Degree-2 Harmonics of the Earth's Mass Load Estimated from GPS and Earth Rotation Data

Richard S. Gross,[1] Geoffrey Blewitt,[2,3] Peter J. Clarke,[3,4] and David Lavallée[2]

Abstract. A fluid, mobile atmosphere and oceans surrounds the solid Earth and upon its land surface lays a continually changing distribution of ice, snow, and ground water. The changing distribution of mass associated with the motion of these surficial fluids changes the Earth's rotation by changing its inertia tensor and changes the Earth's shape by changing the load on the solid Earth. It has recently been demonstrated that large-scale changes of the Earth's shape, and hence of the mass load causing the Earth's shape to change, can be measured using the global network of GPS receivers. Here, the degree-2 mass load coefficients determined from GPS data are compared with those obtained from Earth orientation observations from which the effects of tides, winds, and currents have been removed. Good agreement is found between these two estimates of the degree-2 mass load, particularly at seasonal frequencies.

Introduction

The rearrangement of mass within the surficial fluid layers of the Earth, including the atmosphere, oceans, and water stored on land, causes the Earth's gravitational field to change, causes the Earth's rotation to change by changing the Earth's inertia tensor, and causes the Earth's shape to change by changing the load acting on the solid, but not rigid, Earth. Large-scale changes in the Earth's gravitational field have been measured for more than two decades by satellite tracking and more recently by the CHAMP and GRACE satellite missions. Changes in the Earth's rotation have also been measured for more than two decades by the space-geodetic techniques of satellite and lunar laser ranging, very long baseline interferometry, and the global positioning system (GPS). Recently, it has been demonstrated that GPS can also be used to measure large-scale changes in the Earth's shape by precisely positioning the sites of a global network of ground-based GPS receivers [*Blewitt et al.*, 2001; *Lavallée and Blewitt*, 2002; Wu *et al.*, 2002, 2003; *Blewitt and Clarke*, 2003].

Determining the mass load acting on the surface of the solid Earth that is causing the Earth's gravitational field, rotation, and shape to change is important for a number of reasons. For example, the load on the solid Earth in polar regions will change as the mass of glaciers and ice sheets change. Measurements of changes in the mass load in polar regions can therefore be used to study changes in glacier and ice sheet mass. Here, changes in the

[1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena.
[2]Nevada Bureau of Mines and Geology and Seismological Laboratory, University of Nevada, Reno.
[3]School of Civil Engineering and Geosciences, University of Newcastle, Newcastle-upon-Tyne, UK.
[4]On leave at Colorado Center for Astrodynamics Research, University of Colorado, Boulder.

Gross, R. S., G. Blewitt, P. J. Clarke, and D. Lavallee (2004), Degree-2 harmonics of the Earth's mass load estimated from GPS and Earth rotation data, Geophys. Res. Lett., 31, L07601, doi:10.1029/2004GL019589.

degree-2 spherical harmonic coefficients of the global mass load during 1997–2000 are determined from both GPS measurements of changes in the large-scale shape of the Earth and from Earth rotation observations. These two estimates of the degree-2 mass load are intercompared and shown to be in good agreement with each other, especially at seasonal frequencies.

Degree-2 Mass Loads from GPS

The degree-2 spherical harmonic coefficients of the surface density field have been obtained from GPS data by the method given in *Blewitt and Clarke* [2003]. Data from the global network of GPS receivers is used to determine changes in the low-degree and order spherical harmonic coefficients of the shape of the Earth's surface, and hence of the surface density (mass load) that is acting to cause the changes in shape.

Figure 1 shows the degree-2, order-0 and degree-2, order-1 surface density coefficients, given as weekly averages spanning 1997–2000, that have been obtained by this procedure. Possible aliasing effects on the degree-2 coefficients caused by a relatively sparse station network and the resultant need to truncate the spherical harmonic expansion at relatively low-degree [*Wu et al.*, 2002] have been examined here by recovering the degree-2 coefficients using two different truncation levels, namely, degree-4 (shown in red in Figure 1) and degree-5 (shown in black). As can be seen, the degree-2 coefficients obtained using these two different truncation levels are highly consistent with each other, having correlation coefficients in excess of 0.8. Thus, the degree-2 coefficients determined from GPS data used here appear to be reasonably robust to changes in truncation level. In addition, the formal errors of the degree-2 coefficients (not shown) are smaller than those of the other degrees, indicating that it is the degree-2 coefficients that are determined the best from GPS data.

