Automatic Control – EEE 2002 Tutorial Exercise IV

A second order system is given by $G(s) = \dfrac{k}{as^2 + bs + c}$.

1. Write the transfer function as: $G(s) = \dfrac{k'}{s^2 + 2\zeta\omega_n s + \omega_n^2}$.

$$G(s) = \frac{k}{as^2 + bs + c} = \frac{k/a}{s^2 + {}^b\!/\!_a\, s + {}^c\!/\!_a}$$, hence

$$k' = k/a, \omega_n = \sqrt{{}^c\!/\!_a}, 2\zeta\omega_n = {}^b\!/\!_a \Leftrightarrow \zeta = \frac{1}{2}\,{}^b\!/\!_a\sqrt{{}^a\!/\!_c}$$

2. For k'=1 and $\zeta = 0.5, \omega_n = 5 rad/s$ and define a transfer function and use the command "damp" to find the damping factor and natural frequency.

```
>> kp=1; z=0.5; wn=5;
>> num=kp; den=[1 2*z*wn wn^2];
>> g=tf(num,den)

Transfer function:
      1
--------------
s^2 + 5 s + 25

>> [z1, wn1]=damp(g)

z1 =

    5.0000
    5.0000


wn1 =

    0.5000
    0.5000
```

3. Based on the previous answer predict the behaviour of the system for a (unit) step input.

Since the damping factor is positive and less than 1 I expect a stable but initially oscillatory behaviour. The frequency of these oscillations is $\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-0.5^2} = 4.33 rad/s$. The overshoot is going to be:

```
>> mp=exp(-(z/sqrt(1-z^2))*pi)

mp =

    0.1630
```

The peak time is:

```
>> wd=wn*sqrt(1-z^2); tp=pi/wd

tp =

    0.7255
```

And finally the settling time:

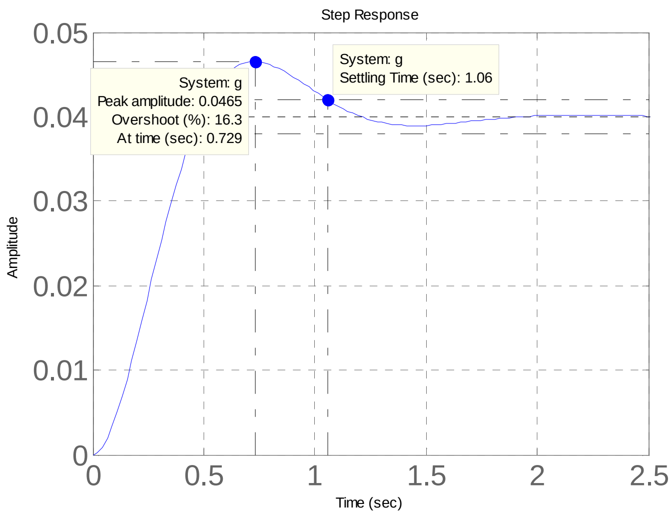
```
>> ts=3/(z*wn)

ts =

    1.2000
```

4. Find the step response of that system using Matlab **AND** Simulink and hence crosscheck your previous answer. Also plot the system's error.

```
>> step(g)
```



Check the properties of the settling time and notice the discrepancy! ????

5. Use the command "*step*" as *[y,t]=step(sys)*, where "*sys*" is the transfer function of the system given in the previous question. Using these two vectors find the overshoot and peak time.

```
clc; clear;
kp=1; z=0.5; wn=5;
num=kp; den=[1 2*z*wn wn^2];
g=tf(num,den);
[y,t]=step(g);
yss=y(end); mp=max(y)
overshoot=100*(mp-yss)/yss
k=find(y==max(y)); tp=t(k)


mp =

    0.0465


overshoot =

   16.1301


tp =

    0.7288
```

6. Using Matlab find the exact value of the steady state error using Matlab **<u>AND</u>** Simulink.

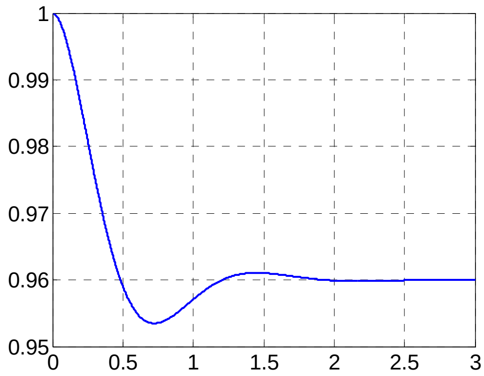
```
>> [y,t]=step(g);
>> yss=y(end); err_ss=1-yss

err_ss =

    0.9599
```

