Effect of gravitational consistency and mass conservation on seasonal surface mass loading models

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Abstract. Increasingly, models of surface mass loads are used either to correct geodetic time coordinates by removing seasonal and other "noise", or for comparison with other geodetic parameters. However, models of surface loading obtained by simply combining the mass redistribution due to individual phenomena will not in general be self-consistent, in that (i) the implied global water budget will not be mass-conserving, and (ii) the modelled sea level will not be an equipotential surface of Earth's total gravity field. We force closure of the global water budget by allowing the "passive" ocean to change in mass. This medium-term passive ocean response will not be a uniform change in non-steric ocean surface height, but must necessarily be spatially variable to keep the "passive" ocean surface on an equipotential. Using existing load models, we demonstrate the effects of our consistency theory. Geocenter motion is amplified significantly, by up to 43%.

1. Introduction

Forward modeling of changes in the Earth's gravity field and geometric shape, due to "known" surface mass loads caused by atmospheric, oceanic and hydrological effects, has frequently been carried out in order to explain variations in the Earth's geocenter and rotation, and displacements of geodetic monuments [e.g. *van Dam and Wahr*, 1998; *van Dam et al.*, 2001]. In this paper, we show that significant changes in the results of this approach arise from conservation of the global mass budget of the (imperfectly) "known" load and from accounting for the long-period response of the ocean to the load's gravitational forcing.

The geometric and gravimetric effects of loading may be computed by convolving models giving the gridded surface mass distribution with a Green's function, describing the unit impulse response of the Earth as a function of load and response location [*Farrell*, 1972]. Typically an elastic Earth model with radial structure, such as PREM [*Dziewonski and Anderson*, 1981] is used, in which case the Green's function depends on load-response separation only. Alternatively, if the mass distribution is expressed spectrally, as coefficients of a spherical harmonic expansion, Love number formalism [e.g. *Grafarend*, 1986] may be used. In either case, the error in the resulting estimates arises principally from that of the mass distribution models, although the effects of lateral heterogeneity in Earth structure and of anelastic deformation may also contribute [*Tamisiea et al.*, 2002].

Typically, surface mass load estimates are obtained separately for the effects of circulatory ocean redistribution, atmospheric surface pressure, and continental hydrology (including ground and sub-surface water, snow, and ice). In this case, there is no guarantee that ocean-continent mass exchange will be representative or that the total load mass will be conserved. In particular, the oceanic component is problematic because the majority of ocean circulation models make use of the Boussinesq approximation, conserving ocean volume rather than ocean mass. Typically, a uniform layer of water is added to the ocean model output so as to conserve oceanic mass [*Greatbatch*, 1994]; however, ocean mass and volume are known to vary seasonally and spatially [*Chen et al.*, 2002; *Blewitt and Clarke*, 2003]. Here, we impose additional constraints to force the total load mass to be conserved and to allow the ocean to respond tidally to the total load's gravitational potential.

2. Theory

The total time-variable load exerted on the Earth, T, includes the loads due to atmospheric pressure, continental surface and ground water storage (including snow and ice), and circulatory changes in ocean bottom pressure, which together may be referred to as the dynamic load, D. In addition to this, the ocean will respond tidally to the total load [*Dahlen*, 1976; *Wahr*, 1982; *Mitrovica et al.*, 1994; *Blewitt and Clarke*, 2003], introducing a "passive" oceanic load, S. As has been previously recognised for the much larger sea-level changes that occur in ocean tides and glacial isostatic adjustment, this response will be such that that the passive load is in the long term in hydrostatic equilibrium with the gravitational potential field due to the total (dynamic plus passive) load. Furthermore, the passive oceanic load must include a degree-zero component, reflecting seasonal ocean-continent mass exchange, that exactly balances that of the dynamic time-varying load so that the total load's mass is conserved.