Figure 1

Degree-2 Mass Loads from Earth Rotation

Theory

The equations governing changes in the solid Earth's rotation and orientation, derived from conservation of angular momentum considerations, are [e.g., *Munk and MacDonald*, 1960; *Wahr*, 1982]:

$$\begin{aligned} \boldsymbol{\chi}(t) &= \mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{d\mathbf{p}(t)}{dt} \\ &= \frac{1.61}{\Omega\,(C-A)} \left\{ \Delta\mathbf{h}(t) + \frac{\Omega\left[\Delta I_{13}(t) + i\,\Delta I_{23}(t)\right]}{1.44} \right\} \end{aligned} \qquad (1)$$

$$\Delta\Lambda(t) = \frac{\Lambda_o}{C_m\,\Omega}\left[\Delta h_3(t) + 0.756\,\Omega\,\Delta I_{33}(t)\right] \qquad (2)$$

where $\mathbf{p}(t) \equiv p_1(t) - i\,p_2(t)$ are the coordinates of the rotation pole with $p_2(t)$ being positive towards 90°W longitude, σ_{cw} is the complex-valued frequency of the Chandler wobble, Ω is the mean angular velocity of the Earth, C_m is the polar moment of inertia of the Earth's crust and mantle, C–A is the difference between the polar and equatorial moments of inertia of the entire

Earth, Λ_o is the nominal length-of-day of 86400 seconds, the factors of 1.44 and 0.756 account for the yielding of the solid Earth to imposed surface loads, and the factor of 1.61 includes the effect of core decoupling. The polar motion excitation functions $\boldsymbol{\chi}(t) \equiv \chi_1(t) + i\,\chi_2(t)$ and changes in the length-of-day $\Delta\Lambda(t)$ are seen to be functions of: (1) changes in the angular momentum $\Delta\mathbf{h}(t) \equiv \Delta h_1(t) + i\,\Delta h_2(t)$ and $\Delta h_3(t)$ due to motion relative to the terrestrial reference frame such as that due to winds and currents, and (2) changes in the angular momentum $\Omega\,[\Delta I_{13}(t) + i\,\Delta I_{23}(t)]$ and $\Omega\,\Delta I_{33}(t)$ due to changes in mass distribution which change the indicated elements of the inertia tensor.

A change $\Delta\rho(t)$ in the density at some location $\mathbf{r}$ within or on the Earth causes a change $\Delta\mathbf{I}(t)$ in the Earth's inertia tensor of [e.g., *Munk and MacDonald*, 1960]:

$$\Delta\mathbf{I}(t) \;=\; \int_{V_o} \Delta\rho(\mathbf{r},t)\,(r^2\,\mathbf{I} - \mathbf{rr})\,dV \tag{3}$$

where $\mathbf{I}$ is the identity tensor. If the change in the density is confined to the surface of the solid Earth, then it can be written as a change $\Delta\sigma(\phi,\lambda)$ in surface density:

$$\Delta\rho(\mathbf{r},t) \;=\; \Delta\sigma(\phi,\lambda,t)\,\delta(r-a) \tag{4}$$

where a is the radius of the Earth, ϕ is N. latitude, and λ is E. longitude. The elements of the inertia tensor required in Equations 1 and 2 can therefore be written in terms of the surface density as:

$$\Delta I_{13}(t) + i\,\Delta I_{23}(t) \;=\; -a^4 \int \Delta\sigma(\phi,\lambda,t)\,\sin\phi\,\cos\phi\,\mathrm{e}^{i\lambda}\,d\Omega \tag{5}$$

$$\Delta I_{33}(t) \;=\; a^4 \int \Delta\sigma(\phi,\lambda,t)\,\cos^2\phi\,d\Omega \tag{6}$$

where $d\Omega \equiv \cos\phi\,d\phi\,d\lambda$ is the element of surface area of a unit sphere.