See also Simulink model, once you run the simulation type:

```
>> plot(error_scope(:,1),error_scope(:,2))
```



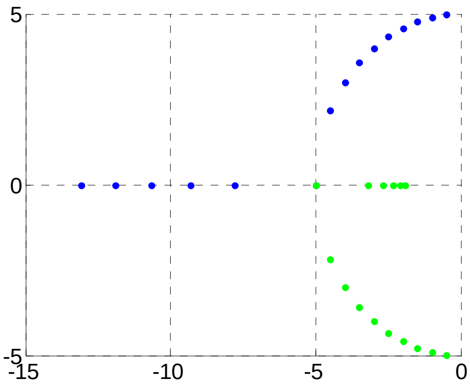

```
>> error_scope(end,2)

ans =

    0.9600
```

7. Assume a system with $k'=1$ and $\omega_n = 5 rad/s$. Use an m-file to calculate and hence plot the pole location in the s-plane for $\zeta \in [0.1, 1.5]$ (use a step size of 0.1).

```
clc; clear;
kp=1; wn=5;
cnt=1;
for z=0.1:0.1:1.5;
den=[1 2*z*wn wn^2];
r(cnt,1:2)=roots(den);
cnt=cnt+1;
end
hold on
plot(real(r(:,1)),imag(r(:,1)),'b.')
plot(real(r(:,2)),imag(r(:,2)),'g.')
```

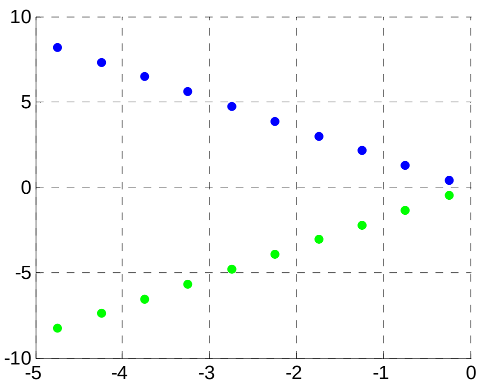


8. Assume a system with $\zeta = 0.5$ and $\omega_n = 5 rad/s$. Use an m-file to calculate and hence plot the pole location in the s-plane for $k \in [0.1, 2]$ (use a step size of 0.5).

As kp is not in the CE no change will be seen in the pole location.

9. Assume a system with k'=1 and $\zeta = 0.5$. Use an m-file to calculate and hence plot the pole location in the s-plane for $\omega_n \in [0.5, 10]$ (use a step size of 1).

```
clc; clear;
kp=1; z=0.5;
cnt=1;
for wn=0.5:1:10;
den=[1 2*z*wn wn^2];
r(cnt,1:2)=roots(den);
cnt=cnt+1;
end
hold on
plot(real(r(:,1)),imag(r(:,1)),'b.')
plot(real(r(:,2)),imag(r(:,2)),'g.')
```