Formally, considering a spherical Earth model and expressing all loads in terms of the equivalent height of a column of seawater, density r_S, the total time-variable load T may be expressed as a function of geographic position Ω (latitude f, longitude l) as

$$T(\Omega) = D(\Omega) + S(\Omega) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{\Phi=C}^{S} T_{nm}^{\Phi} Y_{nm}^{\Phi}(\Omega) \qquad (1)$$

using a notation following that of [*Blewitt and Clarke*, 2003], with un-normalized surface spherical harmonic basis functions

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$Y_{nm}^{\Phi}(\Omega)$. Summation begins at $n=1$ because conservation of mass requires that T_{00} should vanish (although in general $S_{00}=-D_{00}\neq 0$). The resulting change in potential at the reference surface (the initial geoid), due to the effect of the load itself and the accompanying deformation of the Earth, is [*Farrell*, 1972]

$$V(\Omega)=\frac{3g\rho_S}{\rho_E}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\sum_{\Phi=C}^{S}\frac{1+k_n'}{2n+1}T_{nm}^{\Phi}Y_{nm}^{\Phi}(\Omega) \qquad (2)$$

where ρ_E is the mean density of the solid Earth, g is the acceleration due to gravity at its surface, and k_n' is the static gravitational load Love number for degree n. The surface of the solid Earth will change in height by

$$H(\Omega)=\frac{3\rho_S}{\rho_E}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\sum_{\Phi=C}^{S}\frac{h_n'}{2n+1}T_{nm}^{\Phi}Y_{nm}^{\Phi}(\Omega) \qquad (3)$$

where h_n' are the height load Love numbers. Throughout, we use Love numbers derived for PREM (D. Han, personal communication).

We define "relative static sea level" $S(\Omega)$ as the change in ocean bottom pressure expressed as the equivalent height of a column of sea water. In the absence of atmospheric pressure or ocean density and current changes, this will be equal to the true column height of the ocean measured from the solid Earth's surface to the ocean's upper surface. In general, $S(\Omega)$ will follow the "sea-level equation" [*Dahlen*, 1976]

$$S(\Omega)=C(\Omega)\{(V(\Omega)+\Delta V)/g-H(\Omega)\} \qquad (4)$$

where $C(\Omega)$ is the coastline (or "ocean") function, defined as zero over land and unity over the oceans. The spatially-varying term $V(\Omega)/g$ represents the deformation of the geoid, whereas the spatially-constant term $\Delta V/g$ arises because the new instantaneous sea surface will in general have a different potential to that of the original geoid, because of (i) the irregular distribution of oceans and land, and (ii) ocean-continent mass exchange.

The term in braces in equation (4), which we refer to as the "quasi-spectral sea level" [*Blewitt and Clarke*, 2003] may be written as

$$\tilde{S}(\Omega)=\frac{\Delta V}{g}+\sum_{n=1}^{\infty}\sum_{m=0}^{n}\sum_{\Phi=C}^{S}\left[\frac{3\rho_S}{\rho_E}\cdot\frac{1+k_n'-h_n'}{2n+1}\right]T_{nm}^{\Phi}Y_{nm}^{\Phi}(\Omega) \qquad (5)$$

The relative significance of $\tilde{S}(\Omega)$ in comparison with the total load at degree one and higher degrees is controlled by the bracketed ratio in equation (5), which decreases monotonically, falling below the 10% level at degree six (Figure 1). We are therefore able to truncate our estimate of $\tilde{S}(\Omega)$ at some relatively low degree without undue adverse effect on our estimates of the total load. We may then calculate the global representation of the sea level function in the spectral domain

$$S(\Omega)=C(\Omega)\tilde{S}(\Omega) \qquad (6)$$

using a product-to-sum transformation [e.g. *Balmino*, 1984; *Blewitt and Clarke*, 2003]. In particular, the degree-zero term is

$$S_{00}=C_{00}\frac{\Delta V}{g}+\sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\Phi=C}^{S}\frac{C_{nm}^{\Phi}\tilde{S}_{nm}^{\Phi}}{\Pi_{nm}^{2}} \qquad (7)$$

where Π_{nm}^{2} are the normalization constants for the $Y_{nm}^{\Phi}(\Omega)$. Conservation of mass requires that S_{00} is exactly balanced by D_{00}, and therefore