By expanding the surface density in terms of the unnormalized spherical harmonics $Y_{lm}(\phi,\lambda)$:

$$\Delta\sigma(\phi,\lambda,t) \;=\; \sum_{l=0}^{\infty} \sum_{m=0}^{l} \Delta\sigma_{lm}(t)\,Y_{lm}(\phi,\lambda) \tag{7}$$

where the normalization is such that [e.g., *Blewitt and Clarke*, 2003, Appendix A]:

$$\int Y_{lm}(\Omega)\,Y^*_{lm}(\Omega)\,d\Omega \;=\; \frac{4\pi}{(2-\delta_{m0})(2l+1)(l-m)!/(l+m)!} \tag{8}$$

and noting that $Y_{00}(\phi,\lambda) = 1$, $Y_{20}(\phi,\lambda) = (3\sin^2\phi - 1)/2$, and $Y_{21}(\phi,\lambda) = 3\sin\phi\cos\phi\,\mathrm{e}^{i\lambda}$, equations 5 and 6 can be written as:

$$\Delta I_{13}(t) + i\,\Delta I_{23}(t) \;=\; -\frac{4\pi}{5}\,a^4\,\Delta\sigma_{21}(t) \tag{9}$$

$$\Delta I_{33}(t) \;=\; \frac{2}{3}\,4\pi\,a^4\,\Delta\sigma_{00}(t) - \frac{2}{3}\,\frac{4\pi}{5}\,a^4\,\Delta\sigma_{20}(t) \tag{10}$$

Equations 1 and 9 show that changes in the complex-valued degree-2, order-1 harmonic of the surface density cause the Earth to wobble as it rotates, and Equations 2 and 10 show that changes in the degree-0, order-0 and degree-2, order-0 harmonics of the surface density cause the length-of-day to change. In general, for individual components of the Earth system such as the atmosphere, $\Delta\sigma_{00}(t)$ in Equation 10 is not zero since the total mass of that component will change as, for example, water in its various phases cycles through it. However, for global mass loads in which the total mass is conserved, such as those considered here, $\Delta\sigma_{00}(t)$ is zero.

Equations 2 and 10 and, separately, 1 and 9 are now combined to form the final expressions for the unnormalized degree-2 surface density (mass load) coefficients in terms of length-of-day and polar motion excitation variations:

$$\Delta\sigma_{20}(t) = -\frac{1}{0.756}\,\frac{15}{8\pi}\,\frac{C_m}{a^4}\left[\frac{\Delta\Lambda(t)}{\Lambda_o} - \frac{\Delta h_3(t)}{\Omega\, C_m}\right] \tag{11}$$

$$\Delta\sigma_{21}(t) = -\frac{1.44}{1.61}\,\frac{5}{4\pi}\,\frac{C-A}{a^4}\left[\chi(t) - 1.61\frac{\Delta\mathbf{h}(t)}{\Omega\,(C-A)}\right] \tag{12}$$

where $\Delta\sigma_{00}(t)$ has been assumed to be zero.

Observations

The series used here to obtain the degree-2 spherical harmonic coefficients of the mass load from Earth rotation observations using Equations 11 and 12 is SPACE2002, a combination of space-geodetic measurements of the Earth's rotation spanning 1976–2002 at daily intervals [*Gross*, 2003]. To match the sampling interval of the GPS mass load series, weekly averages of the Earth rotation observations were computed that were then linearly interpolated to the epochs of the GPS mass load series. In order to eliminate signals in the observations having periods greater than the four-year-long duration of the GPS mass load series, a high-pass filter with a cutoff frequency of 1/4 cycles per year (cpy) was applied to the observed length-of-day and polar motion excitation series. The effects of the long-period solid Earth and ocean tides on the observed length-of-day values were removed using the models of *Yoder et al.* [1981] and *Kantha et al.* [1998], respectively, and the effects of the long-period ocean tides on the observed polar motion excitation values were removed using the empirical model of *Gross et al.* [1997]. The effects of the winds were removed from the observations using weekly averages of the 6-hour wind angular momentum series computed from the products of the National Centers for Environmental Prediction / National Center for Atmospheric Research (NCEP/NCAR) reanalysis project [*Kalnay et al*., 1996] which was obtained from the International Earth rotation and Reference systems Service (IERS) Special Bureau for the Atmosphere (SBA). As in *Gross et al.* [2004], the effects on the length-of-day of winds above the top of the NCEP/NCAR model (10 hPa) were removed using the wind angular momentum series computed from the products of the United Kingdom Meteorological Office (UKMO) analysis system whose model extends to a height of 0.3 hPa. The effects of the currents were removed using weekly averages of the hourly current angular momentum series computed here following the procedures given in *Gross et al*. [2003, 2004] from the products of the data assimilating ocean model kf047a run at the Jet Propulsion Laboratory (JPL) as part of their participation in the Estimating the Circulation and Climate of the Ocean (ECCO) consortium.