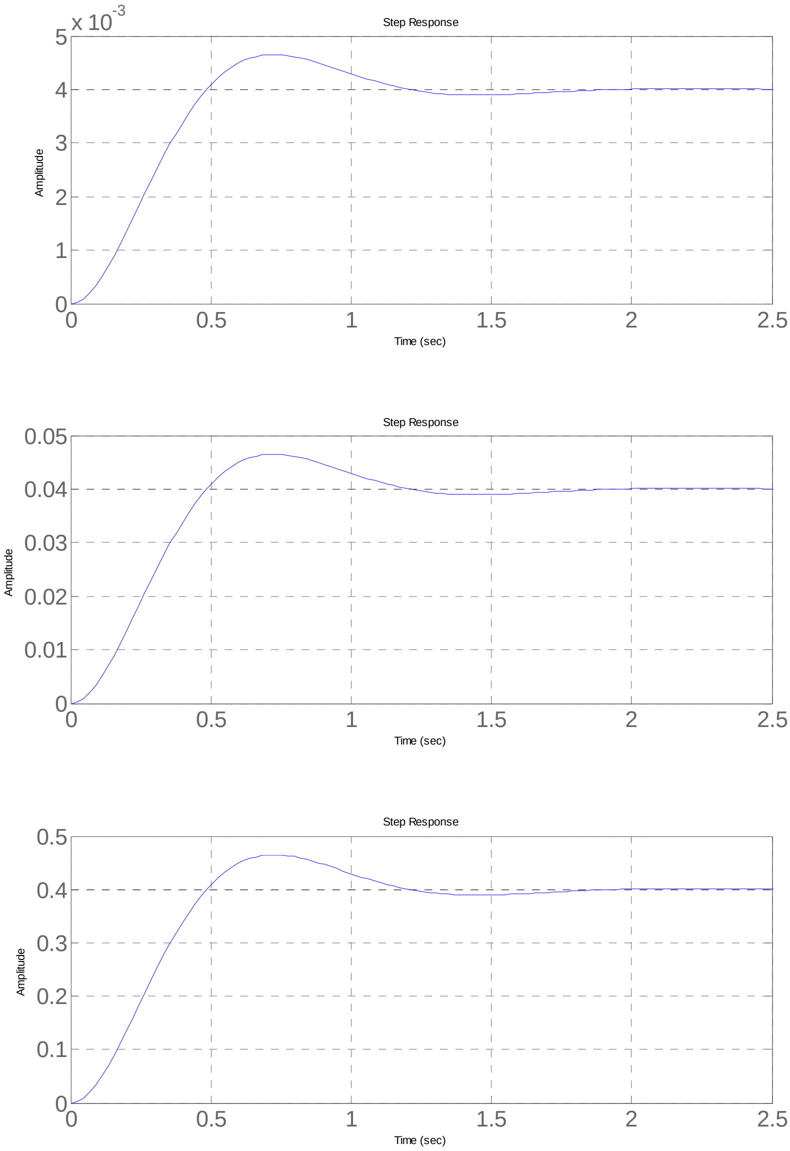


10. Find the unit step response and discuss the results (in connection to your answer given in the previous question) for:

    a. k'=0.1, k'=1 and k'=10 (keep $\zeta = 0.5, \omega_n = 5 rad/s$)clc; clear;

```
num1=0.1; num2=1; num3=10;
z=0.5, wn=5;
den=[1 2*z*wn wn^2];
g1=tf(num1,den); g2=tf(num2,den); g3=tf(num3,den);
subplot(3,1,1); step(g1)
```

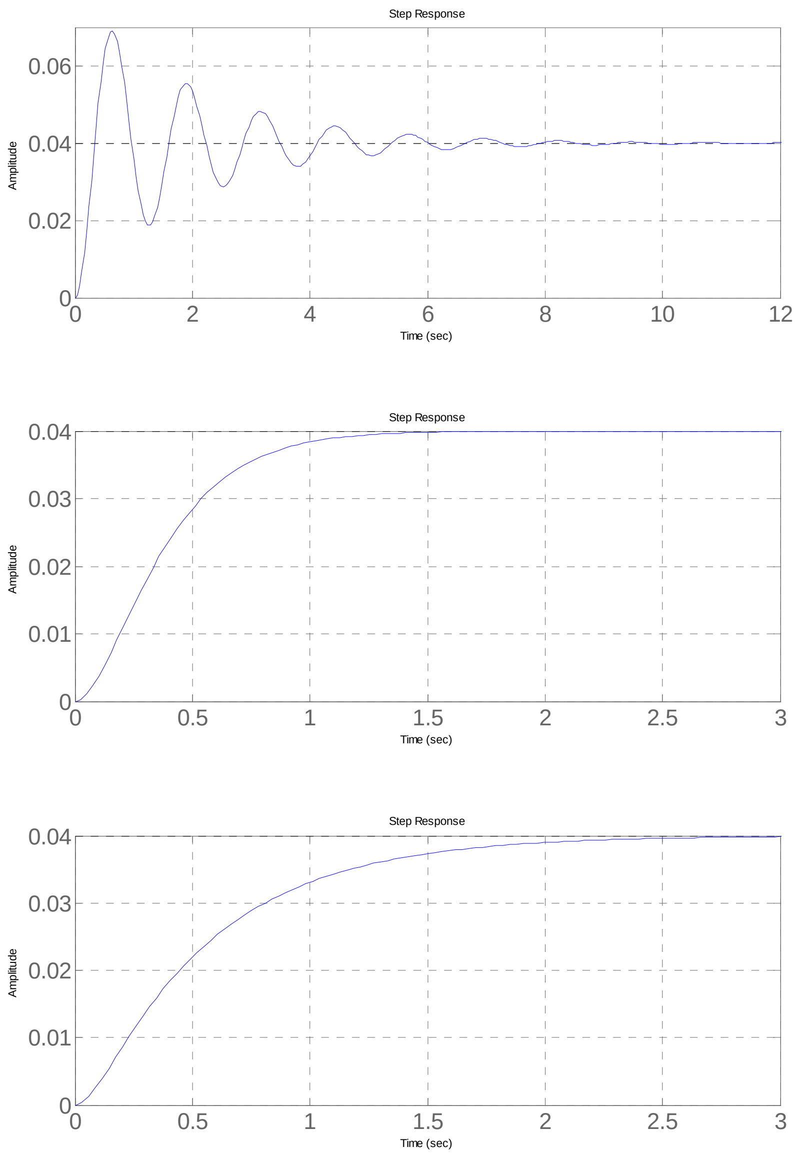
```
subplot(3,1,2); step(g2)
subplot(3,1,3); step(g3)
```