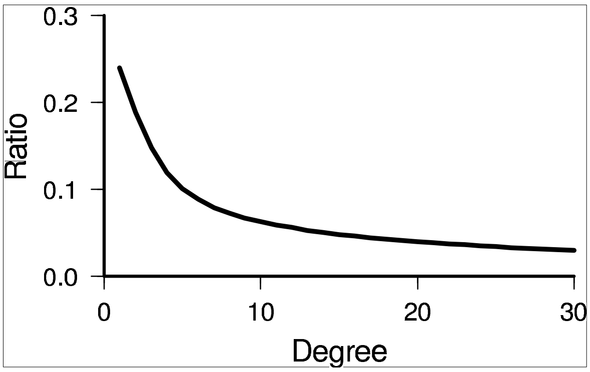

Figure 1. Ratio of passive sea level response to total load (bracketed term in equation (5)) as a function of spherical harmonic degree.

$$\tilde{S}_{00}=\frac{\Delta V}{g}=-\frac{1}{C_{00}}\left(D_{00}+\sum_{n=1}^{\bar{n}}\sum_{m=0}^{n}\sum_{\Phi=C}^{S}\frac{C_{nm}^{\Phi}\tilde{S}_{nm}^{\Phi}}{\Pi_{nm}^{2}}\right) \qquad (8)$$

from which we can establish the full spherical harmonic expansion of $\tilde{S}$, starting from degree zero, and hence apply equation (6) to compute S. Equations (5)-(8) are solved iteratively, starting with the approximation $\tilde{S}=\tilde{S}_{00}=-D_{00}/C_{00}$.

3. Illustration using Mass Load Models

To compute the dynamic load, we take estimates of continental water storage from the LaD climatologically-driven model [*Milly and Shmakin*, 2002], ocean bottom pressure from the ECCO general circulation model [http://www.ecco-group.org], and atmospheric surface pressure data from the NCEP reanalysis [*Kalnay et al.*, 1996]. For the purpose of this paper, each of these datasets may be regarded as being initially expressed as spherical harmonic expansions to high degree and order at weekly intervals. We neglect annual tidal variations in sea level, which are small.

The treatment of the atmospheric load over ocean regions merits special consideration. At time scales greater than a few days, the ocean is expected to respond to atmospheric pressure very nearly as an inverse barometer (IB), so that there will be no net load due to the spatially-varying component of the atmospheric pressure signal over the open oceans. Because the ECCO bottom pressure model is forced by and includes the atmospheric pressure over the oceans, we must set the NCEP atmospheric pressure to zero over the ocean domain. However, we must take care in our interpretation of the total oceanic load, because it will not correspond directly to changes in quasi-static sea level. In particular, the atmospheric pressure load at a point must be removed before sea level at that point can be interpreted, and the mean atmospheric pressure over the oceans must be removed before mean sea level changes can be identified.

From our datasets, we obtain the dynamic load truncated at degree 24, and compute quasi-spectral sea level to degree 12 and hence the passive ocean response up to degree 24, using equations (5)-(8). The results at the annual, dominant period are shown in Figure 2. Clearly, the largest effect is that of ocean-continent mass exchange, imposing an annual variation in global mean sea level (MSL) in the total equilibrated load that is not present in the forced load alone. The mean oceanic total load varies annually with amplitude 10.7 mm, peaking on 18 August,

