Root Locus Example

The given system is:

$$G_{p1}(s)=\frac{1}{s(s+1)(s+2)} \quad G_{p2}(s)=\frac{1}{(s+0.5+0.3j)(s+0.5-0.3j)}$$

And the OLTF: $G_{OL}(s)=\dfrac{1}{s(s+1)(s+2)(s+0.5+0.3j)(s+0.5-0.3j)}$

```
>> gol=tf(1,poly([0 -1 -2 -0.5-0.3*j -0.5+0.3*j]))

Transfer function:
                          1
-----------------------------------------
s^5 + 4 s^4 + 5.34 s^3 + 3.02 s^2 + 0.68 s
```

The OL pole/zero location is

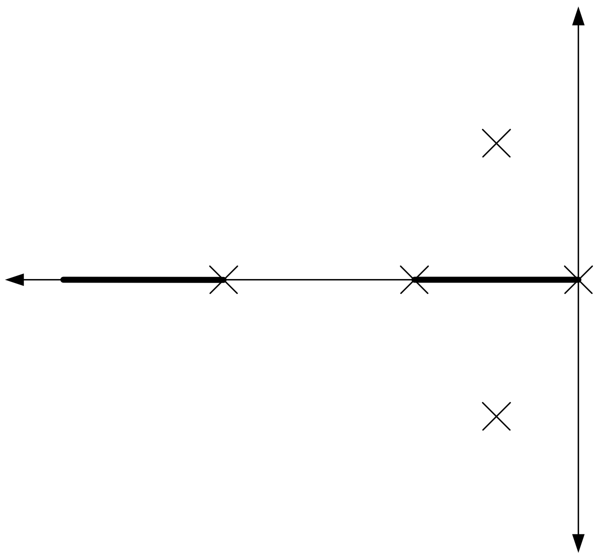

Figure 1

I have 5 asymptotes that intersect at:

$$s=-\frac{\sum_{i=1}^{n_p} p_i - \sum_{i=1}^{n_z} z_i}{n_p - n_z}$$

Now $p_1=0,\ p_2=1,\ p_3=2,\ p_4=0.5+0.3j,\ p_5=0.5-0.3j$

And hence $\sum p_i = 0+1+2+0.5+0.3j+0.5-0.3j=4$ => s=-4/5=-0.8

The angles are

$$\frac{(2k+1)\pi}{5}=\left\{\frac{\pi}{5}=36^o \quad \frac{3\pi}{5}=108^o \quad \frac{5\pi}{5}=180^o \quad \frac{7\pi}{5}=252^o=-108^o \quad \frac{9\pi}{5}=324^o=-36^o\right\}$$

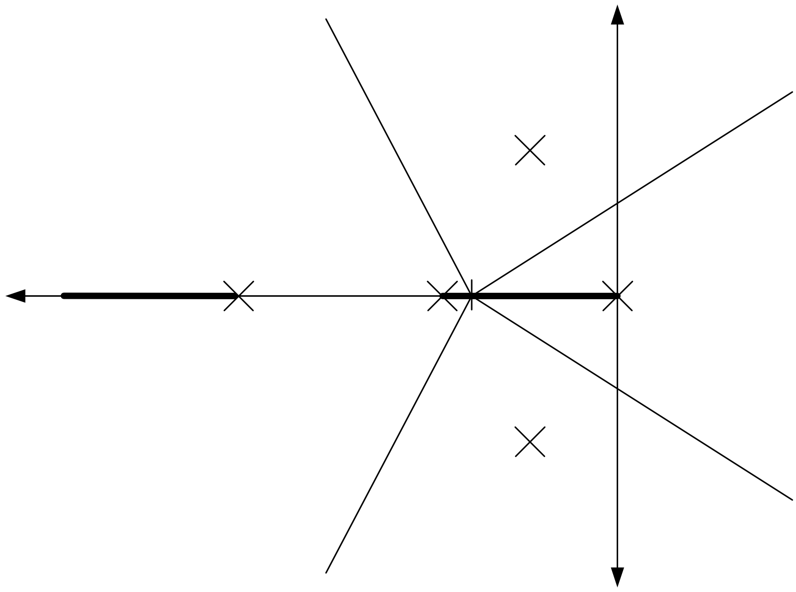

Figure 2

Break in/out points:

$$1 + KG_{OL}(s) = 0 \Leftrightarrow$$

$$1 + K\frac{1}{s(s+1)(s+2)(s+0.5+0.3j)(s+0.5-0.3j)} = 0 \Leftrightarrow$$

$$K\frac{1}{s(s+1)(s+2)(s+0.5+0.3j)(s+0.5-0.3j)} = -1 \Leftrightarrow$$

$$K = -s(s+1)(s+2)(s+0.5+0.3j)(s+0.5-0.3j)$$

We do not have to do this multiplication as it is already given by Matlab:

```
>> [num,den]=tfdata(gol,'v')

num =

     0     0     0     0     0     1


den =

  Columns 1 through 5

  1.00000000000000   4.00000000000000   5.34000000000000
3.02000000000000   0.68000000000000

  Column 6

  0
```

It is easy now to find the derivative but once again we can use Matlab:

```
>> dend=[5*den(1) 4*den(2) 3*den(3) 2*den(4) 1*den(5)];
>> roots(dend)
ans =
  -1.71654571618620
  -0.73740793581773
  -0.55107901321245
  -0.19496733478362
```

We can see that we have 4 roots and only the 1st one is not accepted:

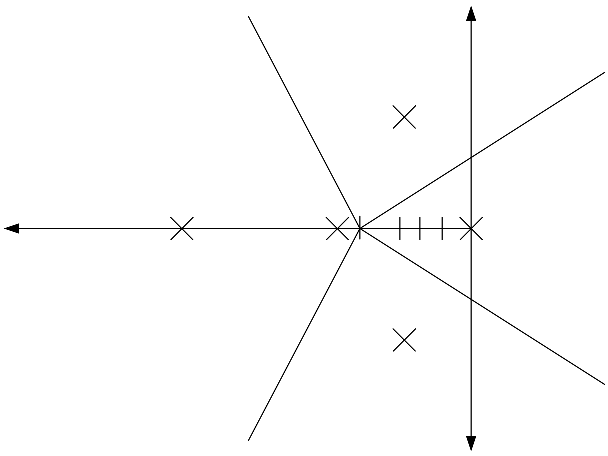

Figure 3

The departure angle from the complex poles is:

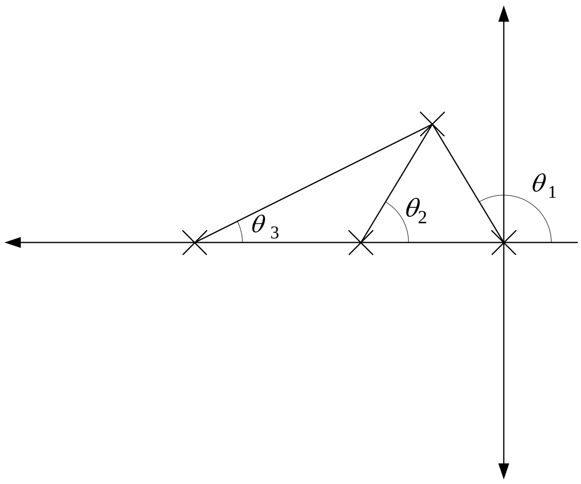

Figure 4

$$\left.\begin{aligned}
&\theta_1 = 180 - \tan^{-1}\left(\frac{0.3}{0.5}\right) = 149 \\
&\theta_2 = \tan^{-1}\left(\frac{0.3}{0.5}\right) = 31 \\
&\theta_3 = \tan^{-1}\left(\frac{0.3}{1.5}\right) = 11 \\
&\text{angle from complex pole} = 90
\end{aligned}\right\} \Rightarrow angle = 180 - 149 - 31 - 11 - 90 = -101 = 259$$