b. $\zeta = 0.1, \zeta = 1, \zeta = 1.5$ (keep k'=1 and $\omega_n = 5 rad/s$)

```
clc; clear;
num=1; wn=5;
z1=0.1, z2=1; z3=1.5;
den1=[1 2*z1*wn wn^2];
den2=[1 2*z2*wn wn^2];
den3=[1 2*z3*wn wn^2];
g1=tf(num,den1); g2=tf(num,den2); g3=tf(num,den3);
subplot(3,1,1); step(g1)
```

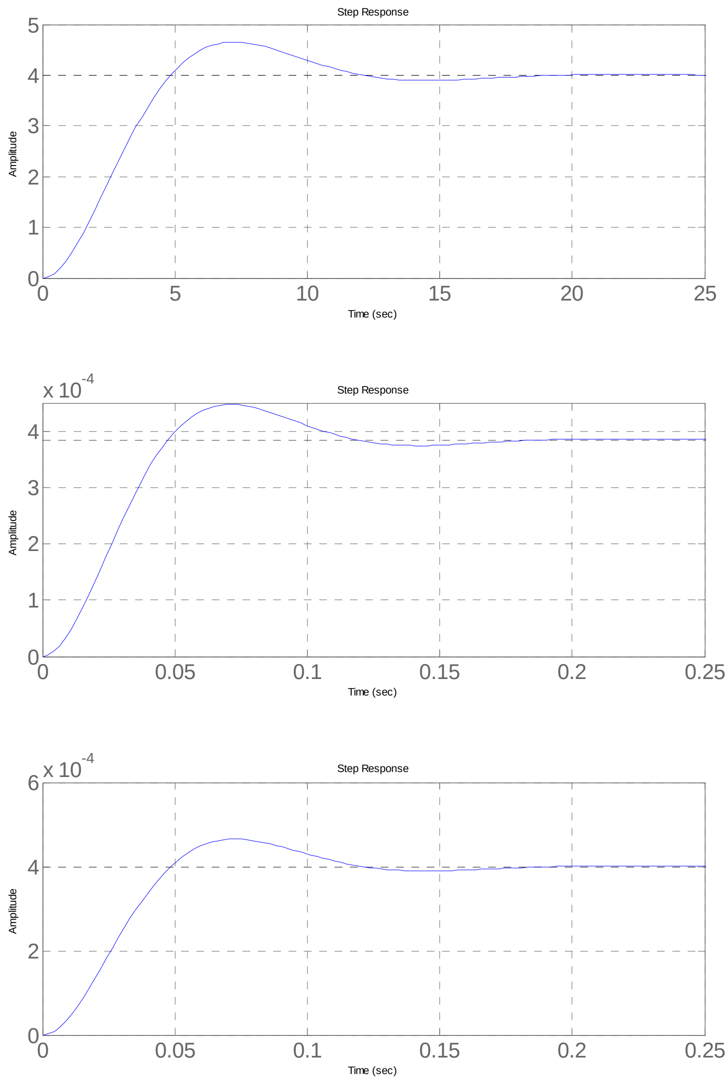
```
subplot(3,1,2); step(g2)
subplot(3,1,3); step(g3)
```



c. $\omega_n = 0.5 rad/s,\ \omega_n = 5 rad/s,\ \omega_n = 50 rad/s$ (keep $\zeta = 0.5, k = 1$)

```
clc; clear;
num=1; z=0.5;
wn1=0.5, wn2=51; wn3=50;
den1=[1 2*z*wn1 wn1^2];
den2=[1 2*z*wn2 wn2^2];
den3=[1 2*z*wn3 wn3^2];
g1=tf(num,den1); g2=tf(num,den2); g3=tf(num,den3);
```

```
subplot(3,1,1); step(g1)
subplot(3,1,2); step(g2)
subplot(3,1,3); step(g3)
```