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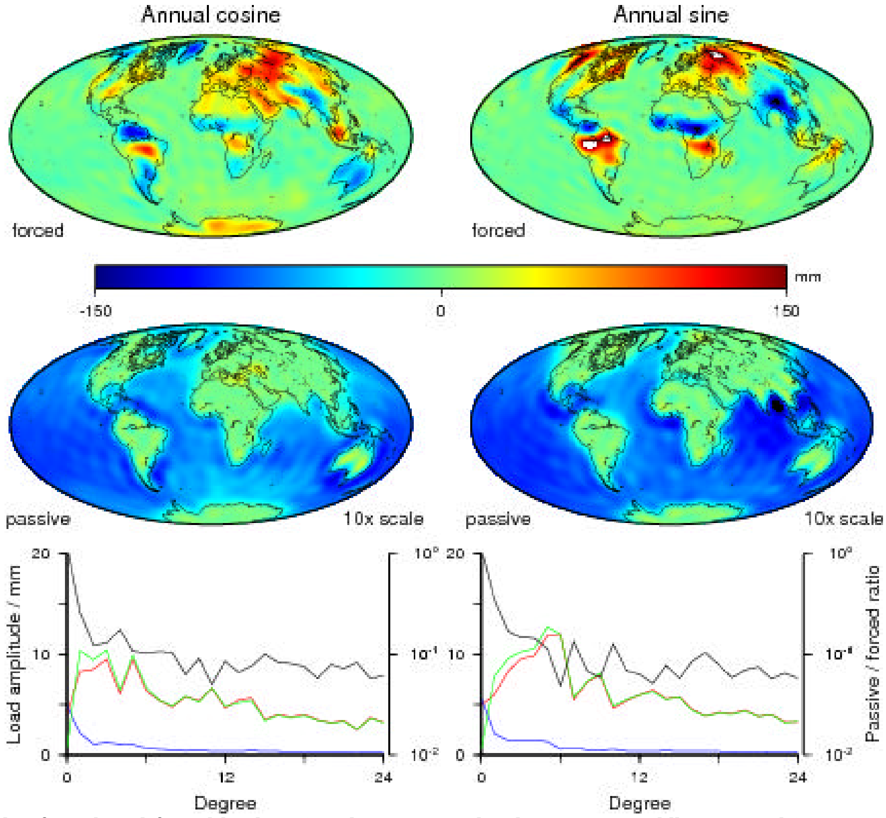

Figure 2. Amplitude of combined forced load (top) and passive sea level response (middle, 10× scale), at annual periods (sine and cosine components). (bottom) Normalized degree amplitude due to forced load (red), passive sea level response (blue) and total load (green) at annual periods (left-hand linear scale), and normalized degree amplitude ratio of passive response to forced load (black, right-hand logarithmic scale).

in phase with but slightly larger than the 8.0 mm inferred using GPS [*Blewitt and Clarke*, 2003]. Correction for the annual variation in mean atmospheric pressure over the oceans (equivalent to a variation in MSL of 5.9 mm amplitude, peaking on 17 July) yields a global MSL annual variation of 6.2 mm, peaking on 11 September (Figure 3, top). This compares well with the TOPEX-Poseidon [e.g. *Chen et al.*, 2002] and GRACE [e.g. *Chambers et al.*, 2004] estimates of annual variation in the range 7-10 mm, peaking mid-September to mid-October. The mass misclosure (degree-zero term) of the unequilibrated model (Figure 3, bottom) varies with an RMS of 3×10^{15} kg, which is reduced to 0.4×10^{15} kg in the equilibrated model. This small residual misclosure is most likely due to our relatively low-degree truncation of surface mass models and the coastline.

In general (Figure 2, bottom) the passive ocean response has magnitude around 10% of that of the forced loading at low degrees. For the annual cosine components, corresponding to seasonal inter-hemispheric changes associated with the largest ocean-continent mass exchange, it is slightly greater than this; whereas for the annual sine components, corresponding to tropical monsoonal changes involving relatively little alteration in ocean mass, it is somewhat less. The forced load dominates in most continental locations, whereas the forced load and passive response are comparable in magnitude over much of the oceans. The load-induced displacement follows a similar pattern to that in Figure 2, although it is progressively attenuated at shorter scales (higher degrees) as the Love numbers decrease. The passive ocean response may be significant for geodetic positioning because it is largely systematic in the way that it affects oceanic sites and, to a lesser extent, near-coastal sites.