Comparison of Degree-2 Mass Load Estimates

The panels on the left-hand-side of Figure 2 compare the resulting degree-2 mass load coefficients determined from Earth rotation data from which tidal, wind, and current effects have been removed (black curves) with those determined from GPS data using a truncation level of degree-4 (red curves). As can be seen, there is an excellent agreement between the sine terms (imaginary parts) of the degree-2, order-1 mass load coefficients which exhibit a large annual signal that is primarily due to the appearance of a high atmospheric pressure system over Siberia every winter [e.g., *Munk and MacDonald*, 1960]. The agreement between the cosine terms (real parts) of the degree-2, order-1 mass load coefficients is also significant at the 99% significance level, but there is very little agreement between the degree-2, order-0 coefficients.

Figure 2

The seasonal cycle, the main spectral component, has been recovered and removed from the mass load coefficients by a least-squares fit for a mean, a trend, and periodic terms at the annual, semiannual, and terannual (3 cpy) frequencies. From the panels on the right-hand-side of Figure 2, which compares the mass load coefficients from which the seasonal cycle has been removed, it is seen that the agreement between the coefficients is at about the same level as it was before the seasonal cycle was removed—the agreement between the sine and cosine terms of the degree-2, order-1 coefficients is still significant at the 99% significance level, and there is still very little agreement between the degree-2, order-0 coefficients.

Discussion and Summary

Changes in the degree-2 coefficients of the surface density field that have been inferred from GPS measurements of changes in the Earth's shape during 1997–2000 have been compared here to those determined from Earth rotation measurements from which the effects of tides, winds, and currents have been removed. When doing this comparison it has been implicitly assumed that after removing the motion effects of winds and currents from the Earth rotation measurements, the residual is dominated by the effects of surface mass loads and that internal processes have a negligible effect on the Earth's rotation. This seems to be a reasonable assumption since most of the internal processes that change the Earth's rotation such as glacial isostatic adjustment, mantle convection, motion within the fluid outer core, or pressure acting on the mantle at the core-mantle boundary, occur on much longer time scales than those considered here and their effects have been removed when the Earth rotation measurements were detrended (see Figure 2 caption). And earthquakes, an internal process that occurs on a rapid time scale, have been shown to have a negligibly small effect on the Earth's rotation [*Chao and Gross*, 1987].

It has been shown here that the agreement between the degree-2, order-1 surface density (mass load) coefficients derived from GPS and Earth rotation data is quite good, especially for the sine term, which exhibits a large seasonal cycle. However, the agreement between the degree-2, order-0 coefficients is quite poor. Length-of-day variations, that component of the Earth's rotation from which the degree-2, order-0 coefficient is determined, are known to be predominantly caused by changes in atmospheric winds [e.g., *Gross et al.*, 2004 and references therein]. The dominating effects of the winds must therefore be removed quite accurately from the observations if they are to be used to study the much smaller influence of mass loads. The lack of agreement between the degree-2, order-0 coefficients may indicate that the effects of the winds have not been removed accurately enough from the length-of-day values.

Even though changes in the surface density field caused by, for example, changes in atmospheric surface pressure and ocean-bottom pressure, have a relatively small effect on length-of-day variations, they have a relatively large effect on polar motion excitation [e.g., *Gross et al.*, 2003 and references therein]. Studying these effects currently relies on using the products of general circulation models of the atmosphere and oceans. The ability of GPS, and of the CHAMP and GRACE satellite missions, to measure changes in the surface density field will finally allow these effects to be investigated using measurements rather than models. This could be particularly important for studies of the Chandler wobble which is thought to be excited primarily by changes in surface and bottom pressure [e.g., *Gross*, 2000]. Direct measurements of the excitation process of the Chandler wobble afforded by GPS, CHAMP, and GRACE will enable better estimates of its period and decay time constant, and hence better understanding of the dissipation processes that are acting in the solid Earth at the Chandler frequency.

Acknowledgments. We thank K. Nurutdinov for performing the weekly IGS analysis that provided the GPS data used here. The work of RSG described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. This work was supported in the U.S. by the National Science Foundation, Geophysics Program, grant EAR-0125575 and by NASA's Office of Earth Science, Solid Earth and Natural Hazards Program; support in the U.K. was provided by the Natural Environment Research Council, grant NER/A/S/2001/01166.

