Control Systems – EEE 8013 Tutorial Exercise I

1. *By using the general form of the analytic solution try to predict the response of the following systems:*

A generic first order system can be written as $\frac{dx}{dt} = ax + b$. This system is stable if *a* is negative (in your handouts a=-k).

Also, the constant term will simply shift the overall solution to –*b*/*a*.

This is because at the steady state (if there is one) $\frac{dx}{dt} = 0$ and hence $ax + b = 0 \Rightarrow x = -b/a$.

If the input is a sinusoid then from theory we know that we must settle to another sinusoid of the same frequency with a difference phase shift and amplitude. The phase shift and the amplitude will depend on the frequency of the input signal.

With these in mind I can answer the question:

- $\boxed{5\frac{dx}{dt} + 6x = 0, \quad x(0) = 0, x(0) = 1, x(0) = -1}$

This is a homogeneous system so if it is stable it will converge to zero. By writing this DE in a general form I have that *a*=-5/6 so the system is stable.

Hence if I start at a nonzero value I will exponentially converge to zero, while if I start at zero I will just stay at zero.

- $\boxed{5\frac{dx}{dt} - 6x = 0, \quad x(0) = 0, x(0) = 1, x(0) = -1}$

In this case *a* is positive (*a*=6/5) so the system will diverge to +/-infinity if it starts from a nonzero initial value.

- $\boxed{5\frac{dx}{dt}+6x=1, \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

As we have seen before this is a stable system so I expect the system to converge (exponentially) to a nonzero value as there is a constant nonzero input. Actually I will converge to 1/6. This is true regardless of the initial condition. Of course if I had started on 1/6 I would have remained there forever.

- $\boxed{5\frac{dx}{dt}+6x=-1, \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

Similarly for this system but now I will converge to -1/6.

- $\boxed{\frac{dx}{dt}+3=0, \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

This is a trivial system as a=0 and the output can e found by a simply integration:

$\frac{dx}{dt}=-3 \Leftrightarrow x(t)=\int 3dt=3t+C$. The value of C can be found from the ICs as x(0)=C.

- $\boxed{5\frac{dx}{dt}+6x=\sin(50t), \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

Again this is a stable system and hence I will not diverge. As the input is a sinusoid I will converge to a sinusoid of 50 rad/s with some phase shift and different amplitude.

- $\boxed{5\frac{dx}{dt}+6x=\cos(50t), \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

Exactly the same here but the phase shift will be $+90^{o}$ as $\sin(\phi+90)=\cos(\phi)$

- $\boxed{5\frac{dx}{dt}+6x=5+\sin(50t), \quad x(0)=0,\, x(0)=1,\, x(0)=-1}$

Again this is a stable system but now I have 2 inputs. One will shift the steady solution to 5/6 and the other will add the sinusoid. So the output will be a shifted sinusoid of 50 rad/s.

- $\boxed{5\frac{dx}{dt}+6x=\cos(50t)+\sin(50t), \quad x(0)=0,\ x(0)=1,\ x(0)=-1}$

The same as before but remember that $c_1\cos(bt)+c_2\sin(bt)=G\cos(40t-\phi)$

where $G=\dfrac{c_1}{\cos\left(\tan^{-1}\left({c_2}/{c_1}\right)\right)}$, & $\phi=\tan^{-1}\left({c_2}/{c_1}\right)$

2. *Using numerical solutions crosscheck your previous results.*

See "tutorial_1.mdl".

3. *Simulate (where possible) the analytic solution and hence crosscheck the results of Q1 and Q2.*

In each case the solution is: $x=e^{at}x(0)+e^{at}\int_0^t e^{-at_1}u\,dt_1$, so based on that see "tutorial_1.mdl"

4. *Reproduce the results of all 2nd order systems that are shown in your lecture handouts. Your answer must include prediction and explanation of response, crosscheck with numerical solutions and simulations (where possible) of analytic solutions.*

   See "tutorial_1.mdl"

5. *Repeat Q4 using damping factors and natural frequencies.*

   Exactly the same as Q4, **no** ".mdl" is provided. Simply replace where $A=2\zeta\omega_n, \quad B=\omega_n^2$.