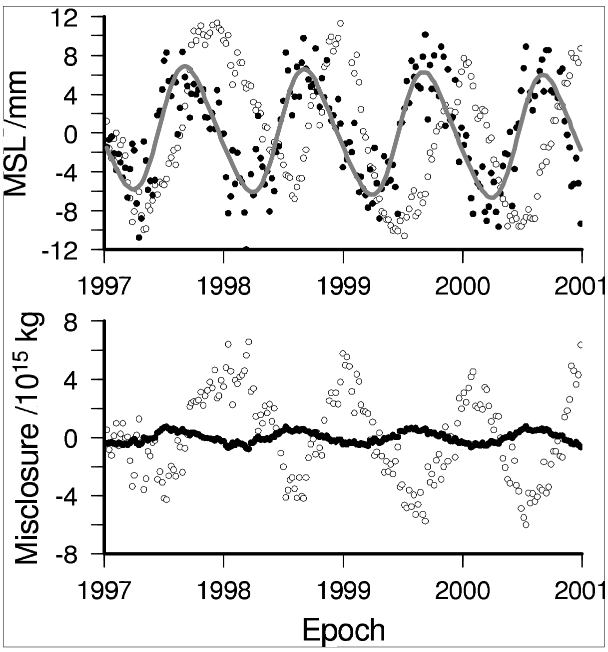

Figure 3. (top) variation in MSL in the unequilibrated model (open symbols) and after forcing of global budget closure (solid symbols, with best seasonal fit). (bottom) misclosure of unequilibrated (open symbols) and total (solid symbols) models.

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An important conclusion is that the predicted geocenter variation (the offset of the center of figure of the solid Earth's surface CF from that of the whole Earth system CM [*Blewitt,* 2003]) will change by up to 43% in amplitude, when the passive ocean response is included. Table 1 shows that the effect of the passive ocean is to amplify the predicted geocenter Y-component variation by 13% and that of the Z-component by 28%. For the X-component, the amplification is greater still, 43%. The amplification arises firstly from the uneven distribution of land on the Earth's surface, which causes the ocean-continent mass transfer resulting from mass conservation to have a degree-1 component, and secondly from the positive feedback due to the gravitational attraction of the original load and this addition. Phase shifts of all parameters are small (<3°). These amplitude changes are comparable in magnitude with the disagreements between recent geodetic observations, or between geocenter variations computed from different surface mass load models. Similarly, the predicted annual variation in Earth's oblateness (proportional to load coefficient T_{20}^{C}) is amplified by 16%. The degree 2, order 1 terms T_{21}^{C} and T_{21}^{S} are amplified by 5% and 7% respectively. Again, phase shifts are small.

Table 1. Effect of load consistency on predicted annual variations in geocenter CF-CM (mm) and degree-2 load coefficients (mm of sea water). The signal *f(t)* is related to amplitude *A* and phase *f* by $f(t) = A\cos(wt - f)$, for angular frequency *w*.

Parameter	Forced load model		Equilibrated model	
	A	*f*	*A*	*f*
r_X	1.34	210°	1.91	209°
r_Y	2.00	145°	2.26	148°
r_Z	3.06	179°	3.70	182°
T_{20}^{C}	3.96	233°	4.60	233°
T_{21}^{C}	0.42	160°	0.44	166°
T_{21}^{S}	1.28	175°	1.37	178°

5. Conclusions

We have shown that the forcing of gravitational consistency and mass conservation on surface mass load models can be used to close the seasonal mass budget of the Earth's fluid envelope. The effects of load consistency will be significant when comparing predictions based on surface mass transfer models with observations of the geocenter displacement (degree 1), Earth rotation and polar motion (degree 2), and time-variable gravity observations from satellite missions such as GRACE (all degrees). The amplification with respect to the forced load is up to 43% at degree 1 and of the order of 5-15% at degrees 2 and higher, so this phenomenon should not be ignored.

The effects of this consistency theory on site displacements will also be appreciable in the ocean basins and coastal areas, compared with the original load. With longer observational GPS datasets and more detailed application of the loading models we expect that these differences will also be detectable in coordinate time series.

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