe, Ramm, Totorelli & Bettess, and on geometrically nonlinear optimization (Mroz, Kamat, Totorelli, Toropov & Gosling) structural optimization combining the affects of material and geometric nonlinearities is a new field of research. It is in this context that EPITOMIZ is set and where significant and important contributions can be made.

PROGRAMME & METHODOLOGY

Overall aim:

1. to develop an effective numerical algorithm for the determination of optimal structural solutions in the presence of both material and geometric nonlinearities arising from large deformations.

Objectives:

1. to formulate a fully nonlinear (material and geometric) numerical model based upon shell theory using position (i.e. material co-ordinates rather than displacements) as the state variable.
2. to derive sensitivities of objective functions (i.e. compliance, principal stresses, etc.) with respect to shape design variables when subjected to constraints in the state and tertiary variables.
3. to determine optimal solutions to problems in structural mechanics exhibiting both material and geometric nonlinearities.

Numerical Model Formulation for Analysis

The basis of the numerical formulation is the assumption of position of the curvilinear abscissa of the beam, plate or shell structure / component to be analysed as the state variable. Generation of the equilibrium equations may be sub-divided initially according to geometric and material nonlinearites.

a) **Geometric nonlinearity**: A nonlinear finite element discretisation technique based on the Gauss integration of a nonquadratic density function is to be developed. The fundamentals of this approach were first described in a paper by Fried (1983) and applied to the two-dimensional elastic case only. Taking a specific case in the (x, y) space, then the direct strain, ε, and curvature, κ, may be written as: $\varepsilon = (x' - y')^{1/2} - 1$ and $\kappa = \dfrac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$, where the prime denotes differentiation w.r.t. the curvilinear abscissa. For an initially curved beam (for example) under the action of distributed loads in the x and y directions respectively, the total potential energy, $\pi(x, y)$, is: $\pi(x,y) = \frac{1}{2} EI \int_0^l (\kappa - \kappa_o)^2 ds + \frac{1}{2} EA \int_0^l \varepsilon^2 ds - \int_0^l (fx + gy) ds$. Assuming a cubic C^1 finite element and an isoparametric mapping such that $s = s_1 + h\xi$, $0 \le \xi \le 1$, expressions for the load vector and elementary stiffness matrix are obtained from the first and second derivatives of $\pi(x, y)$, respectively. Standard techniques are used to generate a structure equivalent equation. Owing to the nonlinearity of the resulting equation, the Newton-Raphson method is used to solve for the equilibrated shape in terms of the position of the element nodes (x_j, y_j).

It is proposed to extend this approach into the (x, y, z) space. In the preceding 2-D approach, a line parametrisation yields a direct strain, ε, and a curvature, κ, which is intrinsically principal. ε is effectively the change in length of a normalised vector in the direction of the curvlinear abscissa. Only a direct strain is defined. κ is given by the variation of the angle subtended by the tangent to the curvlinear abscissa and the x axis with respect to the parametrisation variable. In the 3-D case a surface patch will be adopted with two parametrisations in orthogonal directions. Consistent with plane stress problems, a pair of direct strains and a shear strain will define the membrane action, with the latter given by the change in direction of vectors defining the diagonals of the patch. As in the 2-D case, principal curvatures will be established as a function of variations in surface tangents to define the bending contribution to overall equilibrium.

The adoption of nodal positions as the state variable, rather than the conventional use of displacements, has been shown to be effective in the analysis and optimization of flexible beams (Gosling & Rousselet 1996c&d). Principal advantages of this novel approach include: 1. Effective and clear representation of geometric nonlinearities (either by the inclusion of an additional stretching energy term or by an inextensibility constraint); 3. Explicit definition of the initial configuration; 3. Global updating negated; 4. Intrinsic determination of the shape sensitivities.

