Useful formulas

Chapter 1

$$x = e^{-kt}x(0) + e^{-kt}\int_0^t e^{kt_1}u\,dt_1\ ,\quad \frac{1}{\sqrt{1+\omega^2/k^2}}\cos\left(\omega t - \tan^{-1}\left(\omega/k\right)\right)$$

$$x = C_1x_1 + C_2x_2\ ,\quad x = x_P + c_1x_1 + c_2x_2$$

$$x_1 = e^{r_1t},\ x_2 = e^{r_2t}\ ,\quad x_1 = e^{rt},\ x_2 = te^{rt}$$

$$x_1 = \mathrm{Re}\left(e^{rt}\right),\ x_2 = \mathrm{Im}\left(e^{rt}\right),\quad x_1 = e^{at}\cos(bt),\ x_2 = e^{at}\sin(bt)$$

Chapter 2

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{B}U,\ y = \mathbf{CX} + DU\ ,\quad \mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{BU}(s) + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{X}(0)$$

$$\frac{d}{dt}\Phi_{STM}(t,t_0) = A(t)\Phi_{STM}(t,t_0),\quad \Phi_{STM}(t_0,t_0) = I_{n\times n}$$

Chapter 3

$$|\lambda\mathbf{I} - \mathbf{A}| = 0\ ,\quad (\lambda\mathbf{I} - \mathbf{A})\mathbf{e} = 0\ ,\quad \mathbf{x} = A\times\mathbf{e}_1\times e^{-t} + B\times\mathbf{e}_2\times e^{-6t}$$

$$\mathbf{x} = C_1(\mathbf{e}t + \mathbf{b})e^{\lambda t} + C_2\mathbf{e}e^{\lambda t}\ ,\quad \mathbf{e} = (\mathbf{A} - \lambda\mathbf{I})\mathbf{b}\ ,\quad \mathbf{x} = A_1\Re\mathfrak{e}\left(\mathbf{e}e^{\lambda t}\right) + A_2\Im m\left(\mathbf{e}e^{\lambda t}\right)$$

$$\mathbf{x} = e^{\mu t}\left(\mathbf{a}\left(C_1\cos(\nu t) + C_2\sin(\nu t)\right) + \mathbf{b}\left(C_2\cos(\nu t) - C_1\sin(\nu t)\right)\right)$$

$$e^{\mathbf{A}t} = \mathrm{I} + \mathbf{A}t + \frac{1}{2!}(\mathbf{A}t)^2 + \frac{1}{3!}(\mathbf{A}t)^3 + \ldots,\quad \mathbf{x} = e^{\mathbf{A}t}\mathbf{X}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{BU}\,d\tau$$

$$\Phi(t,t_0) = \left[\phi_1(t,t_0)\quad \phi_2(t,t_0)\quad \cdots\quad \phi_n(t,t_0)\right],\quad \phi(t,t_0,x_0) = \Phi(t,t_0)\Phi^{-1}(t_0,t_0)x_0$$

Chapter 4

$$\tilde{\mathbf{A}} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},\ e^{\tilde{\mathbf{A}}t} = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix},$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathrm{Re}(\lambda) & \mathrm{Im}(\lambda) \\ -\mathrm{Im}(\lambda) & \mathrm{Re}(\lambda) \end{bmatrix} e^{\mathbf{A}t} = \mathbf{T}\left(e^{\mathrm{Re}(\lambda)t} \begin{bmatrix} \cos(\mathrm{Im}(\lambda)t) & \sin(\mathrm{Im}(\lambda)t) \\ -\sin(\mathrm{Im}(\lambda)t) & \cos(\mathrm{Im}(\lambda)t) \end{bmatrix} \right)\mathbf{T}^{-1}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} e^{\tilde{\mathbf{A}}t} = \begin{bmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{bmatrix}$$

$$\tilde{\mathbf{A}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},\ \tilde{\mathbf{A}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \mathrm{Re}(\lambda_2) & \mathrm{Im}(\lambda_2) \\ 0 & -\mathrm{Im}(\lambda_2) & \mathrm{Re}(\lambda_2) \end{bmatrix},\ \tilde{\mathbf{A}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 1 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

Chapter 6

$$\mathbf{A}_{CL} = (\mathbf{A} - \mathbf{BK}), \mathbf{B}_{CL} = \mathbf{BK}_1, \mathbf{C}_{CL} = \mathbf{C} - \mathbf{DK}, \mathbf{D}_{CL} = \mathbf{DK}_1$$

$$\mathbf{U} = -\mathbf{KX},\ J = \int_0^\infty \left(\mathbf{X}^T\mathbf{QX} + \mathbf{U}^T\mathbf{RU}\right)dt,\ \mathbf{A}^T\mathbf{P} + \mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0,$$

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}$$

$$\begin{bmatrix} \mathbf{N}_x \\ \mathbf{N}_u \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$u = -\mathbf{Kx} + K_1 r_{ss}$$