Choice of Basis Functions for the Representation of Seasonal Surface Loading Signals in Geodetic Time Series

UNIVERSITY OF NEWCASTLE

School of Civil Engineering and Geosciences, University of Newcastle upon Tyne

Peter Clarke[1*], David Lavallée[1], Geoffrey Blewitt[1,2], Tonie van Dam[3]

[1]School of Civil Engineering & Geosciences, University of Newcastle upon Tyne, UK • email *Peter.Clarke@newcastle.ac.uk*

[2]Mackay School of Earth Science & Engineering, University of Nevada, Reno, USA • [3]European Center for Geodynamics & Seismology, Luxembourg

1. Introduction

Forward modelling of geodetic site displacements is frequently performed using gridded surface mass datasets and a Green's function approach, but because these Green's functions are reference frame dependent, it may be difficult to account properly for the effects of geocentre motion (Blewitt, 2003). Spherical harmonic representation allows the transparent use of the correct reference frame dependent Love numbers and so does not suffer from this drawback, but fine-scale (higher-degree) inversion becomes unstable due to the continent-rich, ocean-poor distribution of geodetic displacement data (Wu *et al*., 2002). A further problem, which affects both of these methods but is readily correctable using the spherical harmonic approach, is the appropriate treatment of mass conservation and of the oceanic equilibrium-tide response to the total gravitational field (Blewitt & Clarke, 2003).

The aim of this poster is to show how a modified set of basis functions derived from mass-conserving, tidally-equilibrated, land area-masked spherical harmonics can be used to overcome some of these limitations, and to discuss other possible representations of the data.

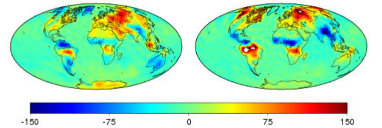

Figure 1. Typical annual cosine (L) and sine (R) terms of surface mass load expressed as the equivalent height of water (in mm), from a combination of the LaD (continental hydrology), NCEP Reanalysis (atmospheric pressure), and ECCO (ocean bottom pressure) models, corrected for overall mass conservation.

2. Forming the basis functions

It is apparent from Figure 1 that the variability in the total continental load is far greater than that over the oceans. If standard spherical harmonics are used as basis functions, much of the information content is "wasted" in maintaining a smooth oceanic load. When inverting GPS displacement data for the load, little of this information content is available, so the solution becomes unstable, even at low degrees, unless *a priori* oceanic constraints are applied (*e.g.* Wu *et al*., 2003). Moreover, the variability in oceanic load is predominantly that due to the equilibrium tidal response to the land load, not that due to other changes in the ocean.

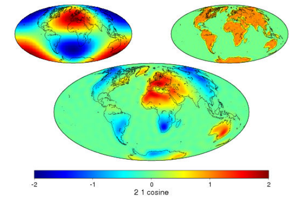

Figure 2. Degree 2, order 1 basis function: (top) initial spherical harmonic and ocean function; (bottom) land-masked and corrected basis function.

We form basis functions $B'_{nm}(\Omega)$ by masking standard spherical harmonics $Y_{nm}(\Omega)$ with the ocean function $C(\Omega)$:

$$B'_{nm}(\Omega) = \{1\text{-}C(\Omega)\} \cdot Y_{nm}(\Omega) \qquad \text{(Eq. 1).}$$

We then correct the raw (land-only) $B'_{nm}(\Omega)$ by adding an oceanic term $S(\Omega)$ to enforce global mass conservation and to allow the ocean to respond to the land load:

$$S(\Omega) = C(\Omega)\{(V(\Omega)+\Delta V)/g - H(\Omega)\} \qquad \text{(Eq. 2)}$$

where $H(\Omega)$ is the vertical displacement of the solid Earth's surface, $V(\Omega)$ is the displacement of the original geoid in response to the total load, and the constant term ΔV accounts for global conservation of mass.

Equations 1 and 2 can be applied to arbitrary degree and order, using Clebsch-Gordan coefficients for multiplication in the spectral domain. Here, we take initial spherical harmonics up to degree 9, truncating the ocean function at degree 36, and our final results at degree 24. Our basis functions are no longer orthonormal (Figure 3), so we must fit them to data by least squares, not global convolution.

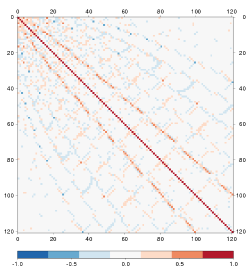

Figure 3. Normalised global convolution integral between corrected land-masked basis functions.

3. Testing goodness of fit

(a) Gridded surface mass data

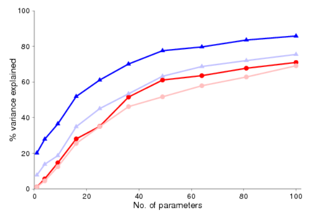

Figure 4. Goodness of fit to annual variation in surface mass of our basis functions (blue) compared with standard spherical harmonics (red). Darker lines are for a gridded dataset based on the LaD hydrological model; paler lines for the CPC model.

We test the efficiency of our basis functions by fitting them to a synthetic dataset formed from ECCO ocean bottom pressure data (http://www.ecco-group.org), continental atmospheric pressure data from the NCEP reanalysis (Kalnay *et al*., 1996), and land hydrology from either the LaD (Milly & Shmakin, 2002) or CPC (Fan & van den Dool, 2004) models, summed and corrected to enforce global conservation of surface mass and self-gravitational consistency (Clarke *et al*., submitted 2004).