References

Blewitt, G., and P. Clarke, Inversion of Earth's changing shape to weigh sea level in static equilibrium with surface mass redistribution, *J. Geophys. Res.*, *108*(B6), 2311, doi:10.1029/2002JB002290, 2003.

Blewitt, G., D. Lavallée, P. Clarke, and K. Nurutdinov, A new global mode of Earth deformation: Seasonal cycle detected, *Science*, *294*, 2342–2345, 2001.

Chao, B. F., and R. S. Gross, Changes in the Earth's rotation and low-degree gravitational field introduced by earthquakes, *Geophys. J. R. astr. Soc.*, *91,* 569–596, 1987.

Gross, R. S., The excitation of the Chandler wobble, *Geophys. Res. Lett.*, *27*, 2329–2332, 2000.

Gross, R. S., Combinations of Earth orientation measurements: SPACE2002, COMB2002, and POLE2002, Jet Propulsion Laboratory Publ. 03-11, 28 pp., Pasadena, Calif., 2003.

Gross, R. S., B. F. Chao, and S. Desai, Effect of long-period ocean tides on the Earth's polar motion, *Prog. Oceanogr.*, *40*, 385–397, 1997.

Gross, R. S., I. Fukumori, and D. Menemenlis, Atmospheric and oceanic excitation of the Earth's wobbles during 1980–2000, *J. Geophys. Res.*, *108*(B8), 2370, doi:10.1029/2002JB002143, 2003.

Gross, R. S., I. Fukumori, D. Menemenlis, and P. Gegout, Atmospheric and oceanic excitation of length-of-day variations during 1980–2000, *J. Geophys. Res.*, *109*, B01406, doi:10.1029/2003JB002432, 2004.

Kalnay, E., M., et al., The NCEP/NCAR 40-year reanalysis project. *Bull. Amer. Met. Soc.* *77*, 437–471 1996.

Kantha, L. H., J. S. Stewart, and S. D. Desai, Long-period lunar fortnightly and monthly ocean tides, *J. Geophys. Res.*, *103*, 12639–12647, 1998.

Lavallée, D., and G. Blewitt, Degree-1 Earth deformation from very long baseline interferometry measurements, *Geophys. Res. Lett.*, *29*(20), 1967, doi:10.1029/2002GL015883, 2002.

Munk, W. H., and G. J. F. MacDonald, *The Rotation of the Earth: A Geophysical Discussion*, 323 pp., Cambridge University Press, New York, 1960.

Wahr, J. M., The effects of the atmosphere and oceans on the Earth's wobble - I. Theory, *Geophys. J. Roy. astr. Soc.*, *70*, 349–372, 1982.

Wu, X., D. F. Argus, M. B. Heflin, E. R. Ivins, and F. H. Webb, Site distribution and aliasing effects in the inversion for load coefficients and geocenter motion from GPS data, *Geophys. Res. Lett.*, *29*(24), 2210, doi:10.1029/2002GL016324, 2002.

Wu, X., M. B. Heflin, E. R. Ivins, D. F. Argus, and F. H. Webb, Large-scale global surface mass variations inferred from GPS measurements of load-induced deformation, *Geophys. Res. Lett.*, *30*(14), 1742, doi:10.1029/2003GL017546, 2003.

Yoder, C. F., J. G. Williams, and M. E. Parke, Tidal variations of Earth rotation, *J. Geophys. Res.*, *86*, 881–891, 1981.

G. Blewitt and D. Lavallée, University of Nevada, MS 178, Reno, NV 89557 (e-mail: gblewitt@unr.edu)

P. Clarke, School of Civil Engineering and Geosciences, University of Newcastle, Bedson Bldg., Newcastle upon Tyne, NE1 7RU, UK (e-mail: Peter.Clarke@newcastle.ac.uk)

R. Gross, Jet Propulsion Laboratory, MS 238-332, 4800 Oak Grove Drive, Pasadena, CA 91109 (e-mail: Richard.Gross@jpl.nasa.gov)