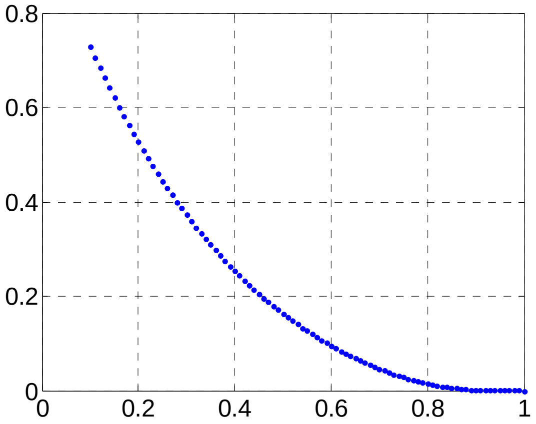


Comments on these responses during the lectures!

11. Using the specific formula plot the overshoot versus the damping factor.

```
clc; clear;
cnt=1;
for z=0.1:0.01:1
mp(cnt)=exp(-(z/sqrt(1-z^2))*pi);
cnt=cnt+1;
end
```

```
plot(0.1:0.01:1,mp,'.')
```

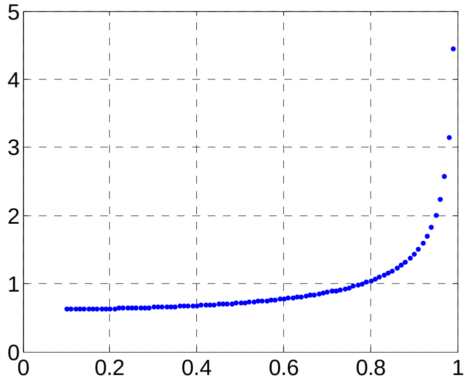


12. Using the specific formula plot the overshoot versus the natural frequency.

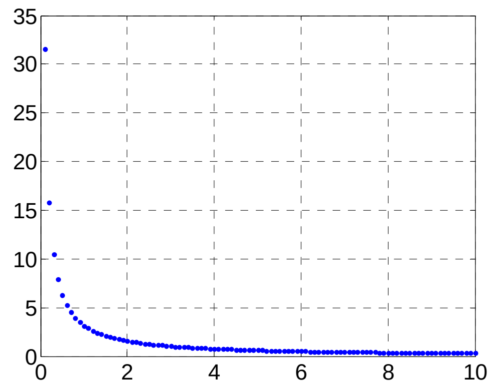
The overshoot is not a function of the natural frequency!

13. Using the specific formula plot the peak time versus the damping factor for wn=5rad/s.

```
clc; clear;
cnt=1; wn=5;
for z=0.1:0.01:1
    wd=wn*sqrt(1-z^2)
tp(cnt)=pi/wd;
cnt=cnt+1;
end
plot(0.1:0.01:1,tp,'.')
```

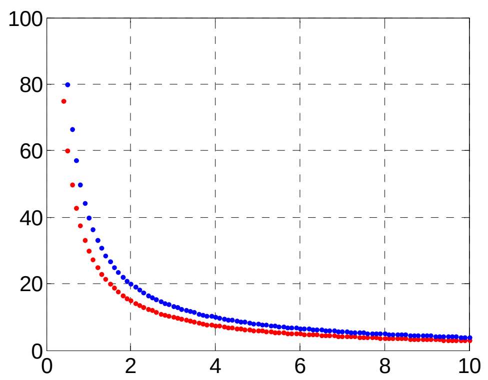


14. Using the specific formula plot the peak time versus the natural frequency for z=0.1rad/s.



15. Using the specific formula plot the settling time (5% and 2% in the same graph) versus the natural frequency for z=0.1rad/s:

```
clc; clear;
cnt=1; z=0.1;
for wn=0.1:0.1:10
    ts5(cnt)=3/z/wn;
    ts2(cnt)=4/z/wn;
cnt=cnt+1;
end
plot(0.1:0.1:10,ts5,'r.'), hold on
plot(0.1:0.1:10,ts2,'b.')
```



16. Using the specific formula plot the settling time (5% and 2% in the same graph) versus the damping factor for wn=5rad/s:

```
clc; clear;
```

```
cnt=1; wn=5;
for z=0.1:0.01:1
    ts5(cnt)=3/z/wn;
    ts2(cnt)=4/z/wn;
cnt=cnt+1;
end
plot(0.1:0.01:1,ts5,'r.'), hold on
plot(0.1:0.01:1,ts2,'b.')
```