The Newton-Raphson method will be used to solve the nonlinear equations of equilibrium. A modified N-R or Quasi N-R solution may prove computationally more efficient in the presence of geometric nonlinearity only. When material nonlinearity in the form of plasticity is represented, the modified N-R method may take an excessive number of iterations to converge, whilst the Quasi N-R method may exhibit a lack of smoothness in the progress of the solution.

b) **Material nonlinearity**: J_2-deformation theory of plasticity will form the basis of the materially nonlinear component of the numerical model. The following assumptions will apply, therefore: (i) the material is initially isotropic; (ii) plastic strain involves only a change in shape but no change in volume, and the elastic strain is related to the stress by Hooke's law; (iii) the principal axes of the plastic strain and the stress coincide; (iv) the principal values of the plastic strain have the same ratios to each other as the principal values of the stress deviator (Chen & Han (1988)). As conventional structural materials will be used (identified by the industrial collaborators) the assumption of isotropy is valid. The "no volume change" plastic response assumption should prove useful in the derivation of shape design sensitivities, certainly in the preliminary stages of the study. Similarly, assumptions (iii) & (iv) are justifiable. A number of hardening rules may be adopted. The simplest work-hardening rule assumes a perfectly plastic material (isoptropic hardening), though the Bauschinger effect will not be represented. Initial stiudies may be limited to single load cases, therefore. This deficiency, if important (a decision based on input from the industrial collaborators) may be overcome by assuming the kinematic hardening rule due to Prager. By adopting a J_2 -material (as commonly used in metal plasticity) it is possible to define an elastic-plastic tangent stiffness tensor C_{ijkl}^{ep} for the mixed-hardening von Mises material which includes isotropic hardening, kinematic hardening and perfectly plastic behaviours (without hardening) as special cases. The flexibility of this model is well suited to the proposed study.

c) **Analysis of results**: Comparisons with analytical solutions. Benchmarking via NAFEMS (National Association for Finite Element Methods & Standards).

Sensitivity Determination

a) A semi-analytical approach will be used initially in the determination of shape sensitivities. (The adjoint variable method is used by some researchers. This method is seen as a specialisation to enhance performance measures in the numerical evaluation of sensitivities. Though important, the area of research associated with the development of the adjoint variable method has not been identified as contributing to the objectives of this proposal.) The semi analytical method is based on the implicit differentiation of the state equations. Silva et al (1997) demonstrated the effectiveness of the semi-analytical method in the case elastic-plastic optimization, recognizing the dependency of the material tangent stiffness tensor on the state variables. The nature of the semi-analytical sensitivities will depend on the potential to define terms explicitly through direct differentiation and the relative computational costs. Extensive use of an algebraic manipulator, namely Mathematic, will be made.

In proposing the adoption of position as the state variables for the analysis component of the study, the numerical efficiency of determining shape sensitivities will be enhanced compared to the conventional displacement variables approach. In a paper by Weichmann, Barthold & Stein (1997) an approach was described in which the sensitivity analysis was formulated to be consistent with the direct analysis solution. In the example cited it was recognised that sensitivity expressions are normally derived from the weak form of the equilibrium equation(s), while the tangent stiffness matrix is obtained by linearising the weak form of the same equation(s) with respect to displacements. Thus, several results of the structural analysis may be used directly in the calculation of design sensitivities, reducing computational effort considerably. This benefit should be enhanced through the adoption of "position" state variables as proposed here.

b) **Analysis of results**: Comparison with analytical sensitivities. Benchmarking against NASA Langley Research Centre's MDO test suite via http://fmad-www.larc.nasa.gov/mdob/.

Development of the optimization algorithm

a) The proposed numerical formulation is to be combined with a penalty function – Lagrangian method to establish the minimum of a function with both equality and inequality constraints. The optimization algorithm to be developed in this task is not intended to represent a significant research contribution. It is more a facilitator to the primary research activities identified in tasks 1, 2 & 4. It is planned to make a parallel grant application at the end of the second year of this project, which will aim to enhance the optimization algorithm with the introduction of adaptive meshing techniques and the multi-point approximation method developed by Dr. Toropov. This is not within the scope of the current project.

b) **Analysis of results**: Optimal solutions to classical problems in nonlinear structural mechanics.

