Chapter 2 Exercises

1. Find the state space model of:

$$\left.\begin{aligned} x^{(4)} &= 3x^{(3)} + 4x'' - 3x' + x + u_1 - 3u_2 + 5u_3 \\ y_1 &= x^{(3)} + u_1 \\ y_2 &= x^{(4)} + 1.2x' + u_3 - u_1 \\ y_3 &= x \end{aligned}\right\}$$

$$\left.\begin{aligned} \dot{x}_1 &= 3x_1 + 3x_2 + u_1 + u_2 + u_3 + u_4 \\ \dot{x}_2 &+ u_1 - 2u_3 \\ y_1 &= x_1 \\ y_2 &= x_1 + 3x_2 + u_1 + u_2 \\ y_3 &= x_1 - 2x_2 + u_3 + u_4 \end{aligned}\right\}$$

2. A state space model is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -0.1 & -0.2 & -0.3 \\ -0.4 & -0.5 & -0.6 \\ -0.7 & -0.9 & -1 \\ -1.1 & -1.2 & -1.3 \\ -1.4 & -1.5 & -1.6 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 2 & 2 & 3 \\ 0 & 0 & 0 & -1 & 152 \end{bmatrix}, \quad \mathbf{D} = \mathbf{0}$$

   a) What is the order of the system?

   b) How many inputs/ outputs do we have in this system?

   c) What are the dimensions of the matrix **D**?

3. Find the transfer function of a system with $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}1\\ 0\end{bmatrix}, C=\begin{bmatrix}1 & 1\end{bmatrix}, D=0$.
4. Find the transfer function of a system with $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}, C=\begin{bmatrix}1 & 1\end{bmatrix}, D=\begin{bmatrix}0 & 0\end{bmatrix}$.
5. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $\mathbf{B}=\begin{bmatrix}1 & 0 & 1\end{bmatrix}^T$. Is this system controllable? Do not use the controllability/observability matrix
6. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $\mathbf{B}=\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$. Is this system controllable? Do not use the controllability/observability matrix.
7. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $\mathbf{B}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$. Is this system controllable? Do not use the controllability/observability matrix.
8. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $\mathbf{B}=\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 1\end{bmatrix}$. Is this system controllable? Do not use the controllability/observability matrix.
9. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $C=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$. Is this system observable? Do not use the controllability/observability matrix.
10. A system is given by $\mathbf{A}=diag\{5,\sqrt{35},-6\}$, $C=\begin{bmatrix}1 & 0 & \sqrt{3}\end{bmatrix}$. Is this system observable? Do not use the controllability/observability matrix.

11. A system is given by $\mathbf{A} = diag\{5, \sqrt{35}, -6\}$, $C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Is this system observable? Do not use the controllability/observability matrix.

12. A system is given by $\mathbf{A} = diag\{5, \sqrt{35}, -6\}$, $C = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Is this system observable? Do not use the controllability/observability matrix.