Cascade Control of DC machines

Permanent magnet dc motor

Armature circuit:
\[ v(t) = i(t)R + L \frac{di(t)}{dt} + E(t) \Rightarrow \]
\[ v(t) = i(t)R + L \frac{di(t)}{dt} + K_E \phi \omega(t) \] (1)

Mechanical dynamics:
\[ \sum T = J \frac{d\omega(t)}{dt} \Rightarrow \]
\[ T_e - T_L - T_F = J \frac{d\omega(t)}{dt} \]
\[ T_e = T_L = 0 \] \[ T_F = B \omega \]

\[ T_e - B \omega = J \frac{d\omega(t)}{dt} \Rightarrow \]
\[ K_T \phi i(t) - B \omega(t) = J \frac{d\omega(t)}{dt} \] (2)

Laplace transform of (1) and (2):
\[ V(s) = I(s)R + LsI(s) + K_E \phi \Omega(s) \]
\[ K_T \phi I(s) - B \Omega(s) = Js \Omega(s) \] (3)

Solve (3a) for \( I(s) \):
\[ V(s) - K_E \phi \Omega(s) = (R + Ls)I(s) \Rightarrow \]
\[ I(s) = \frac{V(s) - K_E \phi \Omega(s)}{(R + Ls)} \]

and replace it in (3b):
\[ K_T \phi \frac{V(s) - K_E \phi \Omega(s)}{(R + Ls)} - B \Omega(s) = J_s \omega(s) \Rightarrow \]
\[ K_T \phi \frac{V(s)}{(R + Ls)} - \frac{K_E \phi \Omega(s)}{(R + Ls)} + B \Omega(s) + J_s \omega(s) \Rightarrow \]
\[ K_T \phi \frac{V(s)}{(R + Ls)} = \left( K_T \phi \frac{K_E \phi}{(R + Ls)} + B + J_s \right) \Omega(s) \Rightarrow \]
\[ K_T \phi V(s) = \left( K_T \phi K_E \phi + (R + Ls)(B + J_s) \right) \omega(s) \Rightarrow \]
\[ \frac{\Omega(s)}{V(s)} = \frac{K_T \phi}{(R + Ls)(B + J_s) + K_T \phi K_E \phi} \]

It is possible to simulate that using the following numerical values:

- \( K_T \phi = 3.65 \text{ Nm/A} \)
- \( K_E \phi = 3.65 \text{ Vs/rad} \)
- \( R = 0.052 \Omega \)
- \( L = 1 \text{ mH} \)
- \( J = 5 \text{ Kgm}^2 \)
- \( B = 0.0001 \text{ Nms/rad} \)

Also we can use a PI controller to improve the motor’s performance:
But this system has a peculiar property which can be seen if we slightly change the model:

\[ V(s) - K_E \phi \Omega(s) = I(s)(R + Ls) \]

\[ K_T \phi I(s) = (B + Js)\Omega(s) \]

So let’s see the current:
This is happening because the electrical time constant \((L/R=0.0192)\) is much smaller (i.e. faster) than the mechanical one (i.e. \(J/B=50000\)). But we can use that difference to break the system into 2 smaller and separated subsystems, i.e. the electrical (A) and the mechanical (B) part and hence to use 2 PI controllers:
This type of control is called cascade control and is very popular in electric drives as we first tune the current loop and then we separately tune the speed loop.