References:

Barthold, F.J., Stein, E., (1996), Continuum mechanical-based formulation of the variational sensitivity analysis in structural optimization. Part I: analysis, Structural Optimization, v.11, n.1-2, 29-42.

Bletzinger, K.U., Ramm, E., (1993), Form-finding of shells by structural optimization, Computers with Structures (New York), v.9, n.1, 27-35.

Cajori, F., (1919), *A History of Mathematics*, Macmillan, New York.

Cardoso, J.B., Arora, J.S., (1988), Variational method for design sensitivity analysis in nonlinear structural mechanics, J. of A.I.A.A., v.26, 595-603.

Fried, I., (1983), Nonlinear finite element computation of the equilibrium, stability and motion of the extensional beam and ring, Comput. Methods in Appl. Mech. & Eng., v.38, 29-44.

Kamat, M.P., Mesquita, L., (1985), Optimization of highly flexible beams, Engineering Optimization, v.9, 61-72.

Klarbring, A., Ronnqvist M., (1995), Nested approach to structural optimization in non-smooth mechanics, Structural Optimization, v.10, n.2, 79-86.

Ko, H.Y., Kwak, B.M., Im,.S., (1996), Contact stress and failure analysis on bolted joints in composite plates by considering material nonlinearity, Proc. Fifth Int. Conf. On Comp. Aided Des. In Comp. Mat. Tech., Udine, Italy, July, 31-40.

Leitmann, G. (editor), (1962), *Optimization Techniques with Applications to Aerospace Systems*, Academic Press, New York.

Lepik, U., (1995), Optimal design of elastic-plastic beams with additional supports, Structural Optimization, v.9, n.1, 18-24.

Lewis, W.J., Gosling, P.D., (1993), Stable Minimal Surfaces in Form-finding of Lightweight Tension Structures, Journal of Space Structures, v.8, n.3, 149-166.

Meek, J.L., Tan, H.S., (1985), Discrete Kirchoff plate bending element with loof nodes, Computers & Structures, v.21, n.6, 1197-1212.

Mroz, Z., Kamat, M.P., Plaut, R., (1985), Sensitivity analysis and optimal design of nonlinear beams and plates, J. Struct. Mechanics, v.13, 245-266.

Silva, C.E.K., Hinton, E., Sienz, J., Vaz, L.E., (1997), Structural shape optimization using elastoplastic stress fields, Proc. WCSMO-2, May 26-30, Zakopane, Poland, v.1, 503-508.

Swaddiwudhipong, S., (1996), Nonlinear finite element analysis of fibre reinforced concrete deep beams, Structural Engineering & Mechanics, v.4, n.4, 437-450.

Vanderplaats, G.N., (1997), Structural design optimization - status and direction, Proc. 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, v.2, 1178-1192.

Weichmann, K., Barthold, F.-J., Stein, E., (1997), Optimization of elasto-plastic structures using the finite element method, Proc. WCSMO-2, May 26-30, Zakopane, Poland, v.2, 1013-1018.

Wu, C.C., Arora, J.S., (1987), Design sensitivity analysis and optimization of nonlinear structural response using incremental procedure. J. of A.I.A.A., v.25, 1118-1125.

Yang, H.T.Y., Saigal, S., Liaw, D.G., (1990), Advances of thin shell finite elements and some applications, Computers & Structures, v.35, n.4, 481-504.

Yoshimura, S., Pyro, C.R., Yagawa, G., Kawai, H., (1994), Finite element analyses of three dimensional fully plastic solutions using quasi-nonsteady algorithm and tetrahedral elements, Computational Mechanics, v.14, n.2, 128-139.