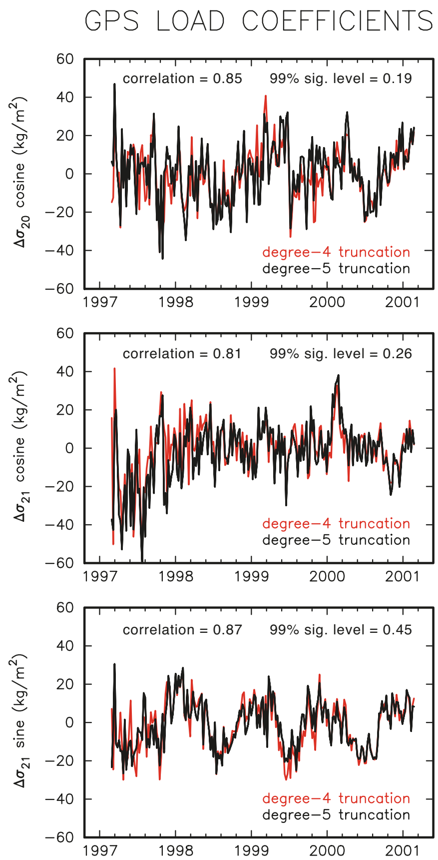

Figure 1. Changes in the degree-2, order-0 (top panel) and degree-2, order-1 (middle and bottom panels) coefficients of the surface density inferred from GPS measurements of changes in the shape of the Earth's surface that have been determined by truncating the spherical harmonic expansion at both degree-4 (red curves) and at degree-5 (black curves). The cosine term is the real part of the complex-valued spherical harmonic coefficient and the sine term is the imaginary part, which is always zero for the zonal (order-0) coefficients.

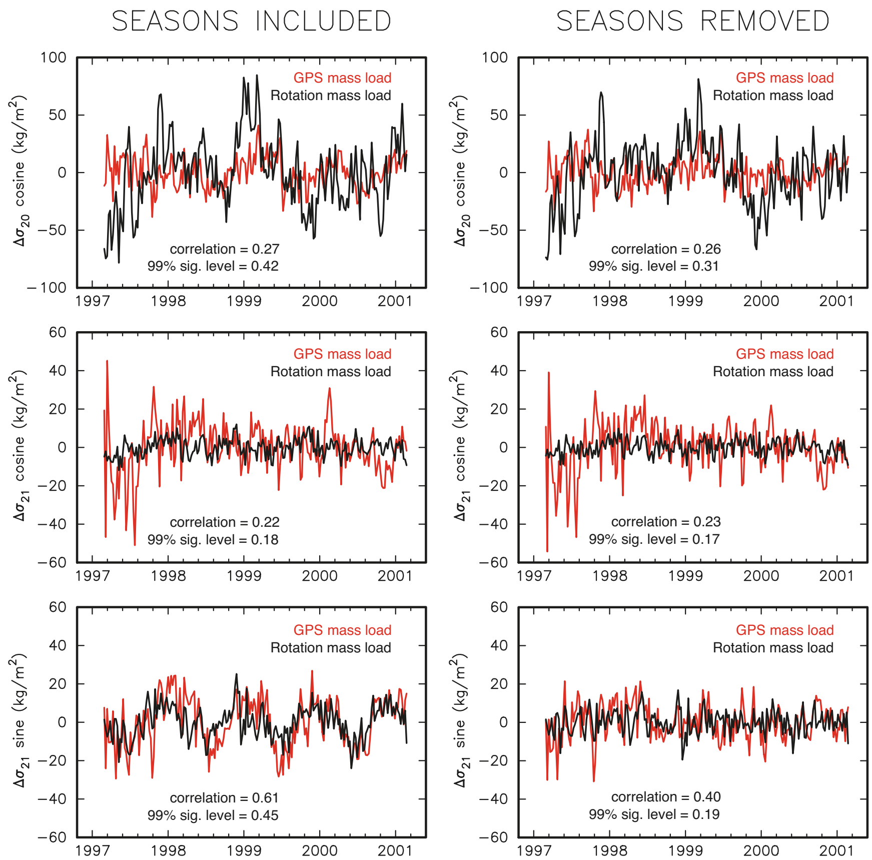

Figure 2. Changes in the degree-2, order-0 (top panels) and degree-2, order-1 (middle and bottom panels) coefficients of the surface density inferred from both GPS measurements of changes in the shape of the Earth's surface (red curves) and from Earth rotation measurements (black curves). All displayed curves have had a mean and trend removed from them. In addition, the right-hand-side panels have also had periodic terms at the annual, semiannual, and terannual (3 cpy) frequencies removed from them.