Our basis functions consistently model a greater fraction of the variance in the synthetic dataset, compared with a spherical harmonic basis set with the same number of fitted parameters. For LaD-based data, the saving in parameters to achieve the same level of fit is typically a factor of three.

(b) Vertical displacement data with real network geometry

Real GPS data do not sample the planet evenly, and the sampling is biased towards continents (particularly Europe and North America). Next, we tested the ability of our basis functions to represent the synthetic dataset when sampled at the actual locations of GPS sites processed each week by IGS Analysis Centres. Network size and distribution have generally improved over time, and may also be improved by using an IGS combined solution rather than an individual Analysis Centre's network. We took three examples, each at early (mid-1997), middle (early 2001) and recent (mid-2004) epochs: (i) the JPL network, relatively small in number but with even distribution; (ii) the SIO network, with many but clustered sites; and (iii) the Newcastle GNAAC combined solution. In each case we fit basis functions describing vertical surface displacement to synthetic displacements. Again, our basis functions are nearly always better able to fit the data than the same number of standard spherical harmonic functions, although the bias in sampling diminishes the advantage.

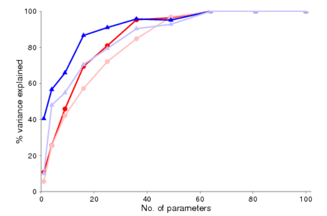

Figure 5. Goodness of fit to annual periodic variation in surface height for our basis functions (blue) compared with standard spherical harmonics (red) at degrees 0–9. Bold lines are for a synthetic dataset including the LaD hydrological model; faint lines for the CPC model. The dataset is sampled at the 60 site locations in the JPL global IGS solution for GPS week 915 (in mid-1997). At degree 7 (64 parameters) and higher, the data are over-fitted.

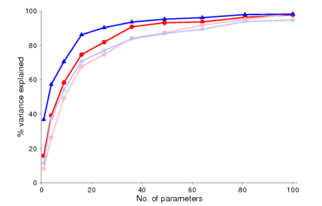

Figure 6. As Figure 5, but sampled at the sites in the 151-station SIO global IGS solution for GPS week 1100 (in early 2001).

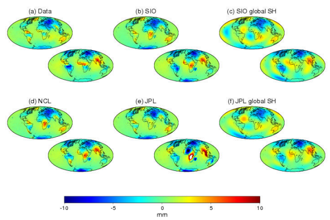

Figure 7. Annual periodic variation in vertical surface displacement for: (a) synthetic dataset using LaD hydrology; (b) fit using 151-site SIO network; (c) standard spherical harmonic fit using SIO network; (d) fit using 109-site NCL GNAAC network; (e) fit using 77-site JPL network; (f) standard spherical harmonic fit using JPL network. All fits are to degree 6; all networks are as at GPS week 1100 (in early 2001). Annual cosine term on upper left of each pair; annual sine term on lower right.

4. Testing global fidelity of fit

As seen in Figure 7, goodness of fit at the sample locations does not necessarily imply global fidelity to the "true" signal. Because we are using a synthetic dataset, we can test the fidelity of our fit, by comparing the actual (synthetic) gridded displacements with those generated from our fitted basis functions. This allows us to compare the network bias that occurs when using our basis functions with the bias that occurs when using standard spherical harmonics. We generally see poor variance reduction at low maximum degrees. This does not necessarily imply that the resulting low-degree coefficients are wrong; rather, we are under-representing the data. Conversely, at high degrees the variance reduction is poor or even negative, because of oscillatory behaviour in unconstrained regions. Our basis functions lead to a far better global fit to the "true" signal, because they incorporate a plausible physical model of oceanic behaviour while allowing the land load to vary in a model-independent manner.

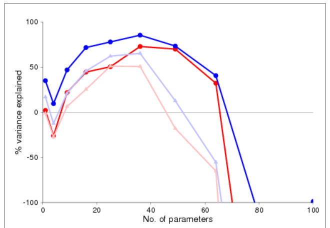

Figure 8. Global variance reduction in annual periodic surface displacement fitted by spherical harmonics (red) and our basis functions (blue) for degrees 0–9 using the 164-site NCL GNAAC network geometry for GPS week 1280 (mid-2004). Dark lines are based on LaD hydrology; faint lines on CPC.

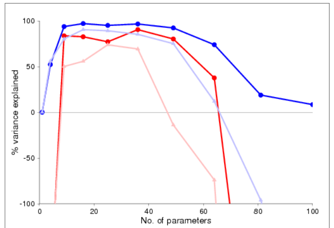

Figure 9. As Figure 8, but calculated after truncating the "true" dataset (and fitted basis functions) to the relevant degree. This leaves the aliasing effect of the network sampling as the sole cause of discrepancy.

5. Discussion and conclusions

We have demonstrated that a physically reasonable set of basis functions, derived from spherical harmonics, can be used to represent the seasonal variation in surface mass and associated displacements of the solid Earth. Our representation achieves better fit to realistic synthetic data than does a spherical harmonic fit with the same degree of freedom, is more robust to the biasing effect of network geometry, and is less prone to oscillation in unconstrained regions. The basis functions directly incorporate the physics of reference frame definition, conservation of mass, and equilibrium ocean response to the land load, whilst parameterising the land load in a model-independent way. A spherical wavelet or spherical cap harmonic representation might achieve a more detailed regional fit where data are available, but would lack most of these advantages.

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