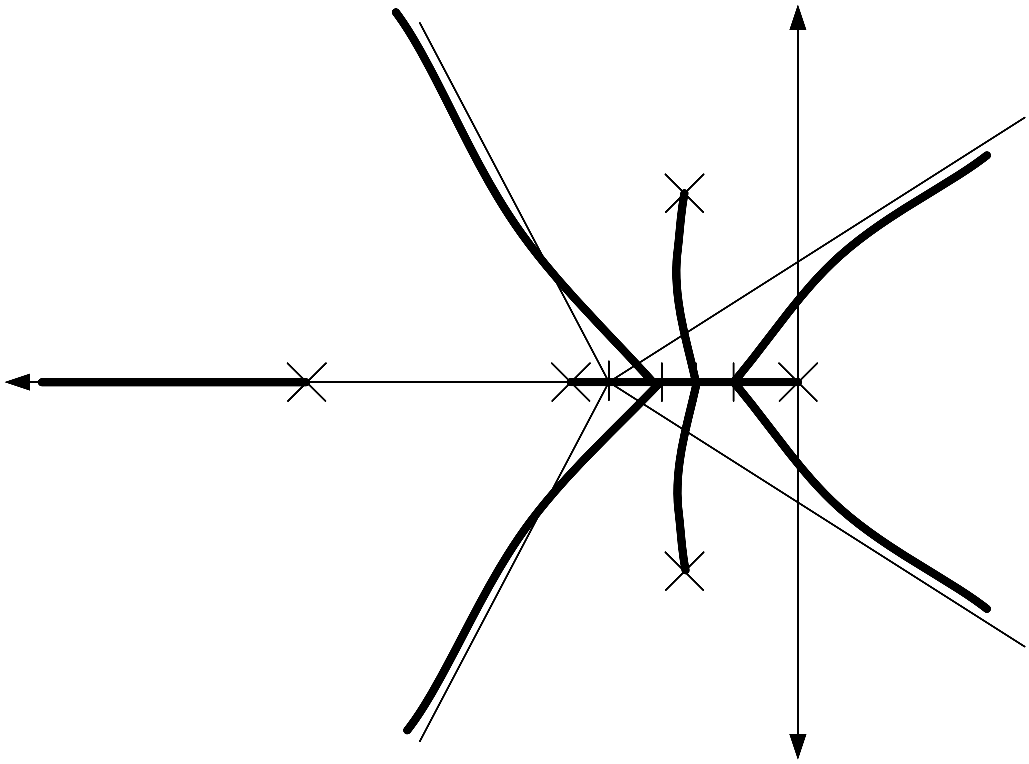

Figure 5

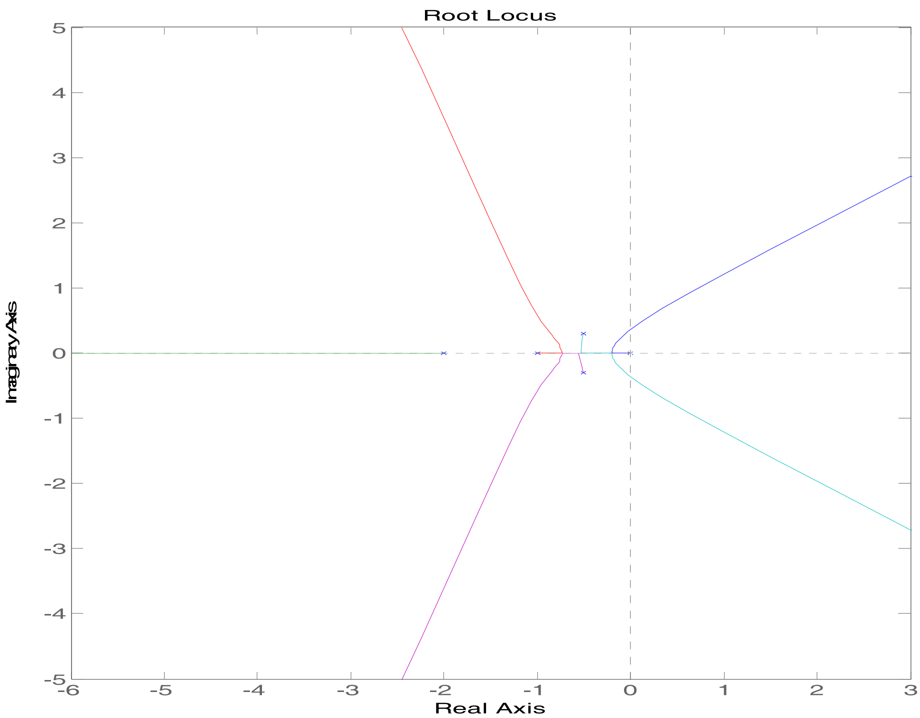

Figure 6